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VITAL STATISTICS

AN INTRODUCTION TO THE SCIENCE OF DEMOGRAPHY

BY

GEORGE CHANDLER WHIPPLE

Professor of Sanitary Engineering in Harvard University
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DEDICATED TO

THE STUDENTS OF VITAL STATISTICS
IN THE SCHOOL OF PUBLIC HEALTH
OF HARVARD UNIVERSITY
AND THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

PREFACE

This book is written for students who are preparing themselves to be public health officials and for public health officials who are willing to be students. It makes no claim to be an exhaustive treatise or a compendium of facts; it is merely a guide to the study of vital statistics, an introduction to the great world-wide science of demography — a science yet in the magmatic stage, not yet crystallized. The Great War is bound to develop this science, because hereafter all the nations of the earth must know each other better, and this knowledge, in order to be usable, must be condensed into statistical forms.

Specifically the book tells what statistics are and what they are not; it shows how to express vital facts by figures, how to tabulate them and how to display them by diagrams; it shows how to compute birth-rates and death-rates and how to analyze a death-rate; it shows how to adjust and standardize death-rates and how to make life tables; it emphasizes the need of using vital statistics with truth, with imagination and with power.

For the convenience of school instruction, exercises and questions to incite further study are given in each chapter. Many subjects worthy of special study, however, are not even mentioned, loose ends have been left in every chapter, illustrations have been chosen as they came conveniently to hand, and the general arrangement has been informal as to its subject matter. The object in all this has been to stimulate the reader to critically analyze all vital statistics as they appear before him from day to day. Although the illustrations have been gathered in a haphazard way, an attempt has been made to set forth the elementary principles of the statistical method in a simple and orderly fashion.

The author wishes to confess that he is not an authority on vital statistics, much less an authority on demography; he is merely a student of the science. He has taken the student's privilege of quoting freely from many writers to whom he wishes to render acknowledgments and thanks. In particular he desires to express his obligations and personal regards to Dr. William H. Davis, Chief Statistician for Vital Statistics, United States Bureau of the Census, who has read the entire proof of this book and given the benefit of his careful criticism.

Just a personal word to the health officers of America. A new day is dawning for you. The care of the public health is becoming a distinct profession. The medical profession alone is not able to cope with it. The young men and women who are to be the executive health officers in the next generation are recognizing the need of special training, based on the principles of preventive medicine, hygiene and sanitation. Schools of public health are coming into existence and receiving warm-hearted support. The health administration of the future will be in the hands of full-time officials, who are adequately paid and protected in their tenure of office, but who in return for these advantages must be adequately trained for their work. The ability to use vital statistics in public health work is an important part of this training. Many of you have been in office for a long time, you have forgotten most of your arithmetic — not to mention algebra. You can see the new era coming and you dread the new methods founded on accurate statistical studies of accident, disease and death. There is no need of this fear. You can use statistics as well as any one, but you must study. This book has been prepared with your difficulties in mind.

GEORGE CHANDLER WHIPPLE

CAMBRIDGE, MASS.
January, 1919.

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VITAL STATISTICS

CHAPTER I

DEMOGRAPHY

Broadly speaking demography is the statistical study of human life. It deals primarily with such vital facts as birth, physical growth, marriage, sickness and death and incidentally with political, social, educational, religious, sanitary, hygienic and medical matters. In a somewhat narrower sense demography is used as a synonym for vital statistics.

The word "demography" is derived from the Greek words *demos*, people, and *grapho*, to write. It is in common use in Europe, but is not as well known or its meaning as well understood in America. High authority for its use is found in the name of that most important triennial gathering of physicians and sanitarians, the International Congress of Hygiene and Demography.

Demography cannot be called a science in the sense that it is a classified body of knowledge from which laws have been developed and established. But all sciences in their evolution go through a descriptive stage in which data are collected and hypotheses tested. So regarded demography may be called a science, — the science of human generation, growth, decay and death as studied by statistical methods.

The principal divisions of demography. — Demography may be said to include the following major subjects:

1. Genealogy, which considers individual ancestries and personal records.
2. Human eugenics, which considers heredity from a scientific standpoint, and is to a large extent the application of the statistical method to genealogy.
3. The census, that is, the collection of social, political, religious and educational facts concerning population, usually by the method of governmental enumeration.
4. Registration of vital facts, such as those concerning birth, marriage, divorce, sickness and death, usually under governmental direction and by the use of individual records.
5. Vital statistics, which is the application of the statistical method to the study of these vital facts.
6. Biometrics, which includes anthropometric studies of human growth, stature, strength, etc.
7. Pathometrics, that is, statistical pathology, which includes detailed studies of diseases and their relations to the human body. These facts are obtained largely in hospitals, by health department laboratories and by life insurance companies.

Demography both old and new. — The word "demography" has come into use during the last generation, and has not even now taken its proper place in the list of recognized sciences; but the gathering together of facts relating to human life and the expression of these facts numerically has been practiced from time immemorial.

Some parts of demography are older than others. Genealogy is very old. "Adam lived an hundred and thirty years, and begat a son in his own likeness, after his image; and called his name Seth: And Seth lived an hundred and

five years and begat Enos: And Enos lived ninety years and begat Cainau; And Cainau lived seventy years and begat Mahalaleel:" And so it goes on. Hundreds of years before Christ enumerations of the people were made for purposes of taxation and for other reasons, as one may read in the histories of Egypt, Persia, Judaea, Greece, Rome or China.

Many fragmentary data relating to births, deaths and marriages were recorded in the old church registers of England. Capt. John Graunt compiled the vital statistics for the city of London in 1662, which attracted much attention at the time. In referring to the Great Plague in London in 1666 Pepys tells about the published "bills," that is, the list of the dead, and gives their statistics.

But the application of statistics and the scientific method to genealogy is relatively modern and so are the developments of biometry and pathometry. Sir Francis Galton and Professor Karl Pearson, of England, have been leaders in this and may almost be said to have founded a new school of statisticians.

Demography, therefore, is both an old and a new science.

History of statistics. — The word "statistics" is nearly two centuries old, being first used by Gottfried Achenwall, who lived in Jena, 1719–1772. Before that we learn of the political arithmeticians in France and Italy and of Aristotle who used statistics in describing and comparing different states. The systematic publication of the details of official statistics owes its origin to Anton Büschvig, 1724–1793, who published a voluminous work on historiography and founded a magazine in which statistics for various countries were brought together and compared. Crome in 1785 published important *Tabellen-Statistik* which contained various data in regard to population in Germany.

Many well-known scientists undertook statistical investigations. Edmund Halley, 1656–1742, the astronomer who discovered the comet which bears his name, compiled in 1693 a series of mortality tables and calculated the expectation of life at each age and thus laid the foundation for scientific life insurance. In 1713 Bernouilli, noted for his hydraulic studies, demonstrated a theory of probabilities which a century later, 1813, was perfected by Laplace in his masterly treatise "Theorie analytique des probabilités."

John Graunt, already mentioned, laid the foundations for vital statistics when in 1662 he wrote his remarkable "Natural and Political Observations upon the Bills of Mortality."

In 1741 Joh. Peter Süssmilch (1707–1767) published an important work on vital statistics from which he attempted to draw some far-reaching moral deductions. He tried to demonstrate statistically the doctrine of the "Natural Order." From the equality of the sexes at marriage (at birth his ratio is 21 sons to 20 daughters) he derives the command of monogamy. From a comparison of urban and rural death-rates (in cities one death to 25 to 32 persons, and in the country, one to every 40 to 45 persons) he censures the unnaturalness, immorality, and luxury of city life, "proving statistically" that these bring down the wrath of God.

With the accumulation of statistical data various divergencies began to appear. The political economists, headed by Adam Smith ("Wealth of Nations," 1776) and followed by Malthus (1804) and others, separated themselves from the realm of general statistics. Ritter (1779–1859) led the study of geography apart. At the end of the 18th century the life insurance companies also drew away from the considerations of general populations, and,

by reason of the accumulation of their own data relating to deaths, began to depend upon them alone. This splitting up of the general science of statistics and the multiplication of the practical applications of statistics led to an increasing laxity in method, a condition which we have hardly yet outgrown.

Quételet, 1796–1874, aroused much enthusiasm over statistics as "the queen of all the sciences." His work on probability was justly famous and was an inspiration to Florence Nightingale. Since his time, however, this branch of the subject has been more commonly considered as a part of pure mathematics and is treated in books on "Least Squares," the law of error, and precision of measurements.

Finally, we come to the brilliant works of Galton, Karl Pearson and others, already mentioned.

The history of statistics is a fascinating one, as it flits around from country to country, now flourishing in Italy, then in France, England, Denmark, Germany, England again. The United States has had many able statisticians but few statistical mathematicians worthy to be compared to Laplace, Quételet or Karl Pearson.

The world's great demographers. — Some of the greatest scientists of the world have been enthusiastic statisticians. In some cases their greatness has been due to their statistical skill. Even at the present time it is safe to say that the most successful health officers are good statisticians, although it does not follow that all good statisticians are successful health officers.

The following is a short list of men, not now living, who have made important contributions to the study of statistics, — especially vital statistics. The student will find it interesting to add to this list.

Capt. John Graunt (1620–1674), of England.
Melchiorre Gioja (1767–1829), of Italy.
Sir Francis Galton (1822–1911), of England.
William Farr (1807–1883), of England.
Louis A. Bertillon (1821–1883), of France.
Alphonse Bertillon (1853–1914), of France.
Edwin Chadwick (1800–1890), of England.
Florence Nightingale (1820–1910), of England.
Edward Jarvis (1803–1884), of Boston, Mass.
Lemuel Shattuck (1793–1859), of Boston.
Samuel Warren Abbott (1827–1894), of Boston.
Carroll D. Wright (1840–1909), of Massachusetts.

Section of Vital Statistics. — The American Public Health Association has always manifested a keen interest in vital statistics. Some of the reports of its committees have had a far-reaching effect. In 1907 a Section of Vital Statistics was organized in this association, and since that date the journal of the association, now known as the American Journal of Public Health, has contained many important articles on the subject. Membership in this section is open to registration officials, statisticians, epidemiologists, sanitarians and other members of the American Public Health Association who are interested in vital statistics.

The "Statistical Method." — Statistics are facts expressed by figures. Strictly speaking a birth reported and recorded officially is not a statistic, but a vital fact; yet inasmuch as reported and recorded births are commonly counted and the results expressed numerically it is appropriate to regard such a birth record as a statistical unit or item, that is, as a statistic. It is not customary, however, to use the word in the singular number.

By expressing facts by figures it is possible to arrange them in various ways for study and comparison, as, for example, in tables and graphs; to classify them; to make generalizations; to use them in logical processes and thus

to draw inferences and conclusions based on the facts. The various mathematical processes used for this purpose are collectively known as the statistical method.

Some of these processes are quite elaborate and involve complicated mathematical methods and conceptions, such as the laws of variation, dispersion, correlation and probability. For many years there has been a discussion as to whether "statistics" should be regarded as a distinct science, ranking with physics, chemistry and biology or merely as a method. Westergaard expresses the truest conception when he says that "it is an auxiliary science in many branches of human thought." "There are some statisticians who are statisticians and there are some statisticians who are mathematicians." There are theories of statistics which comprise a very considerable part of mathematics. Volumes have been written on the Calculus of Probabilities, on Least Squares, on Variation. On the other hand, many of the statistical processes are extremely simple and do not get beyond the bounds of ordinary arithmetic. The simple processes have a wide general use; the more elaborate processes have their place but are not commonly applicable or necessary.

Why we need to use the statistical method. — People who do not like mathematics often say "Oh! Pshaw! Why do we have to study statistics? Of what good are they?" The answer is that in a big world we have to deal with many facts and the statistical method enables us to abbreviate facts, to concentrate them so that we can more readily study and compare them and find out what they mean. If you want to live in a little world and deal with only a few facts then you do not need statistics. The head of a small factory may remember the wages of each one of his employees. Tom gets ten dollars a week, Fred gets twelve, Sam and Bill each get fifteen and Henry

gets sixteen dollars. But the head of a large factory where there are a hundred hands cannot carry all these facts in mind. The bookkeeper of course has a record of them, very necessary for pay-day. The head of the factory may know, however, that ten of the employees get sixteen dollars a week, fifteen get twelve dollars and seventy-five get ten dollars. The factory superintendent needs these statistics. He lives in a large world. The village gossip knows the dates of all the births, marriages and deaths in town since January first, but she lives in a little world. To compare these facts with similar facts for the next town and the one next to that requires that the facts be expressed in figures. Statistics enable one to enlarge his horizon.

Why are statistics thought to be "dry"? — Statistics have the popular reputation of being dry, uninteresting, or, as Shakespeare would say, — "flat, stale and unprofitable." This is very natural, for all figures look alike. If we are considering one hundred and thirty-seven tons of coal we use the figures 137 and if we are talking about the same number of American Beauty roses we also use the figures 137. If we think only of the figures we see no difference between these statistics. It does not take much imagination to visualize 137 roses, their beauty and their odor; it takes more, perhaps, to visualize 137 tons of coal. And if 37 of the roses are said to be yellow, 60 white and 40 red, we can visualize the whole mass even if we know that they are mixed. The reason why statistics are "dry" is because people do not try to visualize them. If you don't try to visualize the statistics the figures are commonplace and of course uninteresting, while if you do try the mental effort is tiring. Moreover, there is a real difficulty and that is our inability to visualize very large figures. I may be able to visualize a hundred

dollars, but I confess not to be able to visualize a million dollars, even though I know that it is one thousand times as much as one thousand dollars. Also visualization is lost, or at any rate confused, when we begin to perform mathematical operations with our statistics.

The way to prevent statistics from being "dry" is to keep in mind that statistics are not merely figures, but are figures which stand for facts.

Is it true that "you can prove anything by statistics"? — We often hear it said "Oh! you can prove anything by statistics." Is this true? Suppose we substitute the meaning of statistics and say "you can prove anything by facts if expressed in figures." Obviously this is not so. Facts are facts whether expressed in figures or not. If the conclusions are wrong the trouble lies not in the statistics but in the way they are used. The drawing of conclusions is the function of logic, a process of reasoning, and fallacious reasoning should not be charged against statistics.

And yet there is something which underlies the popular statement. When figures are used to express facts, and when the logical processes are applied to figures, divorced in the mind from the facts for which they stand, it is easy for fallacies to creep in without being recognized; it is easy to compare things which ought not to be compared, to generalize from inadequate data, and to commit all sorts of illogical errors. Thus the unscrupulous may fool the unwary, and the innocent may fool themselves. Hence to use statistics properly one must be able not only to visualize the facts but to think logically. Students who would be statisticians should therefore study formal logic. Some of the common fallacies in the use of statistics will be considered on later pages. Honesty and conservatism are essential qualities for the makers and users of statistics.

There are numerous works on logic. One of the best is "The Principles of Science," by W. Stanley Jevons. It treats not only of logic but of the scientific method in general.

The national value of "Vital Bookkeeping." — It is of the greatest importance to a nation that accurate records be kept of its vital capital, of its gains by birth and immigration and of its losses by death and emigration, for a nation's true wealth lies not in its lands and waters, not in its forests and mines, not in its flocks and herds, not in its dollars, but in its healthy and happy men, women and children. A well man is worth more to a nation than a sick man; a man in the prime of life is of more immediate worth than an old man or a child, a married man is potentially a greater asset than a single man. Hence, in a nation's vital bookkeeping the number of people, their age and sex and conjugal condition, their parentage, their health, the rate of births and deaths, are matters of great moment. Their environment is also important; their concentration in cities and villages and congested areas, their mode of housing, their occupation, their state of intelligence, their economic condition, their knowledge of sanitation, all contribute to the sum total of their usefulness to themselves and to society.

Vital bookkeeping is carried on much as ordinary bookkeeping; there are daily entries of accessions and losses as they occur, corresponding to receipts and payments; there are weekly statements, monthly statements and annual statements; and at longer intervals there is a taking account of stock, that is, a census. One important difference, however, should be noted. Accounts are accurate records of transactions and if properly kept an exact balance will be obtained. Vital statistics are not always accurate, the individual data are incomplete and subject

to error; the results, therefore, lack the precision of monetary accounts. It is necessary to keep this fact constantly in mind when interpreting the results of statistical studies. An understanding of the principles of the arithmetic of inexact numbers and of the theory of probability is essential.

Vital statistics are useful for many purposes. To the historian they show the nation's growth and mark the flood and ebb of physical life; to the economist they indicate the number and distribution of the producers and consumers of wealth; to the sanitarian they measure the people's health and reflect the hygienic conditions of the environment; to the sociologist they show many things relating to human beings in their relations one with another.

Vital statistics necessary for health officer. — Vital statistics are not to be collected and used as mere records of past events: an even more important use is that of prophesying the future. An engineer in planning a water supply to last for a generation estimates the future population by the previous rate of growth; so also in laying out a system of streets and sewers and transportation service. The whole idea of city planning is fundamentally based on the use of the vital statistics of what has been as a means of estimating what is to be.

The health officer of a city or he whose duty it is to collect and record the vital statistics should study them as soon as received and not wait until some convenient day when other work is slack and then merely tabulate and make averages for formal reports and permanent records. Vital statistics, especially those of morbidity, should be studied in the making, and just as the meteorologist reads his instruments daily in order to forecast the weather and give warnings of the coming hurricane, so the efficient health officer will daily study the reports of new cases of

disease in order that he may be forewarned of an impending epidemic and take measures to check its ravages.

No lighthouse keeper on a rocky coast is charged with greater responsibility than he who is set to watch the signs of coming pestilence from the conning tower of the health department. Making another comparison, we may say that the health service should be organized for rapid work like a fire department, with its rapid facility for learning that a fire exists and its ever ready apparatus for extinguishing the blaze. If the fire alarm is not rung, the blaze will spread, and if cases of disease are not reported the epidemic will likewise spread. The duty of reporting cases of infectious disease rests upon the practicing physicians, and thereby hangs a sad and discouraging tale.

National vital statistics. — It has now become well recognized that the maintenance of accurate records of vital statistics is a proper governmental function, and no nation, state or city can be considered as having a complete governmental equipment which does not provide for the proper collection and permanent record of such statistics. But, as will be seen, even our longest governmental records are relatively short, and for that reason we should be careful in drawing general conclusions from them.

Sweden. — Of modern nations Sweden has a just claim to the longest unbroken series of vital statistics. In 1741 registration of births, marriages and deaths was begun in all parishes and since 1749 a census has been taken each year. The principal data for this long period (1750–1900), were given in a most valuable paper by Sundbärg at the International Congress of Hygiene and Demography in Berlin in 1907.

France. — In 1790 Lavoisier (1743–1794), after the French Revolution, collected extensive data relating to the population of that country, the amount of land under

cultivation, etc., but the first actual enumeration of the inhabitants of Paris was not made until 1817.

England. — In England the old parish records date back at least to 1538, when Henry VIII ordered all parsons, vicars and curates to keep true and exact records of all weddings, christenings and burials. It was not until 1801 that a national census was taken, and it was not until 1851 that a complete census was made.

United States of America. — America is far behind other civilized countries in its records of vital statistics. There is no national registration system, no complete national record of births and deaths. This results from our distributive form of government, the control of such matters being a state or municipal function, not a federal one. The records vary greatly in different parts of the country. Some of the older states like Massachusetts and New Jersey possess fairly accurate records that extend back for several decades, but in some of the western and southern states the records are either absent or so incomplete as to be worthless. At the time of the last census, in 1910, the registration area where the death records were considered accurate enough to warrant their being published included only 58 per cent of the total population of the country. This condition of affairs may be charitably regarded as a youthful sin of omission, but if it is much longer continued it will be nothing less than a national disgrace. The health statistics of our best administered cities are much inferior to the published vital statistics of European cities, as, for example, those of Hamburg, Germany. The United States Census Bureau, now permanent, has become increasingly efficient in recent years, and its reports are of much value, but not until a centralized public health service has been secured will the nation's vital statistics be put upon a high plane of comprehensiveness and accuracy.

The importance of statistical induction. — In using statistics we necessarily employ the methods of logical thinking comprised in what is termed "induction," methods by which general tendencies and laws are drawn out of accumulations of facts.

Statistical induction may be said to be one of the most potent weapons of modern science. Referring to it Royce says that the technique of statistical induction consists wholly in learning how to take fair samples of the facts in question, and how to observe these facts accurately and adequately.

Statistics are being constantly invoked for testing hypotheses in all branches of science. This involves four distinct processes, — *first*, the choice of a good hypothesis; *second*, the computation of certain consequences, all of which must be true if the hypothesis is true; *third*, the choice of a fair sample of these consequences for a test; *fourth*, the actual test of each of these chosen hypotheses.

Deductive reasoning as well as inductive reasoning is involved in the use of vital statistics. It is perhaps the natural order of mental processes for the mind pursuing an inductive study to leap ahead to some conclusion and then fill in the intervening steps by working backward by deduction.

It is by the application of the principles of logic that the statistician is able to keep his conclusion within reasonable bounds.

Choice of statistical data. — First, there is the *complete statistical study* which includes a full count of all the units within the desired area or within the specified time. This method, of course, brings the surest results, but it is often impossible. Second, is the *monographic method*, a procedure in which a detailed and exact study is made of a

particular group. Where the group selected for study is a well-chosen type the application of this method yields valuable results but there is danger in generalizing from monographic researches. The third method is the *representative method*, a study of certain selected parts representative of the whole. This is analogous to the method of the analytical chemist where chosen samples are analyzed and the results applied to the whole. The value of this method depends upon the accuracy of the sampling process quite as much as upon the enumeration of the facts embraced by the sample. The representative method is widely used. There are two general methods of sampling. One is that of *random selection*, the other is that of *mixture and subdivision.* The object in both cases is the same, — to secure a sample truly representative of the whole. The tendency to take samples of the obvious and the accessible is one that must be constantly struggled against.

EXERCISES AND QUESTIONS

1. How can vital statistics be used to determine relative values in public health activities? [See Am. J. P. H., Sept., 1916, p. 916.]

2. Describe the common method used in compiling genealogies. [Consult some systematic genealogy, — say that of your own family.]

3. Prepare a diagram of your own ancestry, giving the names of your father and mother, the dates of their birth (and death) and their birthplaces; also the same information as to your two grandfathers and your two grandmothers; your four great-grandfathers, etc., as far as the information can be readily obtained.

4. Who was Mendel and what is the Mendelian law? [See Rosenau's Preventive Medicine and Hygiene, Chapter on Heredity and Eugenics.]

5. What are the primary laws of heredity and eugenics?

6. What information can you give as to the heights of your father and mother, your grandfathers and grandmothers? Can you illustrate any

of the laws of heredity. as to height, color of hair or any other cl acteristics, from your own family records?

7. Can you suggest a schedule of anthropometric data to be kept each person as a matter of family record?

8. Write a short biographical sketch of some person famous for w in statistics, demography or vital statistics. (Name to be assig by the instructor.)

CHAPTER II

STATISTICAL ARITHMETIC

Statistical processes. — The principal processes used in the study of vital statistics are these:

Collection of the facts.
Classification of the facts.
Generalization from the facts.
Comparison of the facts.
Drawing conclusions from the study of the facts.
Display of the facts.

Collection of data. — There are two primary methods of obtaining the data needed in demography — *enumeration* and *registration*. In the first case the statistician goes or sends to get the facts. The persons employed are enumerators or inspectors. This is the method of census taking and is described in another chapter. In the second case the facts are reported to the statistician in accordance with established rules and regulations. For example, physicians and undertakers are required to send notices of deaths and burials to the proper authorities. Some of the methods in common use and the laws which govern the reporting of vital facts are described later on.

It is of vital importance to make sure that the data collected are sufficient in kind and number for the purpose for which the statistics are intended. It saves time and labor in the end to consider carefully at the outset just what data are needed. Where, as is often the case, the statistician has no control over the collection of the data, he

should make every possible attempt to ascertain the reliability of the sources of information and not attempt to draw conclusions not warranted by the conditions under which the figures were collected.

Statistical units. — The basic statistical process is counting. An easy process, — one says; and so it is if we know what to count, and if we know what to include and what to leave out. Here at the very outset we meet our first difficulty.

Before going on stop and define a "dwelling-house." Is a church a dwelling-house if the sexton lives in it? Is a garage a dwelling-house if the chauffeur lives in the second story? Is a building with two front doors one dwelling-house or two? Is a "three-decker" one dwelling-house or three? Or try to define an infant, a birth, a cotton-mill operative or any other unit used in demography.

Statistical units are the things counted and represented by numbers. Obviously every fact, every item, counted must be included within the definition of the unit. No part of a statistical study demands more careful study than the definition of the statistical units to be employed. Each unit should not only be rigidly, accurately and intelligibly defined, it should be steadily adhered to during the investigation. This is by no means easy.

In counting the number of deaths in a city should non-residents be included? Should still-births be included in "births"? Has practice in this matter been constant during the last fifty years? Has pneumonia always meant what it means to-day? And what has become of the causes of death which no longer appear on our lists? It is certainly obvious that all statistics relating to the causes of death must be used with the utmost caution, and this is especially the case if the statistics cover a considerable period of time.

Or, let us take the simple matter of age. What is a seven-year old child? Shall we take the nearest birthday, or the last birthday? Or shall we do as is done in some foreign countries and take the next birthday? In the latter case a child at birth is regarded as of age one. Even the United States census has not always followed the same method of ascertaining age.

Errors of collection. — One of the errors of enumeration is failure to find the units to be counted. In taking a census some persons are never found by the enumerators. They may be accidentally missed, or they may be traveling, away from home or hiding. At the last census in England where the data are collected on a single day, it is said that some of the suffragettes walked the streets for the entire period, so as not to be at home when the enumerators called, arguing that if they could not vote they ought not to be counted. Failure to obtain complete records is still greater when the data are obtained by registration.

The opposite error sometimes occurs, namely over-registration. This is usually due to carelessness, but padded census records have been known to occur.

There are two kinds of errors which need to be distinguished — *balanced* errors and *unbalanced* errors. For example, if a thermometer is correct it may be assumed that a good observer will be as likely to read too high as too low and that in a long series of readings the errors will balance each other. But if the thermometer is at fault all of the readings will be too low or too high, that is, the errors will be unbalanced. Causes of unbalanced errors must be removed if possible or, if not removed, the results must be corrected for them.

In recording such quantities as the height and weight of persons the errors may be regarded as balanced, but physicians in reporting diseases may by their practice of diagnosis

introduce unbalanced errors. Again, the aggregation of the records of various physicians may cause these errors to become more or less balanced.

Finally we have the effect of the *personal equation* of the collector. His mind may have certain grooves through which errors creep into his work. If reading a scale he may have a natural tendency to over-estimate the space between divisions, — if counting units he may have a natural tendency to skip some. What is more serious, he may possess the unpardonable statistical sin of carelessness, or worst of all, he may be dishonest. Ignorance and failure to understand the definition of the units that are to be enumerated are also fruitful sources of error.

Tally sheets. — When many items are to be counted, and especially when there are different units which must be kept apart it is convenient to use some form of tally sheet. Each item is first indicated by a line or a dot and these are afterwards counted. There are two common methods — the *cross-five method* and the *cross-ten method*. In the former every fifth item is indicated by a line which crosses four, making a group of five. In the latter nine items are indicated by dots, the tenth by a cross over the dots. Other devices will doubtless suggest themselves to the reader. (Fig. 1).

Tabulation. — For purposes of study and display the collected data are commonly arranged in tabular form, that is in columns and lines. The preparation of tables is an important part of statistical work and cannot be done too well. The object of a table is to bring statistics together for comparison, to condense information. Essential qualities of good tabular work are clearness, compactness and neatness. Tables are expensive to print, hence the most should be made of each one. The following suggestions, if followed, should yield good results:

1. Each table should have a title which tells clearly what the table contains. Preferably the title should be short, but clearness is the main thing. It is excellent training in the use of words to produce an artistic title.

THE CROSS FIVE METHOD

Disease	Number	
Measles	𝍸 𝍸 𝍸 //	17
Scarlet-fever	𝍸 ///	8
Whooping-cough	𝍸 𝍸 𝍸 𝍸	20

THE CROSS TEN-METHOD

Disease	Jan.	Feb.	Mar.	Apr.	May	June	Etc.
Measles	5	2	9	4	6	1	
Scarlet-fever	2	1	3	9	0	2	
Whooping-cough	20	14	3	19	40	6	

FIG. 1. — Tally Sheets.

2. Each column should have a clear and appropriate heading. As the space for the heading is often small abbreviations may be used, provided they are well understood or well explained in the accompanying text.

3. If the heading is complex, that is, if certain parts of the heading cover more than one column, care should be taken to have this clearly indicated by proper rulings.

Printers call this "boxing." If there are few columns and if the headings are simple, the rulings are unnecessary.

4. If the different columns of a table are likely to be referred to in the text it is convenient to have each column given a serial number from left to right, placed in parenthesis just below the heading.

5. Long unbroken columns of figures are confusing to the eye; especially if the figures of different columns are to be compared on a given line. This trouble can be obviated by leaving horizontal spaces between every few lines or by the use of horizontal rulings. Sometimes, for purposes of reference, each line is given a serial number from top to bottom.

6. The columns of a table should not be widely separated even if there are only a few columns and the page is large. Compactness is a virtue. Much paper is wasted in annual reports by badly arranged tables. On the other hand the type used in tabular work should not be too small.

7. If the figures tabulated have more than three significant figures it is a good plan to separate them into groups of three. Thus, we should not write 6457102, but 6 457 102.

Tables 1 and 2 are given as examples of tabulation and boxing. From this point on students should criticize the tables in this book (a few of which have been intentionally made imperfect), and they should use great care in the preparation of every table involved in the "Exercises and Questions."

TABLE 1

CAMBRIDGE, MASS.

Estimates of Population

Year.	Census.	Estimate based on U. S. census.	Estimate based on U. S. and state census.	Estimate based on local data.	Estimate used by local board of health.	Estimate used in this report.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1890						
1						
2						
3						
4						
5						
6						
7						
8						
9						
1900						
1						
2						
3						
4						
5						
6						
7						
8						
9						
1910						
11						
12						
13						
14						
15						
16						
17						
18						

TABLE 2

CAMBRIDGE, MASS.: BIRTH-RATES

Year.	Population estimate.	Number of Births.		Birth-rate.		
		Total.	Resident.	Gross.	Resident.	As stated by, etc.
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Inexact numbers. — In vital statistics we are usually compelled to deal with data[1] which are not strictly accurate. The figures used to express the results, therefore, should be prepared with this fact in mind. Unnecessary figures should be omitted and only those digits should be included which are supported by the data. Two guiding principles should be followed in making numerical statements of data; — first, to have the figures of the compilation depend upon and indicate the accuracy of the observations; and, second, to carry the final numerical result no further than practical use demands.

Let us take as an illustration the result of the U. S. Census of 1910, according to which the population of the

[1] Do not misuse this word. It is a plural word. The singular number is "datum," but this is seldom used. Do not say, "The data *is* . . . ," but "The data *are*. . . ."

country is stated as 91,972,266. Obviously this figure cannot be strictly true. Let us suppose the possible error to be as much as 200,000. We might write the result "92 million"; but this would be needlessly crude, though accurate enough for some purposes. We might say that the population was between 91.8 and 92.2 million, or we might write 92,000,000 $\pm$ 0.2 per cent. The U. S. Census Bureau publishes the figures as collected, leaving it for him who uses the figures to abbreviate them into round numbers according to the use which is to be made of them.

Experience has shown that very few measurements or observations of anything are accurate to five significant figures, many not to three, and some are doubtful in the second figure.

In tabulating the results of original data it is best to give the figures as obtained. But in discussing the results it is better to use round numbers, the number of significant figures depending on the accuracy of the data and the needs of the problem at hand.

In presenting figures orally to an audience it is especially important to use round numbers. Nothing is more deadening than for a speaker to tire the ear with the reiteration of meaningless digits.

Example. — Let us suppose that the number of bacteria on a plate can be counted within five per cent, plus or minus, and that three different tests gave the following numbers: — 2790, 4220 and 3470 per c.c. the average being 3493. Five per cent of this figure is 175; — hence the true result might conceivably lie between 3318 and 3668. Obviously it would be sufficiently accurate and for many reasons better to state the result as 3500 per c.c. Recognizing these unavoidable errors in our present methods the Committee on Standard Methods of Water Analysis of the American

Public Health Association has suggested that statements of analysis should be limited in significant figures as follows: Unfortunately the rule has not been lived up to.

TABLE 3

RULE FOR STATING THE RESULTS OF BACTERIAL COUNTS IN WATER ANALYSIS

Numbers of bacteria found.	Records to be made.
1 to 50	As found
51 to 100	To the nearest 5
101 to 250	" " " 10
251 to 500	" " " 25
501 to 1,000	" " " 50
1,001 to 10,000	" " " 100
10,001 to 50,000	" " " 500
50,001 to 100,000	" " " 1,000
100,001 to 500,000	" " " 10,000
500,001 to 1,000,000	" " " 50,000
1,000,001 to 10,000,000	" " " 100,000

Perhaps, sometime, demographers will prepare a similar table for the use of round numbers in vital statistics.

Vital statisticians should at least endeavor to follow the example of the bacteriologists and by concerted action cut out fictitious accuracy from their reports.

Precision and accuracy. — Numerical statements of measurements are accurate as they approach the true value of the thing measured; they are precise as they approach the mean of the measurements. Accuracy takes into account unbalanced as well as balanced errors; precision is concerned with balanced errors only. It is possible for results to be precise and yet be erroneous.

Combinations of inexact numbers. — When data which differ in precision are combined it is possible that faults may be obscured. Let us take the case of a simple addition of the three items in column (1).

TABLE 4

EXAMPLE OF COMBINATION OF INEXACT NUMBERS

Item.	Percentage error.	Possible error in item.
(1)	(2)	(3)
47,386	2	± 948
9,453	5	± 473
843,782	0.5	±4219
Sum 900,621		±5640, or 0.6%

The true value of the sum may lie between 895,000 and 906,000. The result may be written, therefore, 900,000 ± 0.6 per cent. The percentage error of the sum would not, of course, be the sum or even the average of the figures in the second column.

Ratios. — The ratio between two numbers may be expressed as a common fraction or may be indicated by the ratio symbol, the colon (:). Thus we may write $\frac{4}{8}$ or 4 : 8. If the figures are small the difference between the two numbers can be visualized, but if they are large, as for example $\frac{165}{217}$, or 165 : 217, it is difficult to appreciate their meaning. If common fractions are used to indicate ratios they should be limited to those in which the denominator is below 10, or is some round number, such as a multiple of 5 or 10. Thus we might speak understandingly of a $\frac{1}{4}$ or a $\frac{1}{6}$ or a $\frac{1}{25}$ or a $\frac{1}{200}$, but not of a $\frac{1}{17}$ or a $\frac{1}{265}$.

For most purposes in statistical work decimal fractions are to be preferred to common fractions. They facilitate printing as they occupy only one line and do not require the use of smaller type. It must never be forgotten, however, that a decimal fraction is composed of two parts, just as a common fraction, namely the figures which are printed and

unity (or one) which is not printed. Thus $\frac{3}{4} = \frac{0.75}{1} = 0.75$. A decimal fraction is therefore just as much a ratio as a common fraction.

In statistical work we are constantly obliged to compare facts on the basis of their ratios. Let us suppose that we desire to compare cases and deaths from typhoid fever in three different places and that the data are as follows:

TABLE 5

Place.	Cases.	Deaths.
(1)	(2)	(3)
X	541	46
Y	672	53
Z	247	30

In order to make the comparison we must select either one or the other quantity as a base, either cases or deaths. If we select one death as the unit base we have the following ratios:

TABLE 6

Place.	Number of cases to one death.
(1)	(2)
X	11.8 *i.e.* 541 ÷ 46
Y	12.7 672 ÷ 53
Z	8.2 247 ÷ 30

If we select one case as the unit base we have the following ratios, expressed as decimals:

TABLE 7

Place.	Number of deaths to one case.	
(1)	(2)	
X	0.085 *i.e.*	46 ÷ 541
Y	0.079	53 ÷ 672
Z	0.121	30 ÷ 247

We might however select 100 cases as the base unit, in which case the figures are 100 times as large and we have

TABLE 8

Place.	Per cent of cases which resulted in death
(1)	(2)
X	8.5
Y	7.9
Z	12.1

Rates. — Now rates are merely ratios referred to some round number as a base. When 100 is used as the base we have a percentage rate, that is a rate per one hundred; but we may use 10 or 10,000 or even 100,000 or 1,000,000 as the base, and very often do so. In many cases we use *one* as the base. Thus we speak of " gallons of water per day," meaning the number of gallons of water for *one* day, the "number of persons per square mile," meaning, of course, one square mile. All of these rates, where only two quantities are compared may be called simple rates. Simple rates have only one base.

Compound rates are those which have two bases. Thus we speak of " gallons of water per capita per day," meaning

the number of gallons of water used by one person in one day. The "number of births per 1000 marriages per annum" would also be a compound rate. Most of the rates used for comparison in vital statistics are compound rates as they involve both number and time, the latter often being understood as one year, the calendar year perhaps.

Misuse of rates. — Fictitious accuracy in the use of rates and ratios should be avoided. If 35 out of 57 balls were white the percentage of white balls would be 61 404 per cent. The smallest possible error, *i.e.*, 1, would change the percentage to 59.65 per cent or 63.16 per cent. To use two or even one place of decimals is here absurd. Clearly for figures less than 100 fractions of per cents are illogical. In the same way death-rates for populations of less than 1000 are useless beyond the third significant figure. Comparisons of averages of fictitious values are also to be avoided.

Changes of base in the computation of rates should be kept in mind in order to avoid error of statement. Here is a well-known illustration: In the year 1880 the receipts of a water company were $400,000; between 1880 and 1890 they increased 10 per cent, that is, they became $440,000; between 1890 and 1900 they decreased 10 per cent, that is, they became $396,000 (not $400,000). It is said that a strike once resulted from this fallacy. A company found it necessary to reduce wages 20 per cent for a certain period, promising to raise the wages 20 per cent at the end of the period. Naturally the men who were reduced from $2.00 a day to $1.60 thought they would have their pay restored to $2.00 but found that the company wished to give only $1.60 + 20 per cent or $1.92. The base used should be stated in words if it is not perfectly clear from the context.

When interpreting ratios it should be carefully noted whether or not the numerator bears a direct relation to the

denominator. In proportion as it fails to do so any inference from it is less valuable. The ratio between the number of births and the total population is less close than that between the number of births and the number of married women of child-bearing age.

Ratios are sometimes necessarily used in an indirect way. Thus the average annual exports and imports are taken to represent the business condition of a country. Here, a part is taken for the whole. The method is proper if, in the interpretation, it is recognized that it is a part. Or the typhoid fever death-rate of a city is taken as an index of the sanitary quality of the public water supply. It may indeed be such an index, but it is not the only one.

In the same way crude death-rates based on total population regardless of sex or age are less useful in studying relative hygienic conditions than when these factors are taken into account.

Index. — When it is not possible to find a simple direct ratio between two quantities, it is sometimes possible to combine several ratios which taken together give a better indication of the conditions than any one ratio used alone. Thus the prices of various standard commodities sold in any one year may be combined to give a single figure which will indicate the state of trade during that year. This combined result compared with a similar result for the following year will enable one to compare the state of trade in the two years. When several quantities are thus combined the result is called an Index, or an Average Index. Obviously there are various ways in which a combination may be made. Sometimes the weighted average of several quantities is used.

The index has not come into use to any extent in the study of vital statistics, but it would seem logical to use it in comparing the relative hygienic conditions of different

cities. This is partially accomplished when crude death-rates are "corrected" or adjusted to take into account the composition of population as to age, sex and nationality.

Some attempts to compute a satisfactory sanitary index will be referred to later on.

Computation of Rates. — The computation of a death-rate for a city is merely an example in long division. As most health officials and some college students will have forgotten their arithmetic by the time they read this book a few words as to computation may be pardoned. The computation sheet should show a record of what has been done and should bear the date and the name or initials of the computer.

Let us suppose that in a city of 34,691 people, as shown by the census of 1910, the number of deaths in that year was 549; what was the death-rate per thousand of population? In the first place how many thousands of population were there? Answer, by pointing off three places, 34.691. All that is necessary then is to divide 549 by 34.691. This may be done in several ways

The operation of long division may be done in full, thus:

```
34.691)549.000(15.82 = death-rate per 1000.
       34691
       202090
       173455
        286350
        277528
          88220
          69382
```

If we are content to be a little less accurate we may shorten the work by leaving off one decimal of the population, thus:

```
34.69)549.00(15.82 = Answer
      3469
      20210
      17345
       28650
       27752
         8980
         6938
```

The result is not changed. If we write 34.7 instead of 34.69 we shall get

```
34.7)549.0(15.82 = Answer
     347
     2020
     1735
      2850
      2776
        740
        694
```

Still no change. Suppose we try 35 as a round number for the population instead of 34.7 or 34.69 or 34.691. We then get

```
35)549(15.7 = Answer
   35
   199
   175
    240
    245
```

This is evidently incorrect in the decimal. We have gone too far in using a round number for the population.

By using discretion in omitting decimals from the population divisor much work may be saved. It is pitiful to see the energy and time wasted by some health officers in using unnecessary decimals in performing long-division operations, especially as there are so many labor-saving devices

available. An easier way is to use a table of logarithms, and a still easier way is to use a slide-rule, a mechanical device for applying logarithms where approximate results will suffice.

The desirable degree of accuracy of death-rates is discussed on a later page.

Logarithms. — Of course you have forgotten how to use logarithms. Let me remind you.

If you multiply 10 by 10 you get 100. You have put two tens together, and you might write them thus 10^2 and say that $10^2 = 100$. If you put three tens together you get 1000. So that $10^3 = 1000$. And so on. Now, ten is the base of logarithms, and we say that the log (meaning logarithm) of 100 is 2, because 2 tens multiplied together makes 100. And the log of 1000 is 3 and the log of 1,000,000 is 6. So also the log of 10 is 1, the log of 1 is 0, and the log of 0.1 is minus 1, *i.e.*, -1, and so on down. Now if the log of 10 is 1 and the log of 100 is 2, what is the log of 20? It is between 1 and 2; it is 1 plus something. Just what this something is you can find from a table of logarithms. A short table (five places) gives for the log of 2 the figures .30103, so that the log of 20 is 1 plus .30103, or 1.30103. In the same way the log of 200 is between 2 and 3; in fact it is 2.30103. And so we can find the logarithm of any number, taking the decimal from the printed table, and putting down the figure to the left of the decimal point according to the size of the original figure, remembering that for figures

Between	0	and	10,	the log is	0...........
	10	"	100,	"	1...........
	100	"	1000,	"	2...........
	1000	"	10,000,	"	3...........

We use those logarithms in this way. Suppose we wish to multiply $100 \times 10{,}000$. We might do this in the regular way,

$$\begin{array}{r} 10{,}000 \\ 100 \\ \hline 1{,}000{,}000 \end{array} = \text{Answer.}$$

But the log of 100 is 2 and the log of 10,000 is 4. If we add these logarithms we get 6, and 6 is the log of our answer. That is by adding the logs of two number, the sum will be the log of the product of the numbers.

And also if we subtract the log of one number from the log of another the difference will be the log of the dividend obtained by dividing the second number by the first. Thus in our death-rate problem the log of 549 is 2.739572 and the log of 34.691 is 1.540216. Hence,

$$\begin{array}{l} 2.73957 \\ 1.54022 \\ \hline 1.19935 \end{array} \text{ is the log of } 15.82 = \text{the answer.}$$

It must be remembered that the logarithm table contains only the decimals. That is we look up the number which corresponds to the decimal .199356 and find the figures to be 1582. The whole number of the log being 1 tells us that the result is between 10 and 100, and therefore must be 15.82.

In this way the use of logarithms may save the statistician much time.

A table of logarithms of numbers from 1 to 1000, carried to five decimal places, may be found in the Appendix. Tables in which there are six or seven places of decimals can be purchased and are in common use.

Those who do not feel confidence in themselves in using logarithms should consult a textbook of algebra.

The slide-rule. — The slide-rule is a mechanical device for adding and subtracting the logarithms of numbers,

and therefore it enables one to multiply the numbers for which the logarithms stand. It does not add or subtract the numbers themselves.

In using the slide-rule it is first necessary to understand the scale. The logarithms of the numbers from 1 to 10 are as follows:

TABLE 9

LOGARITHMS OF NUMBERS: 1 TO 10

Number.	Logarithm.	Number.	Logarithm.
(1)	(2)	(3)	(4)
1	0.00000	6	0.77815
2	0.30103	7	0.84510
3	0.47712	8	0.90309
4	0.60206	9	0.95424
5	0.69897	10	1.00000

and above 10 the decimals repeat themselves, thus

20	1.30103
30	1.47712

etc.

If these are plotted on a uniform scale we get the result shown in Fig. 2A. It will be noticed that on the number scale the divisions grow smaller as the numbers increase. It is this number scale which appears on the slide rule. There are many subdivisions. The space from 1 to 2 is divided into 10 parts, and so are the other spaces. The space between 1 and 1.1 is also divided into ten parts; but above 2 there is not room for so many lines, so the values of the divisions change and one must be on his guard not to make an error in scale reading. It should be remembered that just as the main divisions between 1 and 10 are unequal, so are the subdivisions be-

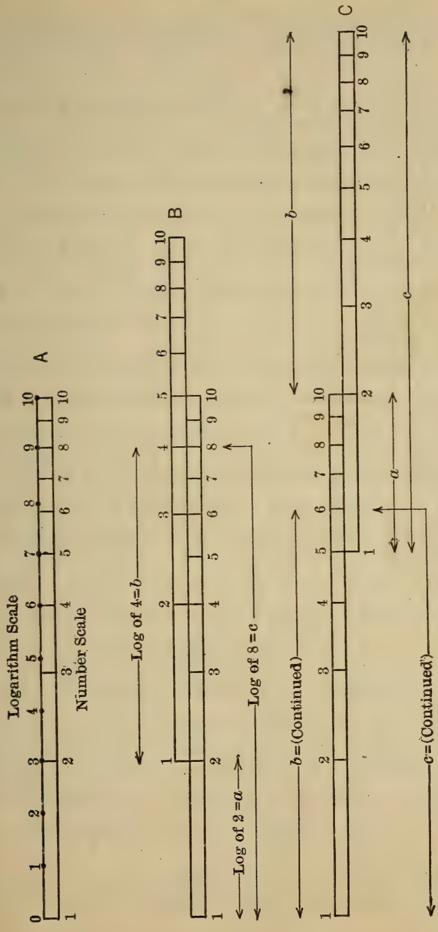


FIG. 2. — Use of the Slide-rule.

tween 1 and 2 unequal. The minor subdivisions are also unequal but the eye cannot distinguish these small differences.

Let us first learn how to multiply two numbers — say multiply 2 by 4. We use the lower scale on the slide and the lower scale on the rule under it. The two scales are just alike. If the left-hand end of the slide is set on 2 of the rule, then the distance (*a*) along the slide is the log of 2, and the distance (*b*) along the slide is the log of 4. The sum of (*a*) and (*b*), *i.e.*, (*c*) is the sum of the logs of 2 and 4 and therefore is the log of their product. And so we find that the distance (*c*) from the end of the rule gives us 8, the result, under the figure 4 of the slide.

Suppose, however, that we want to multiply 2 by 6. The distance (*c*) would then extend to 6 on the slide, or beyond the scale of the rule. That is the product is more than 10. Remembering that the log numbers repeat themselves above 10, all we have to do is to set the right-hand instead of the left-hand end of the slide on the figure 2, of the rule and then read on the rule the number under 6 of the slide. It is 12.

The process of division is just the reverse of that of multiplication. To divide 8 by 4, set 4 of the slide over 8 of the rule and read 1 (the end) of the slide on the rule (*i.e.*, 2).

The upper marks on the ordinary slide and rule are not needed for simple multiplication and division. The movable wire is used as a guide and reference mark.

To return to our death-rate problem (above) we may divide 549 by 34.69 by setting 3469 on the slide over 549 of the rule and reading 1 of the slide on the lower scale of the rule. The result is 158+ as before. It is difficult to set 3469 exactly, so it is impossible to read the result to more than three significant figures.

The slide-rule does not give us the decimal points. That had best be determined by inspection. (There are indeed

rules for the decimal point, but they are hard to remember and one should not attempt to do so.) Inspection shows that 34 goes in 549 more than 10 and less than 20 times; consequently the slide-rule result is 15.8+.

Slide-rules are made in many different lengths, from three or four inches up to twenty inches. A ten-inch rule is best for general use. The twenty-inch rule is easier on the eyes and can be read closer, but it cannot be carried in the pocket. Celluloid rules are the best, as the marks are clear, but cheap wooden rules are satisfactory for some purposes.

Books of instruction accompany most of the high-grade rules and can always be purchased.

Every statistician ought to know how to use logarithms and how to read a slide-rule. Life is too short and time nowadays is too precious to depend upon the old methods of long division and multiplication if much work is to be done.

Classification and generalization. — For purposes of study it is usually necessary to sort out the various data, divide them up into classes, groups or series and to make generalizations in various ways. Some of these processes are very simple; others are rather complicated. The methods used vary according to the nature of the problem at hand. As far as possible the simple methods should be preferred to the more complex procedures.

Classes, groups, series and arrays. — Collections of units which differ from other collections by characteristics which cannot be expressed in figures are properly termed sections or *classes*. Thus, populations are divided into classes according to sex, nationality, conjugal condition, civil divisions.

Collections of units which differ from other collections by characteristics which can be expressed in figures are called *groups*.[1] As an example populations are divided into age

[1] This distinction is not universally made, but if rigidly adhered to it would result in greater clearness of expression.

groups, or into groups of persons having different weights or heights.

Data are also arranged in *series* according to some natural sequence or some order of magnitude or chronological order. When all of the items of a given group are arranged in order of magnitude from small to large, or large to small, they are said to be placed in *array*. Companies of soldiers arranged with the tallest man at one end of a rank and grading down to the smallest man at the other end form an array.

Classes of data. — Little need be said about classification except that the definitions of classes should be clearly and accurately stated, and so drawn as to be mutually exclusive, that is, it should not be possible for an item to appear in more than one class.

Generalization of classes and groups. — The *average*, although a convenient device for generalizing the facts in a class or in a group of observations, has a number of shortcomings. It does not give a true picture of the different items. Two groups may have the same average and yet be composed of very different items. Thus:

	6	1
	6	1
	7	2
	7	3
	9	28
Sum	35	35
Average	7	7

In a large number of items there may be one important item of large magnitude which might be concealed by the average. On the other hand a large item, if erroneous, might unduly raise the average and give a false generalization. Another name for the average is the *mean*.

Some other forms of generalization, therefore, are necessary in statistical work.

The array and its analysis. — If the items are arranged in order of magnitude with the smallest at one end and the largest at the other they are said to be in *array*. If the number of items is not too great this gives an excellent picture of the group. Thus Fig. 3 shows at a glance that the two groups on page 40 are different from each other.

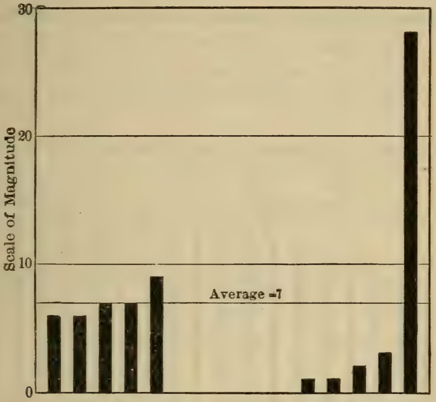


FIG. 3. — Example of Differing Groups which have the Same Average.

In an array the magnitude of the middle item is called the *median*. This is a very important unit in statistical analysis. The means are the same for the above-mentioned groups, *i.e.*, 7, but the medians are different, *i.e.*, 7 and 2. The median may be the same as the mean, — in fact, it usually is near the mean, — but it need not be the same.

The *mode* is the magnitude of the item which is most common among the items. A modish bonnet is one very commonly seen; it is the fashionable one. In one of our two groups there are two sixes and two sevens and we

cannot tell which is the mode. They are tied for first place. In the other group the mode is clearly *one*.

The magnitude of the item halfway between the median and the upper limit is called the *upper quartile*, and the corresponding item towards the lower end, the *lower quartile*. A quartile is one-quarter of the way from one end of the array to the other. See Fig. 4.

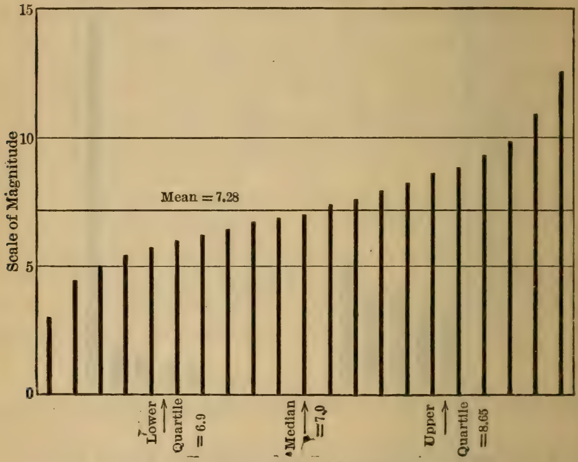


FIG. 4. — An Array of Observations.

The magnitude of the item one-tenth of the way from the lower to the upper limit of the array is the lower *decentile*. And so there may be *quintiles*, and other " iles."

These various units help very much to give one a picture of an array. They are used in various combinations, and ratios are made up by using them.

The average, together with the maximum and minimum, offers a common form of generalization. The median,

together with the upper and lower decentiles, is sometimes used. The quartile difference, that is the difference between the two quartiles, is used.

Again the ratio of the maximum (or minimum) to the mean, the ratio of the quartiles to the median, the ratio of the mean to the median, and other ratios have been used.

Still another way is to find the extent to which the different items differ from the mean and study these differences.

This subject, which involves such matters as variation, dispersion and the like, takes us into the very heart of the statistical method and will be treated at length in Chapter XII.

Groups. — The problem of arranging statistical data into groups is a troublesome one, — troublesome because there are several ways in which groups can be made and defined.

Let us take the case of nine persons whose illness from a certain disease lasted respectively 13, 11, 6, 9, 12, 10, 8, 17 and 13 days. We will consider these merely as whole numbers and try to arrange them in groups. A common way would be:

	(1)	(2)	(3)	(4)
Days............	0–5	5–10	10–15	15–20
Number of persons	0	4 (or 3?)	4 (or 5?)	1

There is confusion here because one does not know whether to put the item 10 into the second or third group. The groups are not clearly stated. They are not mutually exclusive.

Another way would be to arrange the groups thus, making the upper and lower limits both inclusive.

	(1)	(2)	(3)	(4)	(5)
Days..............	0	1–5	6–10	11–15	16–20
Number..........	0	0	4	4	1

A better way would be this:

	(1)	(2)	(3)	(4)
Days....................	0–4	5–9	10–14	15–19
Number................	0	3	5	1

The last two methods are both used. The means for the four groups in the last method would be respectively 2 (average of 0 to 4), 7, 12 and 17. The means for the five groups in the next to the last method would be 0, 3, 8, 13 and 18.

Let us next take a case where we have to deal with whole numbers and fractions, — say to the nearest quarter, — and where the items are 54, $52\frac{1}{4}$, $51\frac{1}{2}$, 57, $50\frac{1}{4}$, $54\frac{3}{4}$, $51\frac{1}{2}$, $56\frac{1}{4}$, 58 inches. We may group them thus:

	(1)	(2)	(3)
Inches..................	50, $50\frac{1}{4}$, $50\frac{1}{2}$, $50\frac{3}{4}$	51, $51\frac{1}{4}$, $51\frac{1}{2}$, $51\frac{3}{4}$	52, $52\frac{1}{4}$, $52\frac{1}{2}$, $52\frac{3}{4}$
Group limits, inches.......	$50–50\frac{3}{4}$	$51–51\frac{3}{4}$	$52–52\frac{3}{4}$
Mean of group...........	$50\frac{3}{8}$	$51\frac{3}{8}$	$52\frac{3}{8}$
	and so on		

With measurements of quarters it is not possible to devise a grouping such that the mean of each group is an even number. Neither $50\frac{3}{4}–51\frac{1}{2}$ nor $50\frac{1}{2}–51\frac{1}{4}$ would give 51 as the mean.

If, however, we had observations in which the fractions were thirds, or fifths, or with some other odd-numbered denominator, we might do so. Thus if we had $50\frac{2}{3}–51\frac{1}{3}$ the mean would be 51; or if we had $50\frac{3}{5}–51\frac{2}{5}$ the mean would be 51. Sometimes it is an advantage to arrange the group so that the mean of the group is a whole number, but often this does not matter.

Again let us suppose we are dealing with whole numbers and decimals (to tenths only). Here the denominator is not an odd number. We might arrange the groups thus:

	(1)	(2)	(3)	(4)	
Limits............	0	0.1–1.0	1.1–2.0	2.1–3.0	etc.
Mean of group.....	0	0.55	1.55	2.55	

or

	(1)	(2)	(3)
Limits......................	0–0.9	1.0–1.9	2.0–2.9
Mean of group................	0.45	1.45	2.45

If the observations were made to the nearest hundredth we might have

	(1)	(2)	(3)	(4)
Limits...........	0	0.01–1.00	1.01–2.00	2.01–3.00 etc.
Mean...........	0	0.505	1.505	2.505

If we had observations of much greater accuracy we would approach the following round numbers as the means of the groups:

	(1)	(2)	(3)	(4)
Limits..	0	0. + . . . 1.0	1. + . . . 2.0	2. + . . . 3.0
Mean...	0	0.5	1.5	2.5

Group designations. — In describing groups it is technically proper to designate the upper and lower limits of the group. For whole numbers this is perfectly simple. Thus in our table we may give

	Age
(1)	0–4
(2)	5–9
(3)	10–14
(4)	15–19 etc.

If the whole numbers are followed by fractions we may assume that any fractions are attached to the whole numbers and that the maximum figure includes the largest possible fraction less than one. Thus $19\frac{1}{2}$ would go in the fourth group, 14.641 would go in the third group. The sign (–) here stands for " to," *i.e.*, 0 to 4.

Sometimes to save space in printing only one group limit is given, the other being understood. Thus in the report of the Registrar General of England we find the following age groups tabulated:

Age	
0–	Meaning 0 to 4 plus fractions
5–	Meaning 5 to 9 plus fractions
10–	etc.
15–	

Where the groups differ by *one*, this method is the only practicable one. Thus

Age
0–
1–
2–
3–

Here we could not state an upper limit without using fractions.

A better nomenclature perhaps would be to use the plus sign instead of the dash, indicating that any fractions were attached to the whole number. Thus:

Age
0+
1+
2+
3+
etc.

Let us compare two groupings, *a* and *b*, the limits of which are stated as follows:

a	*b*
4+	$4\text{–}4\frac{3}{4}$
5+	$5\text{–}5\frac{3}{4}$
6+	$6\text{–}6\frac{3}{4}$

The inference would be that the first group of *a* included items of magnitude 4 and of 4 plus any fraction attached to it however small. The average of the items in this group would be 4.5. In the case of *b*, however, the inference would be that the measurements were made to the nearest $\frac{1}{4}$, and that the items in the first group would be only 4, $4\frac{1}{4}$, $4\frac{1}{2}$ or $4\frac{3}{4}$, the average of which would be $4\frac{3}{8}$.

Percentage grouping. — It often happens that what is wanted is not so much the number of items which fall in each group as the relative number in the different groups. In this case we take the total number of items as 100 per cent and find the per cent which the number of items in each group is of the total, that is, we make a percentage grouping, or a percentage distribution.

In a certain outbreak of typhoid fever the cases were distributed according to age as follows:

TABLE 10

AGE DISTRIBUTION OF TYPHOID FEVER CASES

Age group.	Number of cases in group.	Per cent of cases in group.
(1)	(2)	(3)
0–4	42	8.3
5–9	77	15.3
10–14	82	16.3
15–24	140	27.7
25–34	85	16.8
35–44	45	8.9
45–	34	6.7
Total	505	100.0

The figures in the third column are computed from those in the second. The use of the slide-rule greatly facilitates such computations. The author made the above compu-

tation of percentages with the slide-rule in less than two minutes. For comparison he made the same computations by long division, finding that it required three times as long.

Cumulative grouping. — A cumulative or summation group is one which includes the data for previous groups, that is, all of the data from the beginning of the series up to the group limit. An illustration will make this clear.

TABLE 11

AGE DISTRIBUTION OF CASES OF POLIOMYELITIS

Brooklyn, N. Y., 1916

Age group.	Per cent of cases in group.	Age group (cumulative).	Per cent in group.	Age.	Per cent less than stated age.
(1)	(2)	(3)	(4)	(5)	(6)
0–	8.5	0–	8.5	1	8.5
1–	22.0	0–1	30.5	2	30.5
2–	23.9	0–2	54.4	3	54.4
3–	19.0	0–3	73.4	4	73.4
4–	7.2	0–4	80.6	5	80.6
5–	6.6	0–5	87.2	6	87.2
6–	3.7	0–6	90.9	7	90.9
7–	2.5	0–7	93.4	8	93.4
8–	1.5	0–8	94.9	9	94.9
9–	1.3	0–9	96.2	10	96.2
10–	0.8	0–10	97.0	11	97.0
11–15	2.0	0–15	99.0	16	99.0
16–	1.0	0–	100.0		100.0
	100.0				

The figures in the fourth column were obtained by successive additions of the figures in the second column. It is more common perhaps to state the results of cumulative grouping in the manner shown in columns five and six. If there are 30.5 per cent in the cumulative group 0–1, it is obvious that 30.5 per cent of the cases were younger than 2 years.

The summation table is very useful in many statistical problems.

Averages. — The simplest, most common, and in general the most useful method of generalizing the results of a set of observations is the *average*, or arithmetic *mean*. The word mean is practically synonymous with the word average, but some writers apply the former to the generalization of a group, using the latter to indicate the arithmetical process.

The average is found by dividing the sum of the magnitudes of a number of items by the number of items. The average of 13, 19 and 25 is $\frac{13 + 19 + 25}{3} = \frac{57}{3} = 19$. The average of 12, 14, 10, 5 and 9 is $\frac{12 + 14 + 10 + 5 + 9}{5} = \frac{50}{5} = 10$.

Now what is the average of all the items in both of these groups? Without thinking we might say that it is $\frac{19 + 10}{2} = 14\frac{1}{2}$, but this would be wrong. To prove it add together the items and we have

$$\frac{13 + 19 + 25 + 12 + 14 + 10 + 5 + 9}{8} = \frac{107}{8} = 13\tfrac{3}{8},$$

which is the true answer. The reason why we cannot take the average of the two averages is because the second group has five items and the first group only three. The second group being larger ought to be given a greater weight in combining the two.

Suppose that we give the second group greater weight than the first in proportion to the relative numbers of items in the two groups. We then have

19 (the average of the first group)	$\times$ 3 =	57
10 (the average of the second group)	$\times$ 5 =	50
The sum is		107

and $107 \div 8 = 13\frac{3}{8}$.

This is what is called a "*weighted average*." It is often very useful. Let us take another example of this.

If one man in a factory earned \$30 per week, three earned \$20 and one hundred earned \$10, what is the average wage per man? Certainly not $\frac{30 + 20 + 10}{3}$. It is

$$\begin{array}{rrl} \$30 \times & 1 = & \$\ \ 30 \\ \$20 \times & 3 = & \ \ \ \ 60 \\ \$10 \times & \underline{100} = & \underline{1000} \\ & 104 & \$)\overline{1090} \\ & & \$10.48 \end{array}$$

In reality this is merely an abridgment of the labor required to add together the wages of each particular workman.

Sometimes it is required to find the average of a series of observations arranged by groups. Let us assume that in the following table the observations are made only to the first decimal place.

TABLE 12

Group. (1)	Number in group. (2)	Average of group. (3)	Product of (2) and (3).
0–0.9	21	0.45	9.45
1.0–1.9	17	1.45	24.25
2.0–2.9	12	2.45	29.40
3.0–3.9	8	3.45	27.60
	58		58)90.70
			1.56 = average

The *geometric mean* of two numbers is the square root of their product. If we have two numbers a and b, the geometric mean is $\sqrt{ab}$. It is also called the *mean proportional* between two numbers, because if we let it be represented by x, then $a : x = x : b$, *i.e.*, a is to x as x is to b. By algebra, from this equation $ab = x^2$. $\therefore x = \sqrt{ab}$.

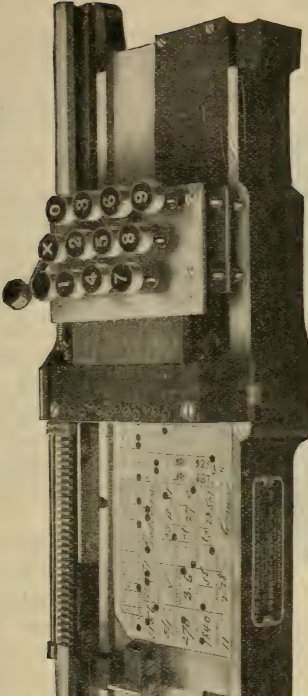


Fig. 5. — Machine for Punching Card.

If there are three numbers the geometric mean would be the cube root of the product of the three numbers; and so for larger numbers.

As compared with the arithmetic mean the geometric mean minimizes the effect of very large numbers and increases the effect of very small numbers on the final results. For instance, the arithmetic mean of 4 and 20 is $\frac{4 + 20}{2} = \frac{24}{2} = 12$. The geometric mean would be $\sqrt{4 \times 20} = \sqrt{80} = 8.95$. The arithmetic mean of 2 and 100 would be 51, the geometric mean 14.1.

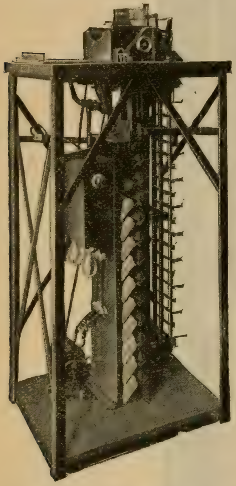


Fig. 6. — Machine for Sorting Cards.

Economists often use the geometric mean in combining the prices of different commodities to obtain an index of trade conditions. It has not been much used in demography, but there are places where it might well be used.

There is another kind of average known as the *harmonic mean*. A man travels two miles, the first at a rate of 10 miles per hour, the second at a rate of 20 miles per hour, what was his average rate of travel? The obvious answer, *i.e.*, 15 miles per hour, is not correct, for the man did not travel for two hours but for two miles. Actually he traveled the first mile in $\frac{1}{10}$ of an hour, or 6 minutes, and the second in $\frac{1}{20}$ of an hour, or 3 minutes. His average time, therefore, was $\frac{6 + 3}{2} = \frac{9}{2} = 4.5$ minutes per mile, and

his average rate $\frac{60}{4.5} = 13.3$ miles per hour. The statistician seldom has occasion to use this. Algebraically the harmonic mean of two numbers, a and b, is $\frac{2\,ab}{a + b}$.

In the study of data arranged in series, the items of which fluctuate up and down but which nevertheless show cyclical variations, the *moving average* is often computed in order to obtain a series from which the local fluctuations have disappeared. The moving average is a series of averages, each based on the same number of items, but each group of items, as it advances, adding one new item and dropping one old one. If for example we have items in this order: — 16, 14, 18, 17, 18, 17, 19, 15, 13, 14, 11, 12, 10, 11, 8, the moving average based on successive groups of three items would be $\frac{16 + 14 + 18}{3} = 16$; $\frac{14 + 18 + 17}{3} = 16.3$; $\frac{18 + 17 + 18}{3} = 17.7$; $\frac{17 + 18 + 17}{3} = 17.3$; and so on. Sometimes groups of five items are taken, or nine, or twenty-one, but usually some odd number. An example of the moving average may be seen in Fig. 44. Some one has said that the moving average is so named because the large amount of work required moves one to tears. Any one thus affected should know that there are shortcuts to the results which may be found described in works on general statistics. The *moving median* might be used if the groups chosen contained many items. This would require somewhat less work than the moving average.

Mechanical devices for statistical work. — It would not do to close this chapter on statistical arithmetic without calling attention to the mechanical devices now available for performing the operations of addition, subtraction, multiplication and division. Where statistical operations are

constantly going on these instruments more than pay for their cost. They are too well known to need description here.

The tabulating devices of the Hollerith and Powers types are not as well known, but they have become an established feature in the U. S. Bureau of the Census and in the statistical departments of large commercial and industrial corporations. Three separate devices are required for this work — a card punching machine, a sorting machine and a counting machine. In keeping records

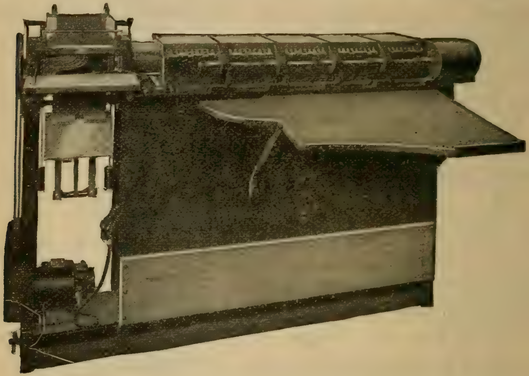
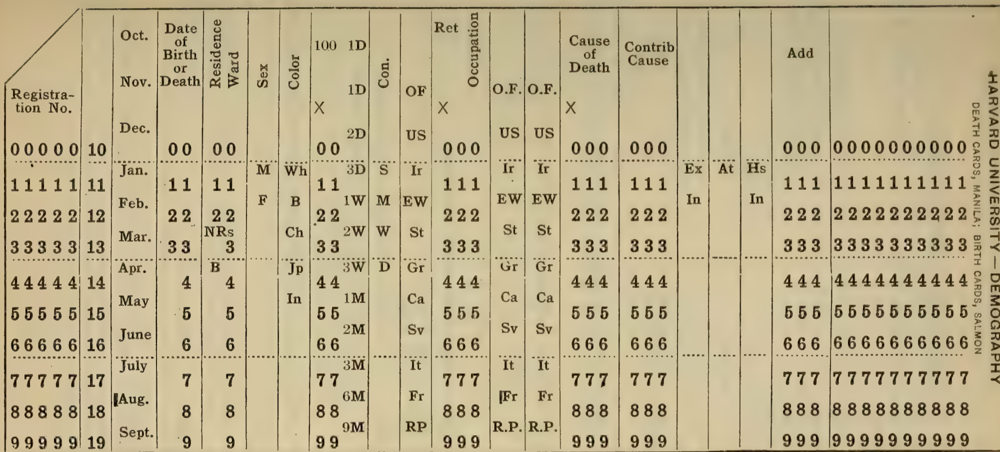


FIG. 7. — Machine for Sorting Cards.

of deaths, the data from each death certificate are transferred to a card, each fact being indicated by number, a hole being punched in the proper column. These holes serve as the basis of sorting in the second machine. By feeding the cards into the sorting machine they can be quickly divided into piles according to age, or sex, or cause of death, or into other groups or classes. The third machine counts the cards.[1]

[1] Information concerning these devices may be obtained from the Tabulating Machine Co., 111 Devonshire St., Boston, Mass. The author is indebted to this company for figures 5, 6 and 7.



(p. 55)

FIG. 8. — Card for Recording Data on Death Certificates.

EXERCISES AND QUESTIONS

1. Define the following statistical units as used by the U. S. Bureau of the Census.

a. A family.
b. A birth.
c. A death.
d. An infant.
e. A dwelling house.
f. A colored person.
g. A farmer.
h. A cotton-mill operative.
i. An urban community.
j. A rural community.
k. The population of a place.
l. Communicable disease.
m. Suicide.
n. Age.
o. A citizen.
p. An industrial accident.
q. A sleeping room.

2. Criticize the tables in the annual reports of any health department (as assigned by the instructor), as to title, form, boxing, abbreviations, etc.

3. Discuss the tables in the reports of the U. S. Bureau of the Census. Should they be taken as models?

4. Is it good form to use the following abbreviations?

a. "No. of Days," for Number of Days.
b. "Pop." for population.
c. "Av." for average.
d. "Ty. rate" for death-rate from typhoid fever.
e. "T. B. rate" for death-rate from tuberculosis.
What other ill-advised abbreviations have you observed?

5. In one ward of a city 517 births were reported, it being estimated, on the basis of past experience, that this figure was within 8 per cent of the truc number; in a second ward the report was 730 births, with an estimated error of 20 per cent; in a third the corresponding figures were 910 and 25 per cent; in a fourth, 604 and 18 per cent; what was the probable number of births in the city? And what was the probable percentage error of the total number of reported births?

6. If the death-rate in a certain city was 20 per thousand in 1910, if it decreased 10 per cent the next year, increased 10 per cent the year after, decreased 20 per cent the next year, increased 20 per cent the next year, what was the death-rate in 1914?

7. Multiply the following numbers by the arithmetic process, by the use of logarithms and by the use of the slide rule. Note the relative accuracies of the result.

a. 17 × 215.
b. 95 × 847.
c. 2161 × 1050.
d. 9230 × 40,373.
e. 10,072 × 736.
f. 54,672 × 93,721.
g. 4.7 × 1573.
h. 0.231 × 1.29.
i. 0.507 × 0.062.
j. 432.1 × 13.41.

8. Similarly perform the following divisions:

a. 342 ÷ 17.
b. 9467 ÷ 872.
c. 473,561 ÷ 2395.
d. 100,262 ÷ 730.
e. 0.517 ÷ 2.43.
f. 20,073 ÷ 98.
g. 763.05 ÷ 40.39.
h. 8999 ÷ 1101.
i. 30,500 ÷ 10.07.
j. 0.03 ÷ 76.

9. Given the following items: Find the mean, the median, the mode, the upper quartile.

a. 6, 7, 6, 2, 8, 4, 9, 6, 7, 2, 1, 2, 1, 9, 8, 7, 3, 6, 6.
b. 71, 3, 2, 0, 0, 1, 9, 5, 6, 3, 0, 2, 7, 7, 0, 4, 0, 2, 8.
c. 2, 12, 2, 14, 3, 13, 9, 16, 1, 0, 40, 90, 3, 22, 7, 15.

10. Arrange each of the sets of figures in the last question in groups as follows and find the average of each set from these groups.

	(1)	(2)	(3)	(4)	(5)
Group limits (inclusive)	0–4	5–9	10–14	15–19	20 and above
Number of items in group	....	....	....	....	

11. Find the arithmetic and geometric means of:

a. 71 and 19. *b.* 421 and 7. *c.* 21, 7 and 11.

CHAPTER III

STATISTICAL GRAPHICS

Use of graphic methods. — Statistics are numerical expressions of facts. When the facts are few in number it is not necessary to use figures to represent them, but as the number of facts becomes larger a point is reached where memory of individual facts must be supplemented by generalizing them, by letting a number stand for a class or a group of facts. In the same way when the numerical processes become complicated, when the figures become unwieldy or attain magnitudes beyond the ordinary range of familiarity, it is useful to resort to another process and represent the figures graphically. And even when the facts are few and simple their representation by diagram is often a distinct aid to the mind in grasping their meaning and fixing them in the memory.

There are two distinct uses of graphic methods and it is important to keep these in mind in preparing diagrams. The first use is for study. The relations between different groups, classes and series of facts can often be understood better from diagrams than from tables of figures. By the use of cross-section paper it is possible to interpolate values between plotted points, to generalize the facts of a series in which the data are more or less irregular, to extend plotted curves ahead of the data, thus enabling statistics to be used as a basis of prediction, to compare different curves and thus establish correlations. Properly used graphic methods will greatly assist the statistician in understanding his data.

It is a great mistake, however, to think that all statistics should be reduced to diagrammatic form, and it must be remembered that not one person in ten is able to read a complicated diagram understandingly. Some regard diagrams as puzzles to be worked out. To such persons diagrams are of little or no practical value.

The other use of graphic methods is for displaying the facts in such a way that they will attract attention, that the general results, regardless of details, will fix themselves in the memory. This use of graphic methods has greatly increased in recent years. We see diagrams of all kinds on bill-boards, in advertisements, in public health reports, in popular and scientific articles, even in moving pictures. The growing importance of the whole subject is shown by the recent publication of a notable book by W. C. Brinton[1] on Graphic Methods for Presenting Facts, which contains several hundred different kinds of graphic representations — a most useful book for statisticians to study.

Thus, on the one hand, we have the diagram forming a part of mathematics, and, on the other hand, we find it merging into the cartoon; hence we may lay down the general principle that graphic methods of depicting statistics must be selected according to the use to which they are to be put.

Types of diagrams. — The word *diagram* may be used in a generic sense to include all of the various kinds of mathematical graphs, plots, charts, maps and pictorial illustrations used by statisticians for the display or comparison of numerical data. These may be roughly classified as follows:

1. One-scale diagrams, in which different items are compared with each other on the basis of a single magnitude scale.

[1] See list of references in Appendix.

2. Two-scale diagrams, commonly known as graphs, in which two magnitudes are involved. One of these is commonly represented by a horizontal scale and one by a vertical scale. These graphs take many forms.
3. Three-scale diagrams. It is difficult to represent three dimensions on a flat sheet of paper, but it is sometimes done by the so-called isometric method.
4. Component-part diagrams, in which a single quantity is shown in sub-division.
5. Pictorial diagrams, or pictograms, a special form of the one-scale diagram used for display.
6. Statistical maps, or cartograms, a special form of the two-scale diagram, in which one scale is area arranged geographically, while the other consists of differently colored or shaded areas.

There are also many miscellaneous types of diagrams with specially devised irregular scales, logarithmic scales, probability scales, etc., and with one scale superposed on another. These are for study and not for display.

The appeal to the eye. — Diagrams are intended as an appeal to the eye, and advantage is taken of the ability of the eye to observe quickly and with fair accuracy:

(*a*) *Distances*, as, for example, the relative heights of different points above a base line or the relative distances of points from some other point or from some axis.

(*b*) *Areas*, as shown by comparison of similar figures, that is by circles, squares, rectangles or even irregular figures.

(*c*) *Volumes*, as shown by comparison of similar cubes, cylinders, spheres and irregular figures.

(*d*) *Ratios*, such as the relative lengths of parallel lines, areas or volumes similar in general shape.

(*e*) *Slopes*, or the relative inclinations of different lines from a base line.

(*f*) *Angles*, as shown by the sub-division of the 360 degrees about a point.

(*g*) *Shades and colors*, as shown by areas on pictograms and maps.

Graphical deceptions. — In preparing diagrams it is well to bear in mind that the eye may be deceived. There may be graphical fallacies as well as statistical fallacies. Some of these may be illustrated by well-known optical illusions.

In Fig. 9 the line *A* appears to be longer than *B*. In reality they have the same length. The shaded area *D* appears to be taller than *C*. In reality they have the same height. Astigmatism is also the cause of optical illusions. Those whose business it is to prepare diagrams for display should study these optical conditions.

But there are other and more important ways in which diagrams may deceive.

In pictograms we sometimes see two objects of different size — say two men, one large and one small, illustrating the relative numbers of persons who have died from two diseases. If the relative numbers are as 2 is to 1, the figures would naturally be drawn with the heights in that ratio. But to the eye the larger man would appear to be more than twice the size of the smaller one, because the eye would here judge not the height alone, but the whole area of the figure This very common fallacy in which one dimension is used for plotting, with no reference to the other dimensions which automatically changes, may be illustrated by the two circles *E* and *F*. The diameter of *F* is only twice that of *E*, but the circle *F* seems to be much more than twice as large as *E*. This fallacy may be called that of plotting by line and seeing by area.

Similarly when a polar diagram is made to illustrate

the seasonal distribution of some disease, the number of cases per 1000 persons being indicated by the distance of each plotted point from the center, an incorrect idea is obtained. In Fig. 21 the death-rate for April and May

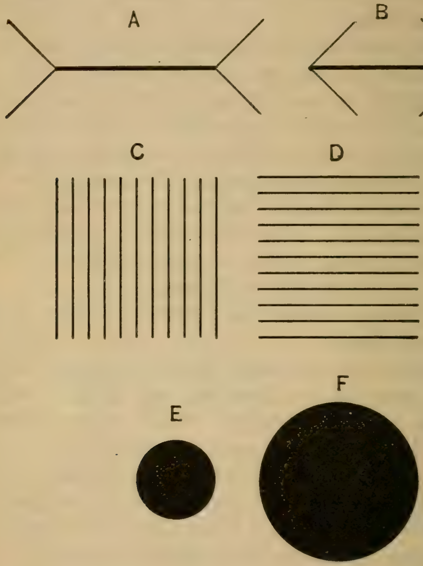


FIG. 9. — Optical Illusions.

was in reality only three times that for August and September, but from the diagram it looks to be more than three times as much. The reason is that the diagram was drawn as a line diagram, but the eye sees the area as well as the lines and the area embraced by the enveloping lines increases as the points become farther from the center.

Other fallacies connected with the choice of scales will be pointed out in the consideration of that subject.

Essential features of a diagram. — Every diagram, save the very simplest, should have a title; one or more scales, plainly indicated; a background of cross-section, or coördinate lines; the points, lines or areas representing the data plotted, marked for identification; and any necessary notes or explanations. As a rule diagrams should be self-contained, that is, they should tell the facts without regard to the accompanying text.

The title may be entirely outside of the frame of coördinate lines, with the idea that if the diagram is published the printer will set up the title in type. This simplifies somewhat the construction of the diagram, but if a lantern slide is made it may be that the printer's type will be found to appear disproportionately small. If the title is placed within the frame of coördinate lines these lines must be discontinued and not allowed to run through the letters of the title. On machine-ruled paper this rule cannot hold as the coördinate lines cannot be erased. It is possible to place the title on a piece of white paper and paste it over the cross-section lines. In the case of machine-ruled tracing cloth, the lines may be removed by the use of xylol, or gasolene, and a clear background obtained for the title.

In designing the title it is not necessary to use the words "Diagram showing the . . . " any more than it is necessary to say "Table showing the . . . ."

The size and shape of the diagram will depend in great measure upon the scales chosen, but as diagrams are very often reproduced, even though not drawn primarily for publication, it is always well to prepare them as if for publication.

For the purposes of a typewritten report, diagrams should be kept within the limits of a rectangle 7 by $9\frac{1}{2}$ in.

The standard typewritten paper is $8\frac{1}{2} \times 11$ in., but there should be margins of 1 in. on the top and left and $\frac{1}{2}$ in. on the bottom and right for binding and trimming. The paper containing the diagram should be cut $8\frac{1}{2}$ by 11 in. Larger diagrams may of course be desirable or necessary.

For reproduction most diagrams have to be reduced in size. When this is done the diagram as a whole is not only made smaller but the letters are made smaller and every line made thinner. Care should be taken therefore that the letters and figures used are not too small and that the lines are not too thin.

As a rule letters and figures should be so placed that they can be easily read from the bottom or the right-hand edge.

The coördinate lines are used to guide the eye and to enable one to read from the scale with accuracy and minuteness. For display purposes, however, no more coördinate lines should be used than are necessary, as too many are confusing. The coördinate lines should be lighter in weight than the plotted points or lines in order that the latter may stand out conspicuously.

Too many plotted lines should not be used in the same diagram as confusion may result. If there is more than one plotted line each should be clearly marked. This is especially important if the lines cross or meet at any point.

Often it is desirable to have the diagram include within its boundaries not only the graphic representation of the figures, but the figures themselves.

One-scale diagrams. — The simplest diagram is one where the magnitudes of the different items are represented by the relative lengths of lines or by narrow rectangles of constant width. They are easy to understand and are useful for many purposes. The magnitudes represented by the lines may be stated in figures or there may be a scale shown

for comparison. See Figs. 10 and 11. The lines may be drawn horizontally or vertically.

An important principle in line diagrams is that all of the lines should start from the same base. If this is not done comparison is difficult. In the case of Fig. 11, which shows the birth-rates and death-rates for two European countries,

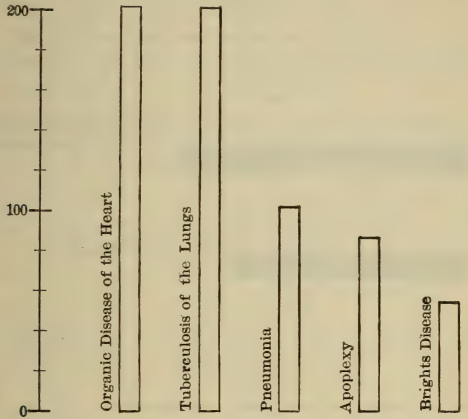


Fig. 10. — Numbers of Deaths from Five Most Important Causes. Cambridge, Mass., 1915.

it is easy to compare the births, shown by the total lengths, and the deaths, shown by the black, because they start from the left-hand line, but it is difficult to compare the natural increase of population in the two countries, shown by the white, because they have no common base. If the natural rate of increase is important it is better to use separate lines for births, deaths and increase as shown in Fig. 11, *b* and *c*.

It is also difficult to compare two lines which, though they have a common base, extend in opposite directions from the base. This, however, is often done with a fair degree of satisfaction. See Fig. 42.

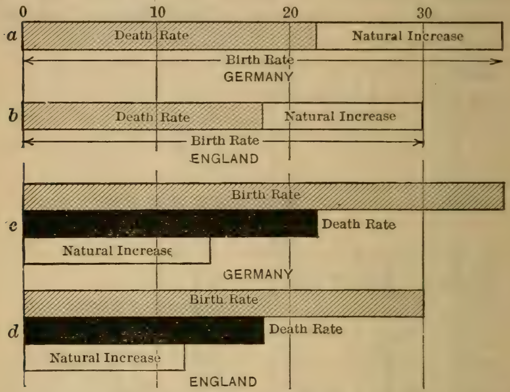


FIG. 11. — Comparison of Birth-rates, Death-rates and Rates of Natural Increase.

Diagrams with rectangular coördinates. — Most of the diagrams used to illustrate statistics are of the two-scale type. There is a horizontal scale with magnitudes increasing from left to right and a vertical scale with magnitudes increasing from bottom to top. It is customary also to rule in a sort of checker-board consisting of parallel vertical and horizontal lines to guide the eye in following the scales across the paper. To further assist the eye heavy lines are used for the round numbers of the scale and finer lines for sub-divisions. It is good practice also to always use for the zero line a line as heavy as the plotted line. Usually this would be the bottom line and the left-hand

line. If there be no zero, as in the case of a scale of years, the heavy line would not be used. In the case of percentage diagrams both the zero per cent line and the hundred per cent line should be heavy. The numerical values for the sub-divisions of the scale are shown in figures, preferably at the bottom and left side of the diagram. Sometimes they are placed also at the top and right. Thus the zeros of both scales are supposed to be at or near the lower left-hand corner; but circumstances may compel some different arrangement.

In diagrams of this kind *time*, whether in years, months or days, is generally expressed by the horizontal scale and always runs from left to right. Such diagrams are sometimes called historigrams, sometimes merely graphs. The distances measured along the vertical scale are known to mathematicians as *ordinates*, the distances on the horizontal scale as *abscissæ*.

There are several ways of plotting with two scales. One way is to use the vertical scale as a measure of the length of certain vertical lines, each of which represents the magnitude of an item, and to use the horizontal scale to indicate the occurrence of the item. Thus in Fig. 12 we have a daily record of the rainfall for one month. Each rainfall is represented by a line of appropriate length, the position of the line showing when the rain occurred. This method is especially adapted to events which occur intermittently, and without regular gradations, that is to discrete series.

The rainfall data might have been indicated by dots, or crosses placed at the tops of the lines, the latter being left out. This would be misleading, however, unless similar dots or crosses were placed on the zero line for the days of no rainfall. This would not look well, and it is never done.

The vertical line method or *ordinate plotting* is sometimes

used for plotting data in series, the horizontal scale representing time. Thus we may compare the death-rates for different years by a diagram such as that shown in Fig. 13 *A*. This, however, is a continuous series and may be plotted

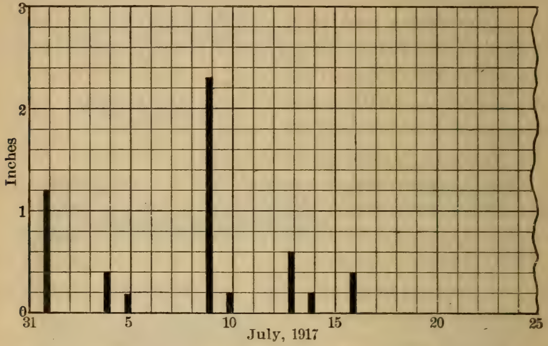


FIG. 12. — Example of Plotting a Record of Rainfall.

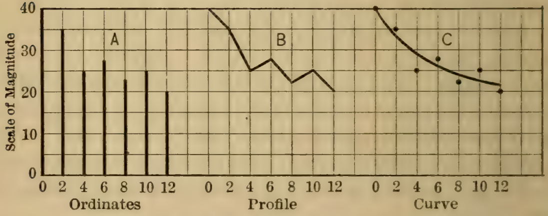


FIG. 13. — Example of Simple Plottings.

as a broken line, known as a profile line, which shows continuity. See Fig. 13 *B*. For most purposes this profile method is to be preferred to the vertical line method, but the latter is perhaps understood better by persons not familiar with graphic methods.

Still another way would be to plot the data as dots, or crosses, and draw a smooth curve through them to show the trend of events. This implies that the data are subject to errors and that the smooth curve gives a better picture of the true events. See Fig. 13 *C*. The art of smoothing curves is described in most books on statistical technique. In general it may be said that the rules usually laid down are based on the laws of probability.

Use of the horizontal scale. — In the illustrations just given the divisions of the horizontal scale were taken to be definite points of time, namely days and years, each point being plotted directly on a vertical line. This does very well for plotting yearly records which run on continuously, and there is no objection to the method for practical purposes. It is not, however, strictly accurate, for a year is not a point of time, but an interval of time. It is the space between the lines, which represents the year, the vertical lines marking the boundaries. Graphs are sometimes made on this basis.

Let us plot the following numbers of deaths which occurred in the different months of a single year.

TABLE 13

NUMBER OF DEATHS: EXAMPLE FOR PLOTTING

Month.	Deaths.	Month.	Deaths.	Month.	Deaths.	Month.	Deaths.
(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Jan.	40	Apr.	27	July	20	Oct.	17
Feb.	30	May	23	Aug.	17	Nov.	20
Mar.	25	June	25	Sept.	15	Dec.	25

Here the problem is to divide the horizontal scale, which represents a year, into twelve parts, each of which represents a month, and plot one point for each month. Now we get

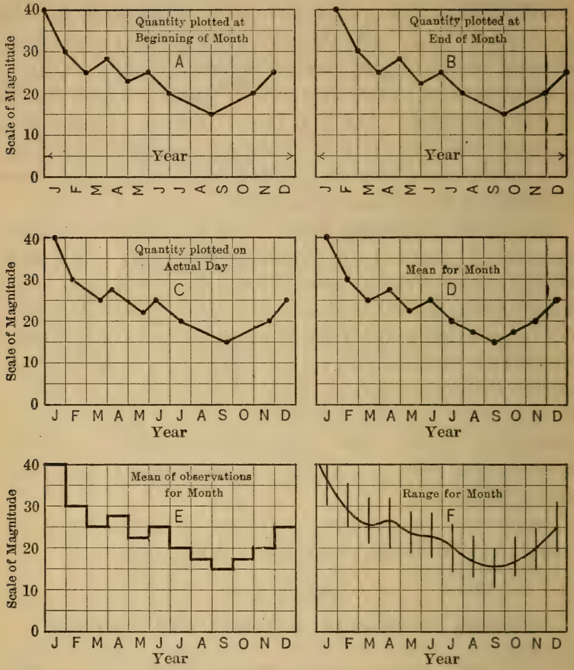


Fig. 14. — Examples of Time Plotting.

into trouble if we plot the data on the lines, for we might do this in two ways, as in Fig. 14, *A* and *B*. In one case we plot the point at the beginning of the month, in the other, at the end of the month. Of the two the latter is to be preferred. It would be more logical to let each month be represented by the space between the lines and to plot the points in the middle of the spaces as in Fig. 14, *C* or *D*. If the figures plotted represent the monthly averages of

several items occurring in each month the method of plotting shown in Fig. 14 *E* is a proper one. Fig. 14 *F* shows how one may plot the mean as well as the maximum and minimum item for each month. At present there is no well-established custom in regard to these methods. Plotting on the line is usually followed simply because it is easier and makes a neater diagram. Its illogical character seldom causes serious misunderstandings.

Plotting figures by groups. — The plotting of individual observations is comparatively easy; but it is difficult to decide how to plot the totals and means of groups, and still more difficult if the groups are irregular. This can best be appreciated by an example. Let us undertake to plot the following data:

TABLE 14

DATA TO BE PLOTTED

Age (last birthday).	Number of cases.	Age group.	Number of cases in group.	Average number of cases for each year.
(1)	(2)	(3)	(4)	(5)
0	1			
1	2			
2	2	0–4	12	2.4
3	4			
4	3			
5	1			
6	4			
7	3	5–9	15	3.0
8	5			
9	2			
10	6			
11	4			
12	7	10–14	25	5.0
13	5			
14	3			
15	4			
16	6			
17	5	15–19	20	4.0
18	3			
19	2			

If we plot the individual items we have the result shown in Fig. 15 *A*. If we plot the total numbers of cases in each group we may do so by the methods *B*, *C*, or *D*. In these the horizontal scale represents not individual ages, but groups. We may indicate this fact by using the hyphens as shown. In *B* we have plotted the figure 12 on the line which indicates the maximum limit of the group 0–4, 15 on the line which indicates the maximum limit of group 5–9, etc. In *C* we have plotted 12, 15, etc., in the middle of the spaces which represent the groups. In *D* the height of the horizontal line above the base is taken to represent the total and extends across the group limits. If we wish to show both the individual observations and the means for the groups we may plot as in *E*.

In plotting by groups care should be taken to make it clear that the horizontal scale stands for groups and that the vertical scale stands for the number in the group.

Plotting irregular groups. — Let us now take the case of irregular groupings. Assume the following data:

TABLE 15

DATA TO BE PLOTTED

Age group.	Number of cases in group.	Average for each year in group.
(1)	(2)	(3)
0– 4	4	0.8
5– 9	6	1.2
10–14	8	1.6
15–19	6	1.2
20–29	7	0.7
30–39	5	0.5
40–59	8	0.4
60–79	6	0.3
80–99	3	0.15

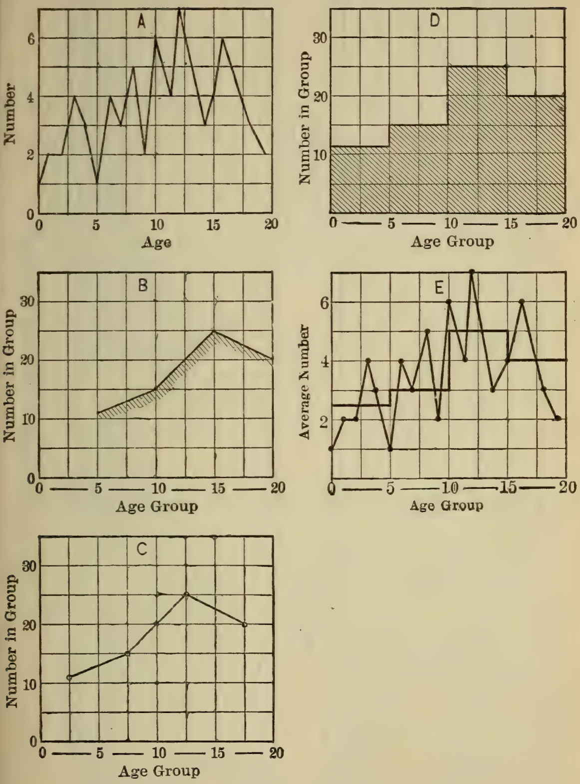


FIG. 15. — Examples of Age Plotting.

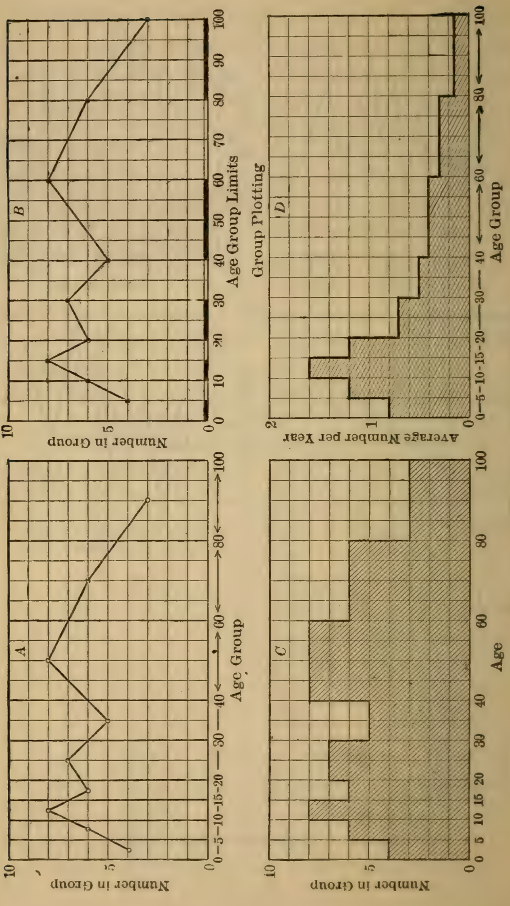


FIG. 16. — Examples of Group Plotting.

In the first place we must find some way to indicate to the eye the varying intervals of the group. The first four groups cover five years, the next two ten years and the last three twenty years each. We might do this as in Fig. 16 *A*, in which the heavy vertical lines indicate the group limits. In *B* the coördinate lines are regular and the group limits are shown by the emphasized horizontal scale. In *C* the blocks indicate the group limits. Not one of these, however, gives an adequate picture of the distribution of the cases according to age, because the groups are not uniform. All three diagrams are fallacious because the ordinates are not strictly comparable. The best way to show distribution by age is to make the groups comparable by reducing all to a common denominator. This can be done by finding the average number of cases for each year in the group. The results are shown in Fig. 16 *D*. Here the irregular grouping on the horizontal scale is maintained, yet a good idea is given of the distribution of the cases according to age.

Summation diagrams. — For many purposes it is desirable to plot the results obtained by the successive summation of the items in preceding groups. This gives what are called summation diagrams, cumulative plots, mass plots or mass curves. This may be illustrated by the data on p. 77. These data are plotted in Fig. 17. Sometimes instead of connecting the plotted points by straight lines a curved line passing approximately through them is sketched in. It should be noticed that in this diagram the horizontal scale stands for age and not for age-groups.

One use which can be made of a plot of this kind is to find the median of the series. There are 53 cases in all. The middle one is the 27th. From the scale this item has a value of 24 years, as shown by the cross. In the same way the quartiles may be found and the decentiles.

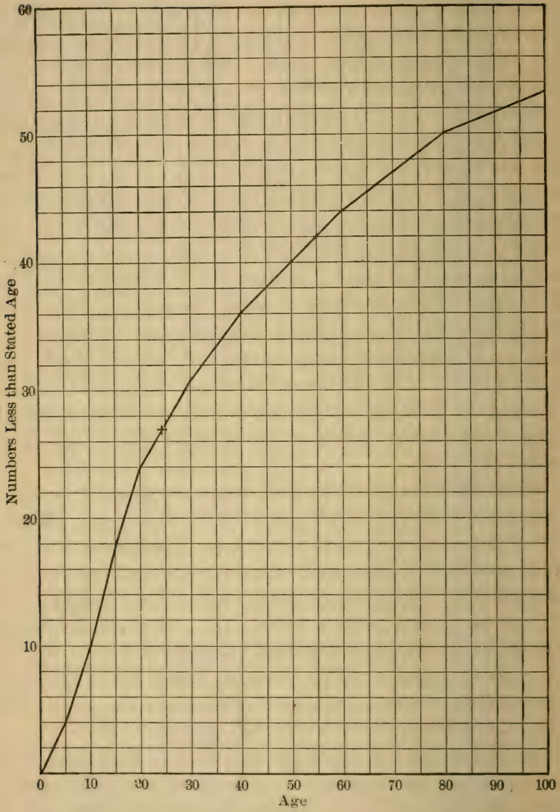


FIG. 17. — Example of Cumulative, or Summation Plotting.

TABLE 16

DATA TO BE PLOTTED

Age-group.	Number of cases.	Summation group.	Number of cases.	Less than age.
(1)	(2)	(3)	(4)	(5)
0–4	4	0–4	4	5
5–9	6	0–9	10	10
10–14	8	0–14	18	15
15–19	6	0–19	24	20
20–29	7	0–29	31	30
30–39	5	0–39	36	40
40–59	8	0–59	44	60
60–79	6	0–79	50	80
80–99	3	0–99	53	100
Total	53		53	

Another use is that of redistributing the cases according to a different age-grouping. Let us suppose that we desire to find the number of cases between the ages of 35 and 45, *i.e.*, in age-group 35–44. From the vertical scale and the plotted curve we find that there are 38 cases below age 45 and 33 cases (approximately) below age 35, hence there are 38 − 33 = 5 cases in age-group 35–44. This principle may be usefully applied in redistributing the population of a city into age-groups in connection with the computation of specific death-rates.

Choice of scales. — The choice of both scales is a matter of great importance, for it not only influences the size and shape of the diagram, but controls the slopes of plotted lines and the apparent differences between plotted points. In Fig. 18 we have the death-rates of Moscow from 1881 to 1910 plotted by five-year groups according to two different scales. The two diagrams look to be quite different, and *B* gives the impression of a greater decrease in rate

than A because on account of the greater vertical scale and the smaller horizontal scale the slope of the plotted line is more.

Sometimes for purposes of comparison two lines are plotted on the same sheet, each having its own vertical scale. Here the choice of the proper scale is all-important.

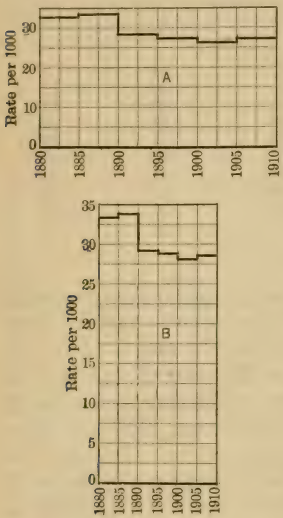


Fig. 18. — Death-rates: Moscow, 1881–1910. Showing Effect of Changing Scales.

It sometimes happens that in order to show the desired variations in a series of plotted ordinates a scale must be chosen so large that the zero point would fall too far below the plotted point to have it appear on the diagram. Right here lurks a graphical fallacy which may be serious. It is best appreciated by studying an actual illustration.

Fig. 19 shows the general death-rate and the tuberculosis death-rate per 1000 inhabitants in Boston, Massachusetts, from 1881 to 1911, the figures being plotted in five-year groups. In A different scales are used and the scales do not extend to zero on the base line. In B the same scale is used for both series of items. From diagram A one would get the idea that the tuberculosis rate was decreasing much faster than the general death-rate, but from diagram B the opposite idea would be obtained.

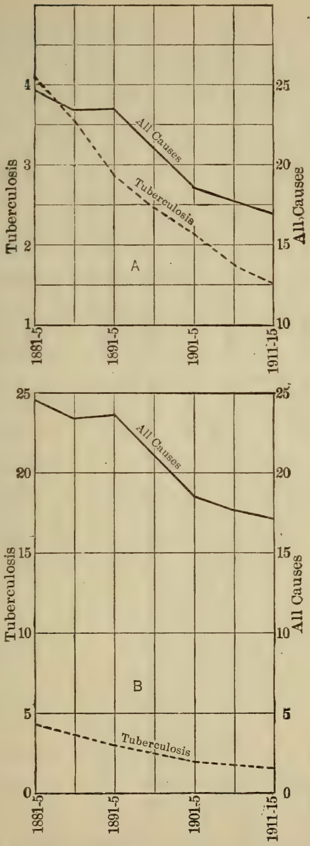


FIG. 19. — Comparison of Deaths from Tuberculosis with Deaths from all Causes: Boston, Mass. A, Incorrect Method. B, Correct Method.

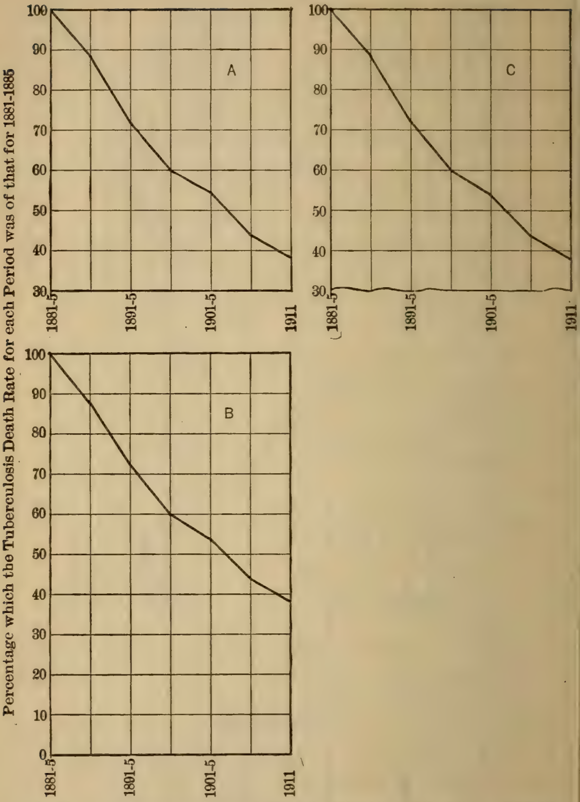


FIG. 20. — Example of Not Carrying Scale to Base Line. Tuberculosis Death-rate: Boston, Mass.

Fig. 20 shows the reduction of the tuberculosis death-rate in Boston expressed in terms of the percentage which the death-rate of each period was of that for the period 1881 to 1885. In *B* the vertical scale is carried down to 0 per cent at the base line. This gives a true picture of the reduction which has taken place and the death-rate remaining. In *A* the vertical scale is not carried to the base line and the diagram gives the optical impression that the reduction has been greater than it actually has been and that the rate at the end of the period was very much less than at the beginning. Brinton has suggested that when the base line does not represent the zero of the vertical scale it should be drawn as a wavy line instead of a straight line, and this idea has much merit. Where two different vertical scales are used, and one goes to zero at the base line while the other does not, the wavy line may extend only half way across the diagram from that side of the diagram where the scale does not go to zero. *C* in Fig. 20 illustrates the appearance of a diagram drawn in this way. The wavy line implies that the lower part of the diagram is omitted.

Diagrams with polar coördinates. — Fig. 21 illustrates a diagram with the ordinates represented by distances from a central point along radial lines, the abscissæ, if we may use the term out of its place, being represented by the angle which the ordinate makes with the vertical measured clockwise around the circle. This form of plotting has a limited application and because of its inherent fallacious character should be abandoned.

Double coördinate paper. — Sometimes it is convenient to use what may be called double coördinate paper. This is illustrated by Fig. 22. Here the plotted line may be read against either set of coördinates. The horizontal lines give the number of deaths from typhoid fever, the scale being at the left. The inclined lines give the death-

rate per 100,000. Thus in 1900 the number of deaths was about 305, the death-rate about 27 per 100,000. The slope of the inclined lines depends upon the increase in population. The black inclined line represents population and this may be read for the censal years from the right-hand scale. It will be seen that the ratio between

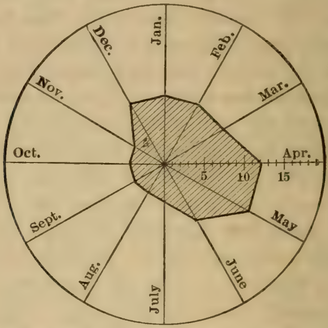


FIG. 21. — Example of Radial Plotting.

the right-hand and left-hand scale for any horizontal line gives the rate for the heavy line, *i.e.*, 200 ÷ 1,000,000 = $\frac{20}{100,000}$, or 20 per hundred thousand. So also 100 ÷ 500,000 = 20 per hundred thousand. Any point on the heavy line, therefore, gives a rate of 20 per hundred thousand. The rate line for 10 per 100,000 is one-half way to the line between the heavy line and the zero or base line, on each vertical line which represents a census. The rate line of 30 is, on each vertical, as far above the black line as the rate line of 10 is below it. And so on.

In the example chosen the typhoid fever rate in Brooklyn has fallen since the date of the last plotting, *i.e.*, 1906.

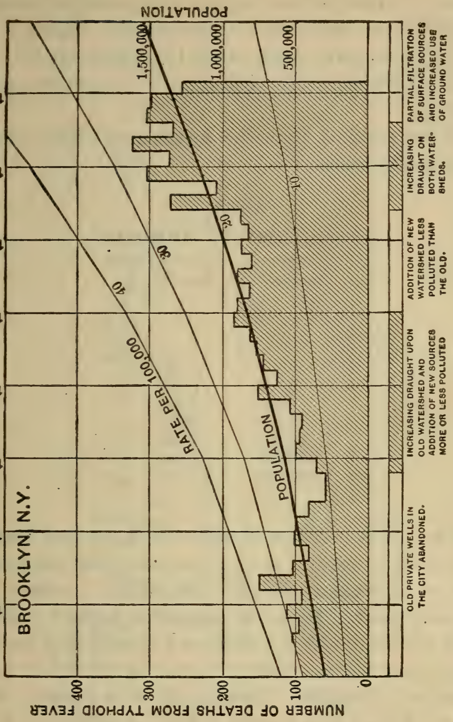


FIG. 22. — Diagram Showing the Number of Typhoid Fever Deaths and the Corresponding Death-rates in Brooklyn, N. Y.

Ratio cross-section paper. — Thus far we have been dealing with regular scales in which the intervals are uniform from one end to the other. It is possible to construct scales with intervals which are not uniform, but which vary in a systematic way. These are used for special purposes. The most common scale of this kind is the logarithmic scale.

Diagrams in which the vertical scale is logarithmic and the horizontal scale uniform are sometimes called "ratio charts." These have been used by engineers for many years, but they are only beginning to be appreciated by statisticians.

It will be recalled that the logarithms of the decimal numbers are as follows:

TABLE 17

LOGARITHMS OF NUMBERS

Number.	Logarithm.
(1)	(2)
1	0.000
10	1.000
100	2.000
1,000	3.000
10,000	4.000
100,000	5.000
1,000,000	6.000

As each number increases tenfold the logarithm increases by one; and in general it may be said that as numbers increase at a regular *rate* the logarithms increase by a regular increment. From the logarithm tables[1] it may be seen that the log of 10 is 1.0000, and that if 10 is increased by 25 per cent and becomes 12.5 the log of 12.5 is 1.0969, an increment of 0.0969. The log of 50 is 1.6990. 50 increased by 25 per cent is 62.5. The log of 62.5 is 1.7959, an increment of 0.0969 as before. The log of 1570 is 3.1959. 1570 increased by 25 per cent is 1962.5 and the log of this is 3.2928, an increment of 0.0969 as before. If, using a uniform scale, we plot figures which increase at a constant rate we shall get a curve as shown in Fig. 23 *A*. Let us

[1] See Appendix.

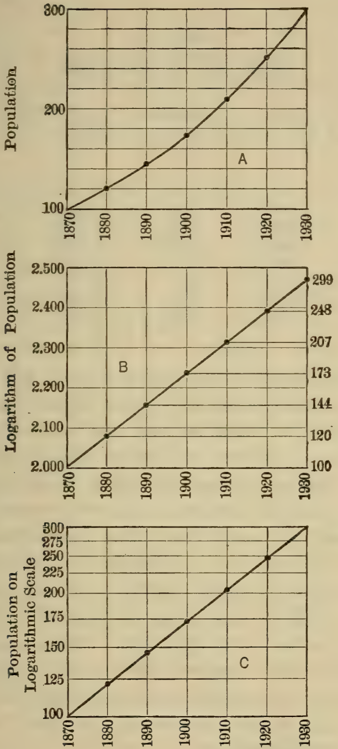


FIG. 23. — Example of Logarithmic Plotting.

start with a population of 100 in 1870 and assume an increase of 20 per cent each decade. We then have the following:

TABLE 18

DATA TO BE PLOTTED

Year.	Population.	Log of population.
(1)	(2)	(3)
1870	100	2.0000
1880	120	2.0792
1890	144	2.1584
1900	173	2.2380
1910	207	2.3160
1920	248	2.3945
1930	299	2.4757

The figures in column (2) are plotted in *A*. If we plot the logarithms of the numbers in column (2) we have a straight line as in *B*. This being so, why not label the horizontal lines with the numbers in column (2) instead of their logarithms? This is done at the right of the diagram. It will be seen that the vertical scale is not made up of uniform intervals, but aside from that fact it is a perfectly good scale. In *C* we have a diagram in which the vertical scale (represented by the horizontal lines) is drawn on this basis, and it will be seen that the figures in column (2) plotted on it fall in a straight line. This is a single logarithmic, or in simpler words, a ratio chart. Figures increasing at a constant rate plot out as a straight line on paper thus ruled, *i.e.*, with a uniform horizontal scale and a logarithmic vertical scale.

There are two uses for single logarithmic paper. One is to show variations in rate. If we plot the population of the United States on ordinary cross-section paper with

uniform scales we obtain an ascending curve, but from this we get no idea of the constancy of the rate of increase. This is shown in Fig. 24 *A*. But if we use ratio cross-section paper, as in *B*, we find that the rate of increase was constant from 1790 to 1860, but that since the Civil War the rate has been nearly constant yet not as great as before. On this paper equal slopes mean equal rates of increase, while on uniform paper equal slopes mean equal increments.

Another use is that of enabling us to plot on one sheet observations which cover a very wide range. If we were using a uniform scale to plot such figures we should have to make the scale so small that individual differences between the small numbers could not be discerned. It will be noticed that on the ratio paper the intervals for the small numbers are larger than for the high numbers, so that if plotted on this paper we can still read differences in the lower part of the scale. The upper part of the scale is foreshortened. In fact we can discern the same percentage differences in all parts of the scale.

Logarithmic cross-section paper. — By logarithmic cross-section paper we usually mean paper on which both the horizontal and the vertical scales are logarithmic. Here the ratios are in both directions. It will be observed that the interval from 1 to 10 is the same as that from 10 to 100, from 100 to 1000 and so on. One objection to the logarithmic scale is that it does not go to zero. The interval below 1 runs from 1 to 0.1, the next from 0.1 to 0.01, the next from 0.01 to 0.001 and so on.

This paper is very largely used in scientific work, but its use for statistical purposes is somewhat limited.

Ruled paper. — It is not difficult to rule your own cross-section paper, although it is tedious work. Many sorts of ruled papers are on the market and can be purchased

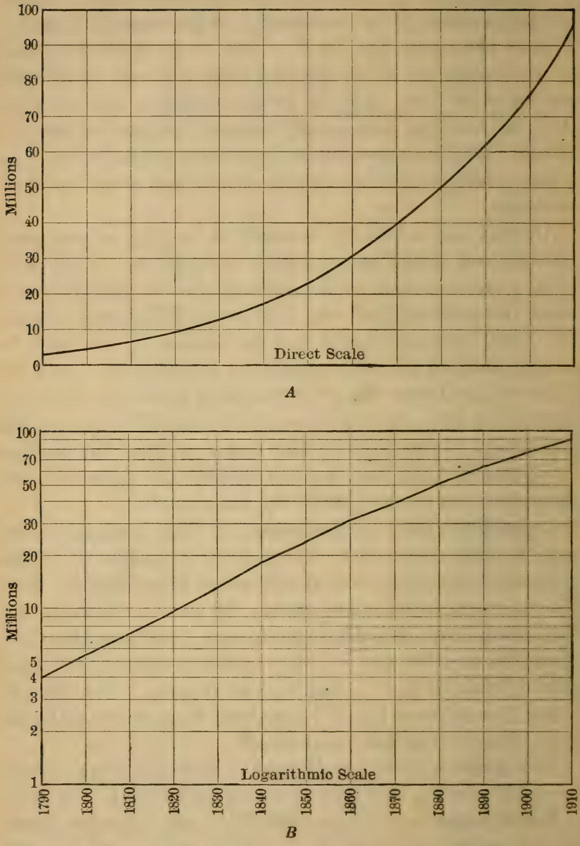


FIG. 24. — Population of the United States shown by Direct and Logarithmic Plotting.

from dealers in engineering drawing materials. The following scales are convenient for ordinary work:

(*a*) Inches subdivided into tenths in both directions.
(*b*) Half inches subdivided into tenths in both directions.
(*c*) Inches subdivided into tenths in one direction and into twelfths in the other direction, — useful for plotting data for the twelve months of a year.
(*d*) Ratio paper, with inches subdivided into tenths in one direction, and with a logarithmic scale from 1 to 10,000 in the other direction.
(*e*) Arithmetical probability paper.
(*f*) Logarithmic probability paper.
(*g*) Paper with horizontal scale ruled for the calendar year, and vertical scale in inches subdivided to tenths.

It is possible to buy tracing cloth ruled in cross-section form, but the kinds of ruling are limited. Such cross-section tracing cloth is sold by the yard, width about 26 in., and may be cut to sheets of desired size.

Mechanics of diagram making. — For making diagrams it is advisable to provide a regular draughtsman's equipment. This should include:

(*a*) A drawing board of appropriate size. For small diagrams a size of about 12 in. by 17 in. is satisfactory.
(*b*) A tee-square long enough to extend across the drawing board.
(*c*) A 30-degree triangle, 10 in. long, celluloid.
(*d*) A 45-degree triangle, 6 in. long, celluloid.
(*e*) A lettering triangle, to give slopes for letters.
(*f*) A ruling pen.
(*g*) One or more scales, steel, celluloid or boxwood, variously ruled in tenths, quarters, etc.
(*h*) Black drawing ink (Higgins).

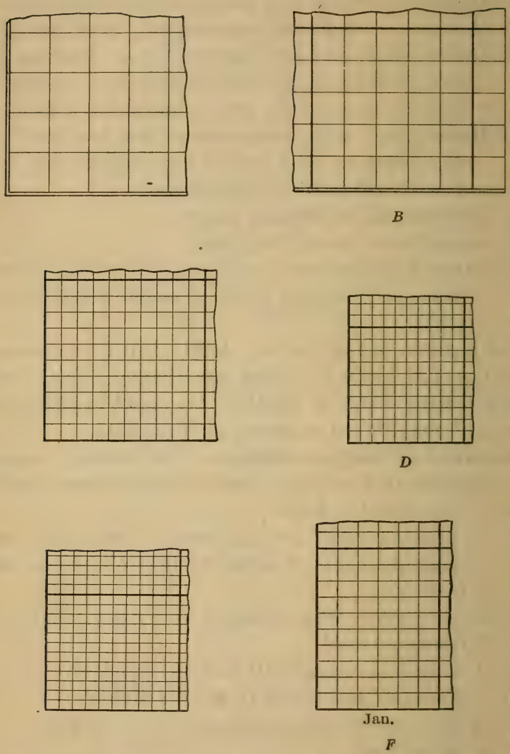


Fig. 25. — Examples of Plotting Paper. Sheets $8\frac{1}{2} \times 11$ inches.

(*i*) Thumb tacks.

(*j*) Brown "detail" paper.

(*k*) Tracing cloth.

Other equipment may be needed according to the nature of the work.

Lettering. — There is much truth in the statement that good letterers are born and not made. Yet it is surprising how much one can improve in lettering by giving attention to a few guiding principles.

For most diagrams it is best to adopt a very simple style of letter. Shaded letters look well on maps, but are out of place on line diagrams. The two styles shown in Fig. 26 are suitable for ordinary work. The choice of a vertical letter or a sloping letter is largely a matter of taste. Most people are more successful with sloping letters. They can be made a little more rapidly, but they are perhaps a little more informal than vertical letters.

It is important that letters appear to be uniform in height and slope. It is well to use guides both as to height and slope. Letters should also appear to be spaced uniformly. The curves of such letters as C, G, O and S should extend slightly above and below the horizontal guide lines. Adjacent straight-line letters such as N, I, U, M, etc., should be spaced a little farther apart than curved letters. Attention should be given to the manner of making the strokes as shown in the plate.

The student should consult a book on lettering such, for example, as that of Reinhardt.

If the title is inset it should be carefully placed. In general the lower right-hand corner is the best place for it, but often its location is governed by available space. The sizes of letters used should follow the important words. Each line should be centered. Write each line on a scrap of paper: count the letters in it: find the middle letter:

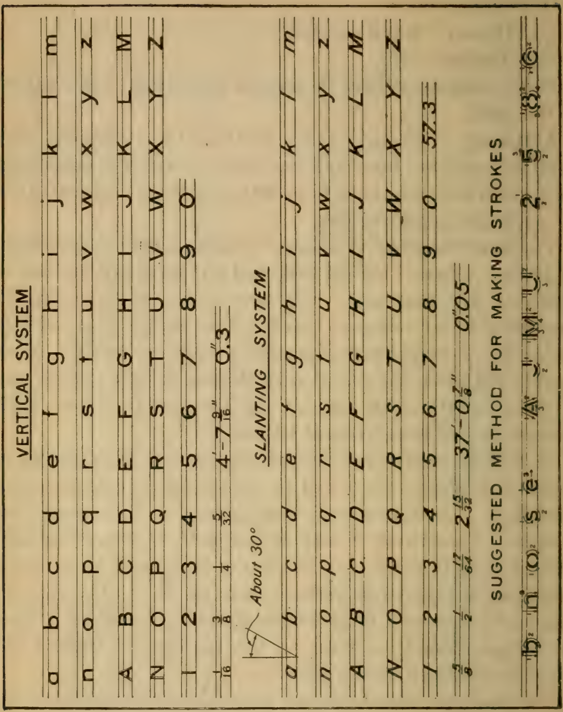


FIG. 26. — Lettering.

put that down first and then letter backwards and forwards. Capitals may be used for the principal lines of a title. In a general sort of way try to arrange the lines so that a line circumscribing the title will be approximately an ellipse.

Label each scale, except that it is unnecessary to do so in the case of months and years. Do not use abbreviations.

If there is more than one plotted line label each one. Be free in the use of explanatory notes. A diagram should tell its own story. In doing this use letters of readable size. It is a good rule never to make a letter or a figure less than $\frac{1}{8}$ inch in height.

Somewhere on the sheet, but outside of the diagram itself, should be placed the initials of the person who made the diagram and the date. This is valuable for identification, but it need not be published.

Wall charts. — Wall charts are much used nowadays in the display of vital statistics. It is not difficult to prepare these, but certain general principles should be kept in mind. They should be simple and clear, of ample size and plainly lettered. If intended to be seen from a distance the letters should be large and the lines heavy. As lettering forms an important part of a wall diagram it is well to know that gummed letters of all sizes can be purchased. Examples of these letters are shown in Fig. 27.

The use of color in diagrams. — Colored lines should be used sparingly if the diagrams are to be published. A sheet must go through the press once for each color and this adds to the cost. The most effective use of color is where a single colored line is made to stand out in contrast to other black lines, and for this purpose red is the best. Color on plotted lines may be avoided by using black lines made in different ways. The following are easily distinguishable:

1. Heavy full line
2. Light full line
3. Heavy broken line
4. Light broken line
5. Dotted line
6. Dot-dash line

For wall charts or posters intended to be viewed from a distance, colors are justifiable.

The cross-section lines on the ruled paper ordinarily sold are colored green or brown or light red. Very bright colors

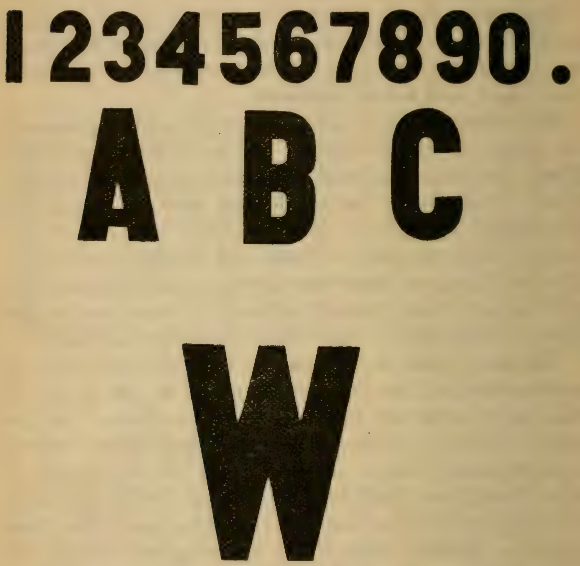


Fig. 27.—Examples of Gummed Letters, Useful for Wall Diagrams.

used for this purpose are exceedingly trying on the eyes. It is desirable however to have a color which can be photographed and also blue-printed. Green is not satisfactory from these points of view. Dull red is much better. Vermilion red should be used, not carmine.

Component part diagrams. — In order to show the component parts of a total number we may subdivide a line or a long rectangle and label each part, or we may subdivide an area, as a square or a circle, indicating differences by

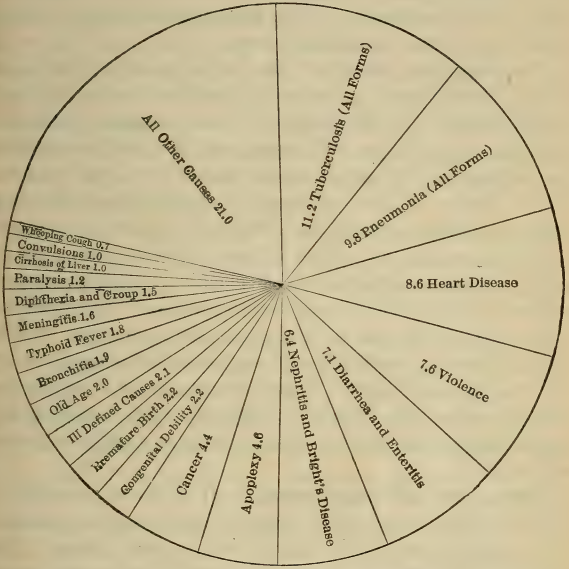


FIG. 28. — Proportion of Deaths from Each Specified Cause in the U. S. Registration Area: 1907.

colors, shades or patterns as in cartography. A circle properly subdivided is perhaps the best type of diagram to show percentages. Here the sectors plainly show the desired differences. This sort of a diagram is not to be confused with plotting by polar coördinates. (See Fig. 28.)

Statistical maps. — The object of statistical maps is to display classes and groups of statistics for different areas. It will be remembered that statistical *classes* involve differences which cannot be expressed in figures, but that statistical *groups* contain facts similar in kind but which differ from each other numerically. This difference should be kept in mind in preparing statistical maps.

The statistical data are shown on maps by different colors, by different patterns of lines and dots or by surface shadings. In the display of data arranged in groups, that is, in accordance with magnitude, it is well to indicate the differences by variations in shade from light to dark. In the display of data arranged by classes it is well to use different patterns or colors. Different shades may be obtained by successive washes of color applied with a brush, or by the use of cross-hatching in which the proportion of surface covered with ink regularly increases. The so-called "Ben Day" system of indicating shades by the use of special devices is well known to printers and engravers.[1]

Sometimes the figures themselves are placed on the maps. If this is done care should be taken to make sure that the boundaries of the areas to which the figures apply are properly defined.

Blue prints and other prints. — It is often desirable to obtain several copies of the diagrams made, and the quickest and cheapest method is that of making blue prints. The process is the same as that of making photographic prints from a negative. Blue-print paper can be purchased; in fact, it can be easily made. A large photographic printing frame is required. The diagram is placed in the frame over the blue-print paper and exposed to the sunlight for a few minutes, after which the paper is washed in water and dried. It is necessary, of course, to have the paper on

[1] See Brinton's Graphic Methods, pp. 216, 233.

which the diagram is made fairly thin and transparent. Paper should be selected with blue-printing in mind. The transparency of paper can be greatly increased by oiling it on the back after the diagram is made. A liquid sold under the name of "transparantine" is satisfactory. The best blue prints of diagrams are obtained by the use of tracing cloth. This has many advantages. It is easy to ink on and erasures may be made. The lines are sharp and photograph well. The cloth does not tear. The cloth is oiled on one side. The drawing should be done on the other. A little powdered chalk should be dusted on and rubbed off before using ink. Pencil lines may be used as guide lines and erased before blue-printing.

In the ordinary blue print the lines are white and the background blue. Additional white lines can be drawn on the blue by using a weak solution of caustic soda in a pen as ink.

It is possible to obtain prints in which blue or brown lines appear on a white ground. This requires the making of a negative, from which subsequent prints are made.

Reproduction of diagrams. — The common method of reproducing diagrams for publication is to photograph them and print from a zinc plate. This is the cheapest and most available method. It is necessary that the original drawing be well made, with lines of the right weight and the letters of the right size. All imperfections are of course reproduced. Usually the drawing should be made at least fifty per cent larger than the published plate, that is, the size is reduced one-third. To have diagrams made by a draughtsman costs something, but, if the photographic process is to be used, it is worth while. The draughtsman should know what the size of the published plate is to be.

Those not skilled in making diagrams ought to know that there is another process of reproduction which does not

require a carefully drawn original, namely, that of wax engraving. In this process the engraver does the work of the draughtsman. A copper plate is used. The lettering in this process can be put in with type. This results in perfect legibility, which is often not the case with photographic work. Reproduction by the wax process costs almost twice that by the photographic process, but if to the latter is added the time and expense of preparing a perfect original the wax process costs no more. Most of the plates in this book were made by the wax process by the L. L. Poates Company of New York. Unfortunately there are not many wax engravers in this country.

Equation of a curve. — Having plotted certain data on rectangular coördinate paper, that is, using a horizontal and a vertical scale, and finding that the points fall on a straight line or on a regular curve, it is sometimes desirable to find the equation of the straight line or curve. This is not difficult, but it requires the use of mathematical principles not considered in this book. The reader is referred to such books as Saxelby's "A Course in Practical Mathematics" [1] or Peddles' "Construction of Graphical Charts." [2]

EXERCISES AND QUESTIONS

1. Describe Ripley's method of preparing statistical maps with different shadings. [Pub. Am. Sta. Asso., Sept. 1899, pp. 319–322.]

2. Construct a graph of the birth-rates and death-rates of Sweden from 1749 to 1900. (See p. 203.)

3. Construct a graph of the natural rate of increase of the population of Sweden from 1749 to 1900.

4. Show by suitable diagrams the data in Tables 100, 106 and 110.

5. Find diagrams in this book which do not conform to the principles described in Chapter III.

[1] Pub. by Longmans, Green & Co., 1908.

[2] Pub. by McGraw-Hill Book Co., 1910.

6. Construct a "devil's checker-board," as follows:

a. Take a piece of cardboard or heavy drawing paper and rule in black ink a rectangle $8\frac{1}{2}''$ wide and $11''$ high. Rule also a horizontal line $1''$ below the top, and a vertical line $1''$ from left-hand edge, in order to leave suitable margins at top and left.

b. Subdivide the $7\frac{1}{2}''$ on the horizontal line into 15 half-inch spaces and rule vertical lines. Subdivide the $10''$ on the vertical line into 40 quarter-inch spaces and rule horizontal lines.

c. Draw in red inclined lines sloping downward to the left, being $\frac{1}{2}''$ apart in a horizontal line and $1\frac{1}{4}''$ apart in a vertical direction.

If the work is done accurately certain of these diagonals will intersect corners of the small rectangles; if the work is not accurate the name of the problem is justified. These guide lines will be found convenient in the construction of tables. The sloping lines will serve as guides for sloping letters.

7. Construct a colored wall chart showing the death-rates from several diseases for some city, using the one-scale type of diagram. Assume the chart is to be read from a distance of twenty feet.

8. Describe the method of construction and the varied uses of ratio cross-section paper. (Quar. Pub. Am. Sta. Asso. June, 1917, p. 577.)

9. Plot the population of some city (assigned by the instructor) using ordinary cross-section paper and ratio paper.

10. Construct a colored component-part diagram (subdivided circle), showing the composition of the population of some city or state (data assigned by the instructor).

CHAPTER IV

ENUMERATION AND REGISTRATION

All civilized nations at regular periods enumerate their populations, that is, take a census. There are various governmental reasons for doing this, two important ones being the adjustment of representation in legislative bodies and the levying of taxes. There are also business, social and sanitary uses to which the figures are put. In considering a census several questions immediately arise; when was it made, what area was included, how were the data obtained, what were the results and where may they be found?

The United States census. — The first general census of the United States was made in 1790, the first year divisible by ten after the founding of the new republic, and a census has been taken every ten years since that date, the census of 1910 being the thirteenth.

The first twelve censuses were made by special commissions created for the purpose and which went out of existence as soon as the task had been accomplished. A permanent Bureau of the Census was created in 1902. At first it was under the Department of the Interior, but in 1903 was transferred to the Department of Commerce and Labor. Its head is known as the Director of the Census. Besides taking the general census of the country every ten years this bureau is charged with the collection of statistics of many kinds relating to the people, vital statistics, financial statistics, municipal statistics, statistics of agri-

culture, fishing, manufacture, transportation, mining, and others.

The census data prior to 1910 were published as a series of special volumes by the commission having the work in charge. Many of the older volumes are out of print, but may be found in large libraries. In 1900 there were three volumes on population and two volumes on vital statistics obtainable by purchase from the U. S. Publication Office at Washington. Bulletins of the census of 1910 may be obtained from the "Director of the Census, Washington, D. C." Lists of available reports and bulletins may be obtained without charge by writing to the director.

In 1910 the report of population comprised four large volumes. The first contained the general data for the country, classified and grouped in many ways; the second and third gave the population subdivided by civil divisions; the fourth, occupations. For some time it has been customary to include in each census report the populations for the two censuses preceding. This is for comparison and to enable estimates of population to be made. Thus, in the thirteenth census will be found the populations for 1910, 1900 and 1890.

A table often consulted was that on page 430 of Vol. I, Part I, of the U. S. Census of 1900, which gave the populations of all cities which were larger than 25,000 in 1900, for every census since 1790. In the 1910 census these figures are given in the second and third volumes mentioned under the head of each state. See also pages 80–97 of the first volume.

These census reports should be in every public library, and in the library of every city government, as they contain a vast amount of important information relative to the growth and condition of our country. Every student of demography should become thoroughly familiar with the U. S. Census reports.

The census date.—For most purposes it is sufficiently accurate to say that the census was taken in a certain year, but for the more exact computations a definite day must be named. The population of the country is constantly changing, even from hour to hour. If we wish to use the figure which best represents the population for any year we should naturally choose the population as it was at the middle of the year, namely July 1st. But it is not practicable to enumerate all of the people on a single day, and July 1st is not the best time to make the enumeration because being in the vacation season many people are likely to be away from home. For practical reasons another day is chosen as the official day for taking the census.

In 1910 this day was April 15th. It took several weeks to make the enumeration, but the data were adjusted to this day so that the statistics are stated "as of April 15th." But it should be noted that in 1900, in 1890, and back to 1830 the official date was June 1. Hence between the census of 1900 and 1910 the interval was not 10 years, but ten years less $1\frac{1}{2}$ months (April 15 to June 1) or $1\frac{1}{4}$ per cent less than ten years. In some computations this introduces an appreciable error and a correction must be made. From 1820 back to 1790 the day of the census was the first Monday in August.

In Great Britain, including Canada and Australia, the national census is taken every ten years, but one year later than in the United States, that is, in 1901 and 1911. This has been so since 1801. The time of the census is "at midnight before the first Monday in April."

It is quite possible to adjust the population of the census year, 1910, so as to find what it was on July 1st of that year, and this has been done by the U. S. Census Bureau and the figures used for the computation of mortality statistics for

that year. The method used is described in the next chapter.

Civil divisions. — The population of the United States is given in the census reports by minor civil divisions. The total population of the nation is subdivided into continental and "non-contiguous territory," the latter including Alaska, the Hawaiian Islands, Porto Rico, and persons in naval and military service stationed abroad. The continental population is subdivided into states; the states into counties; the counties into cities, boroughs or towns; the cities into wards; the boroughs and towns into villages and rural regions. These civil divisions differ somewhat in different parts of the country.

In comparing the figures for different decades it must be remembered that the boundaries of the civil divisions are subject to change. State boundaries are quite permanent, but cities frequently increase by annexation of suburbs, and ward lines change still more frequently according to political exigencies. In most cases changes of boundaries are indicated in the census reports by explanatory notes.

In sending to the Director of the Census for reports of populations by states or for the whole country, the request should be made for that report which gives the facts by "minor civil divisions."

The enumeration schedule of 1910. — In taking the census of 1910 the country was divided into 329 supervisor's districts each under the charge of a supervisor appointed by the President. About 70,000 enumerators were selected by the supervisors, or one for about every 1600 persons. The enumerators were required to visit each dwelling and collect the various statistics included in the schedule.

The enumerators began their work throughout the country on April 15, 1910. The law provided that this

should be completed within two weeks in cities of 5000 or more inhabitants, and within 30 days elsewhere.

The schedule of facts to be collected was printed on sheets of paper, 16 by 23 in., on which were 100 horizontal lines, 50 on each side, and numbered from 1 to 100. The facts for each person occupied one line.[1]

The schedule corresponded closely to those used in the censuses from 1850 to 1880 and 1900. The schedule used in 1890 was somewhat different, a separate schedule sheet 15 by 11 in. being employed for each family.[2]

For purposes of compilation the facts for each person were transferred to a separate punched card. These cards were then sorted by machine.

The data collected by the enumerators for each person were as follows:

At the top of each sheet were given the state, county, township or other division of county, name of incorporated place, name of institution (if any), ward of city, number of supervisor's district, number of enumerator's district, name of enumerator and date of enumeration.

SCHEDULE

Location.

Street, avenue, road, etc.

House number (in cities or towns).

1. Number of dwelling-house in order of visitation.
2. Number of family in order of visitation.
3. *Name* of each person whose *place of abode* on Apr. 15, 1910 was in this family. [Enter surname first, then the given name and middle initial, if any. Include every person living on Apr. 15, 1910. Omit children born *since* Apr. 15, 1910].
4. *Relation.* Relationship of this person to the head of the family.

[1] U. S. Census, 1910, Population, Vol. I, p. 1368.

[2] U. S. Census, 1890, Population, Part I, CCIV.

Personal Description.

5. Sex.
6. Color or race.
7. Age at *last* birthday.
8. Whether single, married, widowed or divorced.
9. Number of years of present marriage.

   Mother of how many children?

10. Number born.
11. Number now living.

Nativity.

Place of birth of each person and parents of each person enumerated. If born in the United States give the State or Territory. If of foreign birth give the country.

12. Place of birth of this person (including mother tongue).
13. Place of birth of father of this person (including mother tongue),
14. Place of birth of mother of this person (including mother tongue).

Citizenship.

15. Year of immigration to the United States.
16. Whether naturalized or alien.
17. *Language.* Whether able to speak English; or, if not, give language spoken.

Occupation.

18. Trade or profession of, or particular kind of work done by, this person, as *spinner, salesman, laborer,* etc.
19. General nature of industry, business or establishment in which this person works, as *cotton mill, dry-goods, store, farm,* etc.
20. Whether an employer, employee, or working on own account. If an employee,
21. Whether out of work on Apr. 15, 1910.
22. Number of weeks out of work during year 1909.

Education.

23. Whether able to read.
24. Whether able to write.
25. Attended school any time since Sept. 1, 1909.

Ownership of Home

26. Owned or rented.
27. Owned free or mortgaged.
28. Farm or house.
29. Number of farm schedule.

Miscellaneous.

30. Whether a survivor of the Union or Confederate Army or Navy.
31. Whether blind (both eyes).
32. Whether deaf or dumb.

One has only to read over this list to see the importance of statistical definitions. What, for example, is meant by the "usual place of abode"? This is the place where he "lives" or "belongs" or "the place which is his home." As a rule it is *where he regularly sleeps*. And then what about those persons who have no place of abode, lodgers in one-night lodging houses, tramps, laborers in construction camps, etc.? Such persons have to be enumerated *where found*. It required a formidable book of instructions to make all these things plain to the enumerators.

Bowley's rules for enumeration. — The English statistician, Bowley, has laid down the following rules in regard to the collection of statistical data by the method of enumeration.

"In practice the enumerator is usually furnished with blanks to be filled out and with questions to be answered. These questions should be:

1. Comparatively few in number.
2. Require an answer of a number or of a "yes" or "no."
3. Simple enough to be readily understood.
4. Such as will be answered without bias.
5. Not unnecessarily inquisitorial.
6. As far as possible corroboratory.
7. Such as directly and unmistakably cover the point of information desired.

These rules apply equally well to the collection of data by registration."

Credibility of census returns. — It is not to be expected that the census figures are strictly accurate. Errors are bound to be made by the enumerators; some persons are

sure to be omitted from the count, especially those traveling; some may be counted twice; and in rare instances the lists have been thought to be padded. Taken as a whole, however, the results may be considered as reliable, and it should be noted that the published data of the U. S. Census are accepted as evidence which may be introduced without proof in courts of record. Unless there is good reason for doing otherwise they should be used instead of local estimates as the basis of computing vital rates. As a rule also they should be used in place of state censuses, but there are some exceptions to this.

Collection of facts by registration and notification. — If it is difficult to secure accurate statistics of population obtained by enumerators hired for the purpose and properly instructed, how much greater the difficulty to obtain complete and accurate statistics by the method of registration, when the returns are made by large numbers of physicians, undertakers, clergymen, nurses and laymen not properly instructed, not interested in the proceedings and not always understanding the law, with inadequate laws, and with governments too easy-going to insist on the enforcement of such laws as exist! And yet most of the vital statistics of the country are collected in this way. Worst of all, the people at large do not appreciate the personal importance of having the most important events in their lives, — birth, marriage and death, — made matters of public record.

By registration is meant the reporting of certain events and associated facts to a governmental authority and the official filing or recording of such facts. The reports are made in accordance with prescribed rules and usually on a blank designed for the purposes.

Most nations in one way or another have endeavored to preserve their history by keeping these personal records. In England the registration of baptisms, marriages and

deaths dates back to 1538 when Thomas Cromwell, Vicar General under Henry VIII, issued injunctions to all parishes in England and Wales requiring the clergy to enter every Sunday, in a book kept for the purpose, a record of all baptisms, marriages and burials of the preceding week. In 1653 this work was assigned to "parish registers." It was not until 1837 that registration of births, marriages and deaths became a civil function. In 1870 it was made compulsory. In parts of Canada the registration of births and deaths is still on a parish instead of a civil basis.

In the early American colonies the practice of recording births, marriages and deaths was instituted. In New England the town clerk figured largely. In Massachusetts a fairly definite law was passed in 1692, according to which the town clerk was required to keep such records, and there were fees to be paid him for so doing, and penalties for those persons who withheld the desired information. This act was altered in 1795. In 1842 a registration act was passed in Massachusetts which made the Secretary of the Commonwealth the custodian of these records. This act, together with an amplifying act in 1844, forms the basis of registration in Massachusetts to this day. It was brought about largely through the activities of Lemuel Shattuck.[1]

The story of the registration of vital statistics is too long to be told here. Many physicians, like Dr. Edward Jarvis, of Boston, and many committees of such organizations as the American Medical Association and the American Public Health Associations have played prominent parts in the movement. At the present time the United States Bureau of the Census is taking the lead in urging necessary reforms in the registration of vital statistics.

The laws relating to the registration of vital statistics are not the same in all states. In Massachusetts a State Reg-

[1] State Sanitation, by George C. Whipple, Vol. I, p. 56.

istrar in the office of the Secretary of the Commonwealth has charge of the matter, but in many states the State Board (or Department) of Health has charge. In order to bring about uniformity a model law was drafted and endorsed by a number of national organizations and this has been adopted by a number of states. Some of the older states, however, still maintain their old arrangement. This model law should be carefully studied. It may be found in the Appendix.

Registration of births. — It is important that the birth of each and every child born be duly registered.

The information desired for the legal, social and sanitary purposes, according to the United States standard certificate approved by the Bureau of the Census, and in use since 1906, is as follows:

1. Place of birth, including State, county, township or town, village, or city. If in a city, the ward, street and house number; if in a hospital or other institution, the name of the same to be given, instead of the street and house number.
2. Full name of child. If the child dies without a name, before the certificate is filed, enter the words "Died unnamed." If the living child has not yet been named at the date of filing certificate of birth, the space for "Full name of child" is to be left blank, to be filled out subsequently by a supplemental report, as hereinafter provided.
3. Sex of child.
4. Whether a twin, triplet, or other plural birth. A separate certificate shall be required for each child in case of plural births.
5. For plural births, number of each child in order of birth.
6. Whether legitimate or illegitimate. (This question may be omitted if desired, or provision may be made so that the identity of parents will not be disclosed.)
7. Date of birth, including the year, month and day.
8. Full name of father.
9. Residence of father.
10. Color or race of father.
11. Age of father at last birthday, in years.
12. Birthplace of father; at least State or foreign country, if known.

13. Occupation of father. The occupation to be reported if engaged in any remunerative employment, with the statement of (*a*) trade, profession, or particular kind of work; (*b*) general nature of industry, business, or establishment in which employed (or employer).
14. Maiden name of mother.
15. Residence of mother.
16. Color or race of mother.
17. Age of mother at last birthday, in years.
18. Birthplace of mother; at least State or foreign country, if known.
19. Occupation of mother. The occupation to be reported if engaged in any remunerative employment, with the statement of (*a*) trade, profession, or particular kind of work; (*b*) general nature of industry, business, or establishment in which employed (or employer).
20. Number of children born to this mother, including present birth.
21. Number of children of this mother living.

The duty of making out this certificate rests upon the attending physician, mid-wife or person acting as such, or in their absence upon the father or mother of the child, the householder or owner of the premises where the birth occurred or the manager or superintendent of the institution, public or private, where the birth occurred, each in the order named. This certificate must be filed with the local registrar within ten days after the date of the birth. A supplemental blank is provided in case the child has not been named when the first report is submitted. The local registrar, or a sub-registrar, must examine this certificate as to completeness and probable accuracy, secure corrections if necessary, keep a record of the birth certificates received, numbered serially as received, and once a month transmit the original certificates to the State Registrar, for permanent preservation. Small fees to local registrars for the recording of births are provided and likewise penalties for failure. Provision is made for giving certified copies of the birth records to persons entitled to receive them.

The period of time within which a birth must be recorded may with advantage be less than the ten days above men-

tioned, especially in cities, in fact it is best that the birth be reported within twenty-four hours. If, as should be the case, the local registrar is connected or closely associated with the local board of health, the prompt information that a birth has occurred enables the health officer to send a visiting nurse to offer advice and assistance in caring for the child. Infant mortality cannot be greatly reduced in cities unless this prompt report is made.

Advantages to individuals of having births publicly recorded. — Legal evidence is thus made available as to: —

Place of Birth, useful to prove citizenship (necessary for pass-ports), to prove residence, to acquire a legal "settlement."

Time of Birth, useful to prove age, to obtain admission to school, to establish right to go to work (legal age), to prove liability for military service, to establish right to vote, to obtain pensions.

Parentage, to prove legitimacy, to inherit property, to obtain settlement of insurance, to establish citizenship.

What are some of the evidences of incomplete birth registration? — Dr. Louis I. Dublin has suggested three simple tests. *First.* The number of births registered in a calendar year should be greater than the number of living children under one year of age. *Second.* The birth-rate does not usually vary greatly from year to year. Wide and erratic variations indicate probable deficiencies in reporting. *Third.* Birth-rates less than 20 per thousand (or less than 25 per thousand in cities which have large foreign populations) are uncommon where registration is complete.

Enforcement of the registration law. — The persons most concerned in the enforcement of the birth registration law are (1) the state registrar, who should be associated with the state department of health; (2) the local registrar, who

should be associated with the local board of health; (3) the physician, whose duty it is to make the report; and (4) the parents of the child and the child himself or herself.

In order that better registration be obtained parents and physicians should be made to understand the benefits which result to individuals and to the community. Facilities in the form of suitable blanks, etc., should be provided, so as not to make the matter of reporting a burden to busy physicians. It might well be that a simple post-card notification, stating that a birth occurred at such and such a place, sent on the day of birth, with a complete certificate filed at a later date would help to solve the problem. No fee should be given to a physician who does not report within the statutory time limit. What is most needed however is a rigid enforcement of the penalty clause. A local registrar once gave the author as the reason for not imposing fines on physicians for failure to report, — "I am too good natured." This spirit is fatal to good government.

Registrars are not without opportunity to obtain evidence of neglect. In the case of reported infant deaths the local registrar should ascertain if the child's birth had been recorded. Church records of baptisms may be compared with birth returns. The checks are not as complete as in the case of death returns, but an ingenious local registrar will have little difficulty in getting good returns if he takes his task seriously.

In Cambridge, Mass., the birth records are so incomplete that annually a house to house canvass is made to ascertain the births for the year. This is a disgraceful admission of incompetence on the part of the local registrar and of the negligence of all concerned. No fines are imposed and some of the payments of fees are, with a proper interpretation of the law, of questionable legality. Unfortu-

nately Cambridge is not alone in this, but is typical of hundreds of other cities.

Registration of deaths. — The facts desired in connection with deaths are as follows, according to the United States Standard Certificate.

1. Place of death, including State, county, township, village, or city. If in a city, the ward, street, and house number; if in a hospital or other institution, the name of the same to be given instead of the street and house number. If in an industrial camp, the name of the camp to be given.

2. Full name of decedent. If an unnamed child, the surname preceded by "Unnamed."

2a. Residence at usual place of abode (ward, street and number), and length of residence in city or town where death occurred in years and months. Also how long in United States if of foreign birth.

3. Sex.

4. Color or race, as white, black, mulatto (or other negro descent), Indian, Chinese, Japanese, or other.

5. Conjugal condition, as single, married, widowed, or divorced.

5a. If married, widowed, or divorced. Name of husband or wife.

6. Date of birth, including the year, month, and day.

7. Age, in years, months, and days. If less than one day, the hours or minutes.

8. Occupation. The occupation to be reported of any person, male or female, who had any remunerative employment, with the statement of (*a*) trade, profession or particular kind of work; (*b*) general nature of industry, business, or establishment in which employed (or employer); (*c*) name of employer.

9. Birthplace; at least State or foreign country, if known.

10. Name of father.

11. Birthplace of father; at least State or foreign country, if known.

12. Maiden name of mother.

13. Birthplace of mother; at least State or foreign country, if known.

14. Signature and address of informant.

15. Official signature of registrar, with the date when certificate was filed, and registered number.

16. Date of death, year, month, and day.

17. Certification as to medical attendance on decedent, fact and time of death, time last seen alive, and the cause of death, with contribu-

tory (secondary) cause of complication, if any, and duration of each, and whether attributed to dangerous or insanitary conditions of employment; signature and address of physician or official making the medical certificate.

18. Where was the disease contracted if not at place of birth? Did an operation precede death? If so give date. Was there an autopsy? What test confirmed diagnosis?

19. Place of burial or removal; date of burial.

20. Signature and address of undertaker or person acting as such.

The first thirteen items are chiefly personal and these facts may be signed by any competent person acquainted with the facts. Items 16 and 17 comprise the medical certificate, which must be made out by the physician, if any, last in attendance. In the absence of medical attendance the undertaker must notify the local registrar who may not issue a burial permit until the case is referred to the local health officer for investigation and certification. In case there is suspicion of neglect or unlawful act the coroner, medical examiner, or other proper officer must conduct an investigation. There are various provisos, differing in different states, which should be known by every physician and nurse and, of course, by every health officer. Finally items 19 and 20 must be signed by the undertaker.

The certificate of death thus made out and duly signed must be filed by the undertaker with the local registrar (in some states the local board of health), and a burial permit or removal permit obtained prior to the disposition of the body. This permit must be delivered before burial to the person in charge of the place of burial. If the body is transported the undertaker must attach a removal permit to the box containing the corpse in order that it may reach the person in charge of the place of burial.

Thus the undertaker is primarily responsible for filing the certificate with the local registrar (or local board of

health), but the physician is responsible for making out certain very essential parts of the certificate.

Records of death certificates and burial permits are of course kept by the local registrar (or local board of health). Thus there is a check on the death certificate, and partly for this reason the registration of deaths is more complete than the registration of births. It is easier to come into the world without public notice than it is to leave it.

The data regarding the deaths are transmitted by the local registrar to the state registrar.

Uses of death registration. — The uses of death registration are legal, economic, and social. It assists in the prevention and detection of crime. It is invaluable in the settlement of life insurance and property inheritance cases. It furnishes the basis of genealogical studies. The statistics based upon these records have been a powerful weapon in studying disease, and therefore in improving the health of the race and lengthening human life. The records may be of great local value in the study and suppression of epidemics and outbreaks of communicable diseases.

Marriage registration. — There is no uniform or "model" marriage law in the United States; state laws differ from each other. The custom is that persons desiring to marry must first obtain a civil license from a designated local official and present it to the authorized person who performs the ceremony. The person officiating is required to register the marriage. The persons responsible for marriage registration are therefore the clergymen and the justices of the peace.

The facts required in the registration of marriages are commonly as follows:

1. Date of the marriage.
2. Place of the marriage.
3. Names of the persons married.
4. Their places of birth.
5. Their residences.
6. Their ages.
7. Their color.
8. The number of the marriage (as the first or second).
9. If previously married, whether widowed or divorced.
10. Their occupations.
11. The names of their parents.
12. The maiden names of their mothers.
13. The date of the record.
14. The signature of the officiating person.
15. His residence and official station.

Morbidity registration. — The compulsory registration of cases of disease dangerous to the public health is comparatively modern. It is true that many years ago such dreaded diseases as smallpox had to be reported, but it is since the organization of modern health departments and the general understanding of the manner in which communicable diseases are spread that compulsory notification has become widespread. In 1874 the State Board of Health of Massachusetts took the lead by arranging a plan for the weekly voluntary notification of *prevalent* diseases. Over a hundred physicians agreed to make this report. Ten years later, in 1884, the state passed a law requiring householders and physicians to report immediately to the selectmen or board of health of the town all cases of smallpox, diphtheria, scarlet fever, or any other disease dangerous to the public health. Other states followed suit.

The requirement of notification of diseases is an act of police power and authority for it resides in the state governments. In Massachusetts legislative authority has been delegated to the State Board (now Department) of Health to determine what diseases are dangerous to the public

health, and such diseases must be reported according to prescribed rules. Power is often delegated to local communities to supplement the state requirements for local reports. At the present time the regulations in the several states differ greatly from each other.

In 1913 a model law for morbidity reports was adopted by a conference of state and territorial health authorities and the U. S. Public Health service. According to this law the state boards (or departments) of health are required to provide machinery for keeping informed as to current diseases dangerous to the public health; physicians are required to report cases of such diseases immediately to the local health authorities having jurisdiction; teachers in schools must do the same; these records must be promptly sent to the state authorities. There are various provisos and provisions for keeping records, and for penalties. The data required are the following:

1. The date when the report is made.
2. The name of the disease or suspected disease.
3. Patient's name, age, sex, color, and address. (This is largely for purposes of identification and location.)
4. Patient's occupation. (This serves to show both the possible origin of the disease and the probability that others have been or may be exposed.)
5. School attended by or place of employment of patient. (Serves same purpose as the preceding.)
6. Number of persons in the household, number of adults and number of children. (To indicate the nature of the household and the probable danger of the spread of the disease.)
7. The physician's opinion of the probable source of infection or origin of the disease. (This gives important information and frequently reveals unreported cases. It is of particular value in occupational diseases.)
8. If the disease is smallpox, the type (whether the mild or virulent strain) and the number of times the patient has been successfully vaccinated, and the approximate dates. (This gives the vaccination status and history.)

9. If the disease is typhoid fever, scarlet fever, diphtheria, or septic sore throat, whether the patient had been or whether any member of the household is engaged in the production or handling of milk. (These diseases being frequently spread through milk, this information is important to indicate measures to prevent further spread.)

10. Address and signature of the physician making the report.

Notifiable diseases. — The following was the list of diseases made notifiable by the model law of 1913. Obviously this cannot be a permanent one. It is being continually revised chiefly by addition. In many states influenza has been added to the list during the last few months (1918). Under present conditions the lists vary in different states.

GROUP I. — INFECTIOUS DISEASES.

Actinomycosis.
Anthrax.
Chicken-pox.
Cholera. Asiatic (also cholera nostras when Asiatic cholera is present or its importation threatened).
Continued fever lasting seven days
Dengue.
Diphtheria.
Dysentery:
(a) Amebic.
(b) Bacillary.
Favus.
German measles.
Glanders.
Hookworm disease.
Leprosy.
Malaria.
Measles.
Meningitis:
(a) Epidemic cerebrospinal.
(b) Tuberculous.
Mumps.
Ophthalmia neonatorum (conjunctivitis of new-born infants).
Paragonimiasis (endemic hemoptysis).
Paratyphoid fever.
Plague.
Pneumonia (acute).
Poliomyelitis (acute infectious).
Rabies.
Rocky Mountain spotted, or tick, fever.
Scarlet fever.
Septic sore throat.
Smallpox.
Tetanus.
Trachoma.
Trichinosis.
Tuberculosis (all forms, the organ or part affected in each case to be specified).
Typhoid fever.
Typhus fever.
Whooping cough.
Yellow fever.

GROUP II. — OCCUPATIONAL DISEASES AND INJURIES.

Arsenic poisoning.
Brass poisoning.
Carbon monoxide poisoning.
Lead poisoning.
Mercury poisoning.
Natural gas poisoning.
Phosphorus poisoning.
Wood alcohol poisoning.
Naphtha poisoning.
Bisulphide of carbon poisoning.
Dinitrobenzine poisoning.
Caisson disease (compressed-air illness).
Any other disease or disability contracted as a result of the nature of the person's employment.

GROUP III. —VENEREAL DISEASES.

Gonococcus infection.
Syphilis.

GROUP IV. — DISEASES OF UNKNOWN ORIGIN.

Pellagra.
Cancer.

Incompleteness of morbidity statistics. — Complete accuracy in securing records of morbidity under any law is impossible. All of the cases existing are not seen by physicians, of the cases seen not all are recognized or correctly diagnosed, of those recognized not all are reported within the required time and some not at all. The chief error is that of incompleteness. Conservative physicians wait until sure of their diagnosis before reporting. A vast number of physicians are careless; a few deliberately shield their patients from possible inconvenience by withholding reports. More and more, however, physicians are coming to realize that in dealing with communicable diseases they have a public as well as a private duty. Death certificates give a partial check on morbidity reports. The ratios between statistics of sickness and death from reportable diseases furnishes a measure of the incompleteness of the reports. Trask has noted this difference between morbidity and mortality returns; death records are usually com-

plete but the cause of death often incorrectly diagnosed, morbidity records are incomplete, but the diagnosis usually correct. This must be kept in mind in dealing with fatality ratios.

Morbidity from non-reportable diseases. — It is much to be regretted that at the present time there is no adequate way of getting the facts in regard to sickness in the community due to diseases which are non-reportable. Sickness surveys are sometimes made, but they give only the facts at a given date, and are, moreover, very expensive to make. Hospital records help us a little, the examinations made by the life insurance companies help a little, the recent examinations of men for the army have helped a good deal, but some day a more universal method must be devised.

Reporting venereal diseases. — For a number of years the matter of requiring physicians to report cases of venereal diseases as diseases dangerous to the public health has been under consideration by public health officials; in a few places it has been attempted. The present war has emphasized the need of such reports and these are now required in many states. For social reasons it is undesirable to have the names of the victims reported, yet under some conditions it is desirable and necessary in the control of disease. The following system was adopted by the Massachusetts State Department of Health in 1918 as a war measure.

1. Gonorrhœa and syphilis are declared diseases dangerous to the public health and shall be reported in the manner provided by these regulations promulgated under the authority of chapter 670, Acts of 1913.

2. Gonorrhœa and syphilis are to be reported (in the manner provided by these regulations) on and after Feb. 1, 1918.

3. At the time of the first visit or consultation the physician shall furnish to each person examined or treated by him a numbered circular of information and advice concerning the disease in question, furnished by the State Department of Health for that purpose.

4. The physician shall at the same time fill out the numbered report blank attached to the circular of advice, and forthwith mail the same to the State Department of Health. On this blank he shall report the following facts:

Name of the disease.
Age.
Sex.
Color.
Marital condition and occupation of the patient.
Previous duration of disease and degree of infectiousness.
The report shall not contain name or address of patient.

5. Whenever a person suffering from gonorrhœa or syphilis in an infective stage applies to a physician for advice or treatment, the physician shall ascertain from the person in question whether or not such person has previously consulted with or been treated by any other physician within the Commonwealth. If not, the physician shall give and explain to the patient the numbered circular of advice, as provided in the previous regulation.

If the patient has consulted with or been treated by another physician within the Commonwealth and has received the numbered circular of advice, the physician last consulted shall not report the case to the State Department of Health, but shall ask the patient to give him the name and address of the physician last previously treating said patient.

6. In case the person seeking treatment for gonorrhœa or syphilis gives the name and address of the physician last previously consulted, the physician then being consulted

shall notify immediately by mail the physician last previously consulted of the patient's change of medical adviser.

7. Whenever any person suffering from gonorrhœa or syphilis in an infective stage shall fail to return to the physician treating such person for a period of six weeks later than the time last appointed by the physician for such consultation or treatment, and the physician also fails to receive a notification of change of medical advisers as provided in the previous section, the physician shall then notify the State Department of Health, giving name, address of patient, name of the disease and serial number, date of report and name of physician originally reporting the case by said serial number, if known.

8. Upon receipt of a report giving name and address of a person suffering from gonorrhœa or syphilis in an infective stage, as provided in the previous section, the State Department of Health will report name and address of the person as a person suffering from a disease dangerous to the public health, and presumably not under proper medical advice and care sufficient to protect others from infection, to the board of health of the city or town of patient's residence or last known address. The State Department of Health shall not divulge the name of the physician making said report.

Sickness surveys. — A new method of securing data in regard to disease has been recently applied in an experimental way in a number of cities, namely that of making a house to house canvas to determine the number of persons ill at the time. The Metropolitan Life Insurance Company has been foremost in this undertaking under the direction of Dr. Lee K. Frankel and Dr. Louis I. Dublin. Spring and fall surveys have been made in several cities, the enumerator for the most part being the collecting agents of the insurance company.

The data sheet used included the age, sex, and occupation of each member of the family; and if sick the disease or cause of disability, its duration and extent, *i.e.*, whether confined to bed, and the kind of treatment, *i.e.*, by physician at home, hospital, or dispensary.

Surveys have been made for Rochester, N. Y., September, 1915; Trenton, N. J., October, 1915; North Carolina (sample districts throughout the state), April, 1916; Boston, Mass., July, 1916; Chelsea neighborhood of New York City, April, 1917; Pittsburg and other cities of Pennsylvania and West Virginia, March, 1917; Kansas City, Mo., April, 1917.

This method obviously has its advantages and disadvantages. Within its natural limitations the data secured ought to be of value and should furnish an excellent check on the results obtained by registration of communicable diseases.

Other methods of securing data. — It will not be possible to describe here all of the many ways in which data bearing on the health of a community may be secured, but mention should be made of the importance of hospital records, life insurance records, records of physical examinations made by the U. S. Army and Navy, records of physical examinations of school children. More and more the systematic physical examination of the people will be extended, until it becomes universal and compulsory. All of this will wonderfully increase our knowledge of vital statistics.

United States registration area for deaths. — The Bureau of the Census keeps records and publishes reports of the mortality of those parts of the United States where the statistics are sufficiently accurate to make it worth while to do so. A so-called registration area for deaths was established in 1880. This included those states and cities

in which satisfactory registration laws were being effectively enforced and where there was good reason to believe that more than 90 per cent of all deaths were being registered. At first the registration area included only two states, Massachusetts and New Jersey, and certain cities in other states. The area has gradually expanded as shown by the following tables. In studying the mortality rates of the country in the published reports it is important to keep in mind this addition of new territory, new populations, from year to year.

TABLE 19

REGISTRATION AREA FOR DEATHS

Year.	Population.		Land area.	
	Number.	Per cent of total.	Square miles.	Per cent of total.
(1)	(2)	(3)	(4)	(5)
1880	8,538,366	17.0	16,481	0.6
1890	19,659,440	31.4	90,695	3.0
1900	30,765,618	40.5	212,621	7.1
1901	31,370,952	40.3	212,770	7.2
2	32,029,815	40.4	212,762	7.2
3	32,701,083	40.4	212,762	7.2
4	33,345,163	40.4	212,744	7.2
5	34,052,201	40.4	212,744	7.2
6	41,983,419	48.9	603,066	20.3
7	43,016,990	49.2	603,151	20.3
8	46,789,913	52.5	725,117	24.4
9	50,870,518	56.1	765,738	25.7
1910	53,843,896	58.3	997,978	33.6
11	59,275,977	63.1	1,106,734	37.2
12	60,427,247	63.2	1,106,777	37.2
13	63,298,718	65.1	1,147,039	38.6
14	65,989,295	66.8	1,228,644	41.3
15	67,336,992	67.1	1,228,704	41.3
16	71,621,632	70.2	1,307,819	44.0
17	75,306,588	72.7	1,349,506	45.4
18	81,786,052	77.7	1,546,166	52.0

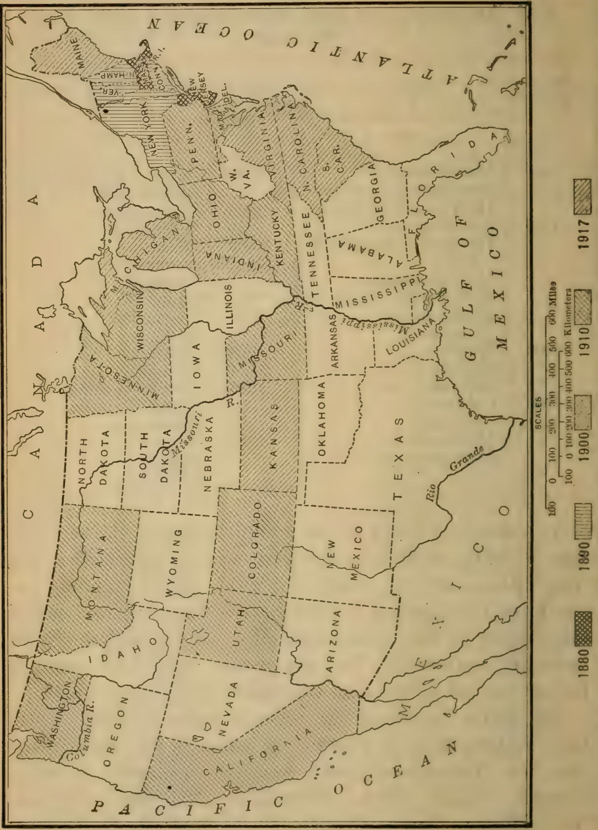
TABLE 20

LIST OF STATES IN THE REGISTRATION AREA FOR DEATHS

Year of Entrance.	State.	Year of Entrance.	State.
(1)	(2)	(1)	(2)
1880	Massachusetts	1906	South Dakota (dropped in 1910)
	New Jersey	1908	Washington
	District of Columbia		Wisconsin
1890	Connecticut	1909	Ohio
	Delaware (dropped in 1900)	1910	Minnesota
	New Hampshire		Montana
	New York		Utah
	Rhode Island	1911	Kentucky
	Vermont		Missouri
1900	Maine	1913	Virginia
	Michigan	1914	Kansas
	Indiana	1916	North Carolina
1906	California		South Carolina
	Colorado	1917	Tennessee
	Maryland	1918	Illinois
	Pennsylvania		Oregon
			Louisiana

As a result of a test of the completeness of the registration of deaths in Hawaii the territory was admitted to the registration area for deaths for 1917, thus extending beyond the Continental United States the area from which the Bureau of the Census annually collects and publishes mortality statistics. The population and land area of Hawaii are *not* included in the figures of the above table.

The states in which the registration of deaths is still too unsatisfactory to warrant inclusion in the registration area are: Alabama, Arizona, Arkansas, Delaware, Florida, Georgia, Idaho, Iowa, Mississippi, Nebraska, Nevada, New Mexico, North Dakota, Oklahoma, South Dakota, Texas, West Virginia, and Wyoming. (1918.)


ATLANTIC OCEAN
PACIFIC OCEAN
GULF OF MEXICO
CANADA
MEXICO
MAINE
N.HAMP.
VER.
MASS.
CONN.
R.I.
NEW YORK
NEW JERSEY
PENN.
DEL.
MD.
VIRGINIA
W. VA.
N. CAROLINA
S. CAR.
GEORGIA
FLORIDA
ALABAMA
MISSISSIPPI
TENNESSEE
KENTUCKY
OHIO
INDIANA
ILLINOIS
MICHIGAN
WISCONSIN
MINNESOTA
IOWA
MISSOURI
ARKANSAS
LOUISIANA
TEXAS
OKLAHOMA
KANSAS
NEBRASKA
SOUTH DAKOTA
NORTH DAKOTA
MONTANA
WYOMING
COLORADO
NEW MEXICO
ARIZONA
UTAH
NEVADA
IDAHO
OREGON
WASHINGTON
CALIFORNIA
Mississippi R.
Missouri R.
Rio Grande
Columbia R.
SCALES
100 0 100 200 300 400 500 600 Miles
100 0 100 200 300 400 500 600 Kilometers
1880
1890
1900
1910
1917

United States registration area for births. — A registration area for births was not established until 1915. For this year the Bureau of the Census published its first annual report of birth statistics based on registration records. The birth statistics published in connection with the regular decennial reports from 1850 to 1900 inclusive were based on enumerator's returns.

The registration area in 1915 included only ten states — Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New York, Pennsylvania, Michigan, Minnesota, and the District of Columbia. In these states the registration of births is believed to include upwards of ninety per cent of the actual numbers. This registration area includes only 10 per cent of the area and 31 per cent of the population of the country. In spite of this unfavorable showing a beginning has been made, and inasmuch as the standard birth certificate has been adopted for 85 per cent of the population and as public sentiment in regard to the importance of vital statistics is rapidly gaining ground, it is likely that the registration area for births will rapidly extend. No state is admitted until the accuracy of its records have been submitted to test.

In 1916 Maryland was added. In 1917, Virginia, North Carolina, Kentucky, Indiana, Ohio, Wisconsin, Washington, Utah and Kansas were added, bringing the population included up to 53.1 per cent.

Need of national statistics. — More and more it becomes obvious that there is need of a national system of keeping records of vital statistics, with uniform state laws, and with proper provision for the local use of the data registered. The excellent work done by the Bureau of the Census has done much to emphasize this need. Likewise interstate barriers must be broken down in the interest of suppressing diseases dangerous to the public health. The

U. S. Public Health Service keeps a record of cases of diseases from data furnished by the states and publishes the same in its weekly Public Health Reports. This is only a part of what is needed. If the time ever comes when the United States establishes a real National Health Department the maintenance of an adequate system of vital records will be one of supreme importance.

EXERCISES AND QUESTIONS

1. Compare the methods of numeration used in taking the U. S. census of 1910, with those used in 1900, 1890, 1880, etc.

2. How do these methods compare with those used in England, France, Sweden?

3. Would there be any advantage in making the census date, "as of January first"?

4. What advantages would come from the adoption of a uniform census date for the entire world?

5. How accurately is the population of China known?

6. To what extent is the keeping of accurate census records and records of vital statistics an index of national progress?

7. How can improvements be made in ascertaining the facts concerning morbidity?

CHAPTER V

POPULATION

Estimation of population. — It is only for the census years that populations can be known with certainty. For the intercensal years, the years between two censuses, it is necessary to depend upon estimates. This is also the case for the postcensal years, namely, the years following the last census. These estimates are only approximately true, a fact which must not be forgotten, but they are sufficiently near the truth for many practical uses.

Estimations of population may be made in various ways. The natural growth of population is like that of money at compound interest except that the interest is being added constantly instead of semi-annually or quarterly. Mathematicians call this geometrical progression. With a given constant rate of interest money in the bank increases more and more each year. It is the same with population. In geometrical progression the basis of our population estimates is the annual *rate* of increase. When dealing with very large populations, and especially when dealing with populations not influenced by emigration or immigration, this method is the most accurate one to use. It has several practical disadvantages, however, and in the present shifting condition of the world's population there are not many places where the natural growth of population is the only factor to be considered.

A simpler method is that of arithmetical progression, which assumes a constant annual increment between two

census years. The increase in ten years divided by ten gives the annual increase. This is practically the method by which money increases by simple interest. The arguments in favor of this method of estimating population are that it is simple and easily understood; that in view of the various disturbing factors due to migration and other causes it gives results practically as near the truth as those obtained by geometrical progression; that the estimates for the whole area of a given district will be equal to the sum of the estimates for all the parts of the district, which would not be the case with the geometrical method. The U. S. Bureau of the Census has adopted this method, and in the interest of uniformity all cities and states should do the same. The method is one which should not be extended far into the future. In vital statistics it is not necessary to extend it beyond ten years from the last census, for ten years always brings another census.

Another method is that of using local data as indices — such as the number of registered voters, the number of new building permits, the number of school children, the number of names in the directory, the bank clearings, the number of passengers carried by the trolley cars, etc. These facts are often obtainable for each year and serve as valuable checks on the census method, but as a rule they should not be depended upon alone. Common sense must be used. What is wanted are the facts, and rigid adherence to a rule when the result is manifestly unfair is absurd. When deviations from accepted practice are made, however, a statement of the method of making the estimates of the population should always accompany the result. Even the U. S. Census utilizes local data to modify its estimates where plainly necessary.

Estimates of population might be made from records of births and deaths if these were accurately kept and if the

migrations of the people were known. Practically this method is useless.

One item in connection with the estimation of the population of cities should not be lost sight of, namely, that of changing boundaries. Cities often grow by extending their area. Increases of population from this cause should not be mistaken for natural increase in population.

Arithmetical increase. — Let us assume that the population of a place in 1900 was 70,000 and in 1910, 100,000. The increase was, by the arithmetical method, 30,000 in ten years, or 3000 in each year. For 1904, therefore, the estimated population would be 70,000 plus four times 3000 or 82,000; and for 1915 it would be 100,000 plus five times 3000 or 115,000. It is assumed that within the ten years following the last census the annual increase will be the same as the average annual increase between the last two censuses. This is the simple and customary way of making the estimates.

It must be remembered that between these particular censuses the interval was not exactly ten years. The census of 1900 was "as of June 1st," that of 1910 "as of April 15th." Consequently, the interval was ten years less a month and a half or $9\frac{7}{8}$ years ($=9.875$ years). The average increase was not, therefore, 3000 per year, but $30{,}000 \div 9.875$ or 3038. This would make the estimated population 82,152 for 1904 and 115,190 for 1915. It will be seen that this difference is not great. Nevertheless, it is a correction which in some cases is of importance. Whether it will have to be made after the next census will depend upon the date decided upon. Strictly speaking, the populations of the census years should be adjusted to the middle of the year before the average annual increments are computed.

Adjustment of population to mid-year. — The census of 1910 was "as of April 15th." What then was the population on July 1st?

On June 1, 1900, the population was **70,000**. The average annual increase was 3038 per year, or $3038 \div 12 = 253$ per month. On July 1, 1900, the population was, therefore, $70,000 + 253 = 70,253$. On July 1, 1910, it was $100,000 + 253 \times 2\frac{1}{2}$ months or 100,633. The increase in ten years was, therefore, $100,633 - 70,253 = 30,380$, or 3038 per year, as before.

This arithmetical method, therefore, is used in adjusting the population for the census years from the day on which the census was actually taken to the mid-year.

For example, on June 1, 1900, the population of the state of Indiana was 2,516,462; on Apr. 15, 1910, it was 2,700,876, an increase of 184,414 in 118.5 months. On July 1, 1910, *i.e.*, 2.5 months later than the census date, the estimated population would therefore be $2,700,876 + \frac{2.5}{118.5} \times 184,414$ or 2,704,767. This is the figure used by the U. S. Census in the Mortality Report of that year.

Geometrical increase. — A simple rule for computing populations by the geometrical method is to use the logarithms of the populations concerned in the same way that the populations are used in computing by the arithmetical method. Let us assume, as before, that the population was 70,000 in 1900 and 100,000 in 1910. The logarithm of 100,000 is 5.0000, that of 70,000 is 4.8451. Instead of subtracting 70,000 from 100,000 we subtract 4.8451 from 5.0000 and get 0.1549. Instead of dividing 30,000 by 10 we divide 0.1549 by 10 and get 0.01549. Then we multiply this by 4 and get 0.0620. Finally, we add this to 4.8451, which is the log of 70,000, and get 4.9071. This is the log of the answer, which is 80,750. The following comparison ought to make this clear:

Example. — The population of a city in 1900 was 70,000 and in 1910, 100,000. What was the population in 1904, in 1915 and in 1925?

TABLE 21

	Arithmetical method.	Geometrical method.
(1)	(2)	(3)
Population in 1910..	100,000	log of 100,000 = 5.0000
" " 1900..	70,000	" " 70,000 = 4.8451
Increase in 10 years.	30,000	0.1549
Increase in 1 year.	3,000	0.0155
Increase in 4 years..	12,000	0.0620
Population in 1900..	70,000	4.8451
" " 1904..	**82,000**	log of **80,750** = 4.9071
Increase in 5 years..	15,000	0.0775
Population in 1900..	100,000	5.0000
" " 1915..	**115,000**	log of **119,500** = 5.0775
Increase in 15 years.	45,000	0.2325
Population in 1900..	100,000	5.0000
" " 1925..	**145,000**	log of 170,800 = 5.2325

It will be noticed that for intercensal years the arithmetical method gives higher estimates than the geometrical, but that for postcensal years the geometrical results are higher. This is illustrated graphically by Fig. 30.

Formula for geometrical increase. — The mathematical formula for geometrical increase is

$$P_n = P_c\,(1 + r)^n,$$

in which P_c is the population at one census, P_n is the population n years after P_c, r is the annual rate of increase and n is the number of years.

Let us apply this to the case already considered. Here we know the two populations P_c and P_n, 70,000 and 100,000,

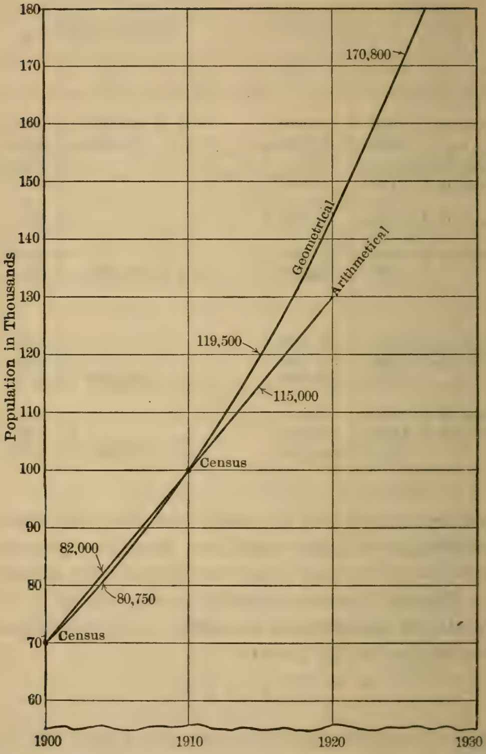


Fig. 30. — Example of Arithmetical and Geometrical Methods of Estimating Population.

and we know that n is 10 years; first we need to find r, the annual rate of increase. According to algebra we may rewrite the above formula, thus:

$$\log P_n - \log P_c = n \log (1 + r).$$

Substituting the values of the logarithms of 100,000 and 70,000 and the value of n we have

$$\begin{aligned} 5.0000 - 4.8451 &= 10 \log (1 + r) \\ 0.1549 &= 10 \log (1 + r) \\ 0.01549 &= \log (1 + r) \end{aligned}$$

and from the tables of logarithms $(1 + r)$ is found to be 1.036, hence $r = 1.036 - 1 = 0.036$, or 3.6 per cent. Therefore, the average annual rate of increase between 1900 and 1910 was 3.6 per cent.

Knowing this rate and assuming it to be constant we can find the population in any other year. Suppose we try 1925, 15 years after 1910. Then we have:

$$\begin{aligned} P_n &= 100{,}000\,(1 + 0.036)^{15}, \\ \log P_n &= \log 100{,}000 + 15 \log 1.036 \\ &= 5.0000 + 15 \times 0.01549, \\ \log P_n &= 5.23245, \\ \therefore \quad P_n &= 17{,}079 \text{ (according to the log. tables.)} \end{aligned}$$

By the use of this formula many interesting problems can be solved. For example, how many years would it take the population in our now familiar example to reach 200,000? We know that the average rate of increase between 1900 and 1910 was 3.6 per cent. Therefore, we have in the formula

$$200{,}000 = 100{,}000\,(1 + 0.036)^n.$$

We want to find the value of n. We have

$$\begin{aligned} \log 2000{,}00 &= \log 100{,}000 + n \log 1.036, \\ 5.30103 &= 5.0000 + n \times 0.01549, \\ 0.30103 &= n \times 0.01549, \\ n &= \frac{0.30103}{0.01549} = 19.45 \text{ years.} \end{aligned}$$

Strictly speaking, we have no reason to use a year or even a month as the basis of compounding, as the population is increasing from day to day and from hour to hour. A more accurate formula may be found in books on calculus. We do not need to use it in this work.

Rate of increase. — The population of the United States on June 10, 1900, was 75,994,575; on Apr. 15, 1910, it was 91,972,266. The increase in $9\frac{7}{8}$ years was 15,977,691 or, 134,833 per month, assuming the increase to have been constant. We might divide this still further and say that the average increase was 4494 per day, or about 3.12 persons per minute. On this basis we might also by computation ascertain that the population of the United States passed the one hundred million mark at 4 o'clock on Apr. 3, 1915. Such statements as this have a fascination for certain people, but they are of idle moment. They merely serve to illustrate the method of computation by the arithmetical method. Had the geometrical method been used the result would have been different. As a matter of fact no one will ever know just when the population passed the hundred million mark.

If we take the above figures for 1900 and 1910 and regard them as representing a ten year period (instead of 9.875 years), the increase amounts to $\frac{15,977,691}{75,994,575}$, or 21 per cent. We may divide this by 10 and say the annual *increment* was 2.1 per cent, or more accurately by 9.875 and say that it was 2.13. But we ought not to use the word *rate* in this connection. As a matter of fact, if the rate of increase in 10 years was 21 per cent, the average annual rate would not be 2.1 per cent. If in the formula for geometrical increase we let $P_c = 100$ and $P_n = 121$, which would represent an increase of 21 per cent in 10 years, then

$$\log 121 - \log 100 = 10 \log (1 + r),$$
$$2.08278 - 2.00000 = 10 \log (1 + r),$$

from which

$$r = 1.92 \text{ per cent, not } 2.1 \text{ per cent.}$$

This assumes that, as we might say, the interest is compounded annually.

This error of dividing the percentage increase in 10 years by 10 to find the annual increase is sometimes made in using the geometrical method of estimating increase. Obviously with compound interest a lower rate suffices to produce a given increase in 10 years than with simple interest. The proper way to find the annual rate is by the use of the formula.

Decreasing rate of growth. — It seems to be generally true that as cities become larger their annual *rate* of growth decreases. A study of six American cities gave the following annual rates of increase when the populations were as indicated.

TABLE 22

DECREASING RATE OF GROWTH OF CITIES

Stage of Population	Annual percentage Increase.
(1)	(2)
100,000	4.85
200,000	3.59
300,000	2.91
400,000	2.48
500,000	2.02
600,000	1.75
700,000	1.66
800,000	1.58

Difference between estimate and fact. — In estimating the population either by the arithmetical or geometrical method we are assuming something which is almost never true, *i.e.*, that the population is increasing *regularly*. As a

matter of fact the increase is not regular from year to year. Therefore, any estimate may be erroneous. In the absence of the facts, however, we are compelled to resort to the method of estimation. Also when we assume that the growth in the present decade is the same as in the last decade we assume a uniformity of conditions which seldom obtains. Let us check a few of our estimates by actual census returns.

In Cambridge, Mass., the census population was 70,028 in 1890 and 91,886 in 1900, a gain of 21,858 in the decade. If the same increase had continued during the next decade the population would have been 113,744. The census of 1910 was, however, only 104,839.

In Detroit, Michigan, the population in 1890 was 205,876 in 1900 it was 287,704, the increase being 79,828. If this increment continued regularly the population in 1910 would have been 365,532; actually it was 465,766. This of course is an extreme case. In most cities the estimates agree fairly well with the facts.

Let us take the case of a larger population, say that of the United States. In 1890 the population was 62,947,714, in 1900, 75,994,575, the decadal increase, 1,304,861. The estimate for 1910 based on these figures would have been 89,041,436 by arithmetrical, or 91,723,000 by geometrical increase. Actually the population in 1910 was 91,972,266. For large populations like this the geometrical method gives closer results.

Revised estimates. — Suppose, however, that as in Cambridge, Mass., the census of 1910 showed that the city had not grown as fast as in the decade from 1890 to 1900, what shall be done with the estimates already made for the years 1901 to 1909, inclusive? Obviously they were not correct even on the theory of steady increase. Yet they have been used as the basis of computing birth-rates and death-rates. The answer is that if the discrepancy is large

the populations for those years should be reëstimated and the birth-rates and death-rates recomputed. Let us see what differences would result.

TABLE 23

REVISION OF POPULATION ESTIMATES

Year.	Census.	Postcensal estimate based on 1890–1900.	Intercensal estimate based on 1900–1910.
(1)	(2)	(3)	(4)
1890	70,028		
1900	91,886		
1		94,072	93,181
2		96,258	94,476
3		98,444	95,771
4		100,630	97,066
5		102,816	98,361
6		105,002	99,656
7		107,188	100,951
8		109,374	102,246
9		111,560	103,541
10	104,839		

Ordinarily the errors are not as great as this and no correction need be made, but the chance of error is so great that old published figures for death-rates should not be accepted at their face value until the population estimates have been carefully examined for errors of this sort.

It would be sound practice to revise all rates based on postcensal population estimates every ten years, *i.e.*, after each new census.

Estimation of population from accessions and losses. — In a place where records of the births and deaths are accurately kept it would be possible to use them in estimating population, but emigration and immigration enter as disturbing factors. Two examples of this method will illustrate the way in which this method works out.

In England and Wales the data were as follows:

Population in 1891	29,000,000
Births, 1891 to 1901	9,160,000
	38,160,000
Deaths, 1891 to 1901	5,560,000
Computed population in 1901	32,600,000
Census gave, for the year 1901	32,530,000
Difference representing excess of emigration over immigration	70,000

In Massachusetts the population for 1910 computed in this way was 3,092,349, but the census gave 3,366,416, an excess of 274,067, which represented in part the excess of immigration over emigration and in part, no doubt, incomplete registration of births.

Estimation of future population. — For some purposes, as, for example, in planning for sewerage or water supply systems, it is necessary to estimate the population of a city for half a century or more in the future. This cannot be safely done by mathematical methods alone, for much depends upon other things not subject to definite analysis. Boundaries may change, business and manufacturing may expand or contract in ways unforeseen, changes in transportation or in methods of housing may influence the problem. Mathematical analyses are helpful, but the conclusions must be tempered with judgment based on a study of local conditions and on the history of other cities similar in size and conditions.

A few examples of unfulfilled estimates may be mentioned. In 1865 Jas. P. Kirkwood, a well known civil engineer, estimated that the population of Cincinnati would be 431,644 in 1890; actually it proved to be 297,000.

At Rochester an estimate made in 1889 claimed that the population in 1910 would be 283,459; actually this city grew to 218,149. At Winnipeg in 1897 a certain estimate of the

probable population in 1907 was made, but when 1907 arrived the population was double the estimated figure.

For long-time estimates the methods already described may be used, but with this difference that the rate of past increase is best obtained, not by taking the results of the last two censuses only, but by considering a longer period. To look farther ahead than 10 years you must begin farther back. This is an important principle which applies to many things in life. It means the use of experience. The public health student who desires to project himself forcefully into the coming era needs to study the past history of the health of the human race. It is equally true in the fields of science, philosophy and religion.

As a rule in United States cities the arithmetical method gives results which are too low, and the geometrical method gives results which are too high. All things considered the graphical method is the most serviceable for long term estimates as it enables data for various cities to be brought together.

Immigration. — The irregular effect of immigration in the United States may be inferred from Fig. 31, which shows the immigration by years from 1820 to 1909. Immigration has occurred in a series of waves, resulting from the relative economic conditions in this country and abroad. When this incoming population has been concentrated in manufacturing cities, as has periodically been the case, it has been followed by an increased prevalence of disease. The subject is, therefore, an important one for sanitarians to consider.

The data for immigration are published in the annual reports of the U. S. Commissioner General of Immigration.

Graphical method of estimating population. — The simplest method of estimating future populations graphically is to plot the populations on cross-section paper using all past

records available, and then sweep the curve forward according to its general trend. It is easy to use poor judgment in doing this. Local conditions should be kept in mind and especially additions of area should be considered.

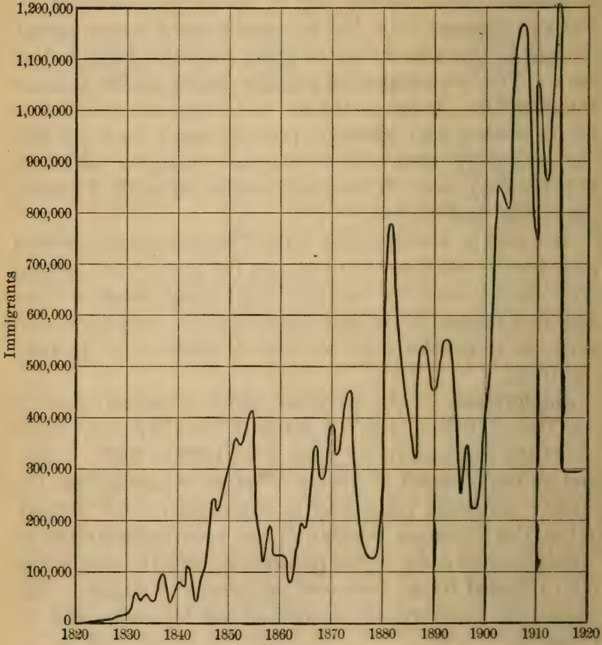


FIG. 31. — Immigration to the United States: 1820–1917. (From report of Commissioner of Immigration.)

A safer way is to plot not only the populations for the city being considered, but for other cities where the conditions are near enough to warrant them being taken as guides.

These cities must be larger than the one for which the estimate is to be made.

A good way is to plot them all on cross-section paper on the same scale and then trace the lines upon a single sheet, adjusting the time scale so that all of the curves meet and cross on some selected year, usually the last census year. In this way there will be a number of population lines extending ahead and these may be used as guides in sweeping in the curve of the city being considered.

Judgment inevitably plays a large part in long-term estimates and statistics are used merely as an aid to that judgment.

Example. — Estimate the future population of Springfield, Mass., using for comparison the cities of Worcester, Mass., Syracuse, N. Y., Rochester, N. Y., and Providence, R. I. From the census reports we have the following data:

TABLE 24

POPULATION OF CITIES

Year.	Springfield.	Worcester.	Syracuse.	Rochester.	Providence.
(1)	(2)	(3)	(4)	(5)	(6)
1860	15,199	24,960	28,119	48,204	50,666
1870	26,703	41,105	43,051	62,386	68,904
1880	33,340	58,291	51,792	89,366	104,857
1890	44,179	84,655	88,143	133,896	132,146
1900	62,059	118,421	108,374	162,608	175,597
1910	88,926	145,986	137,249	218,149	224,326

These data are first plotted as in the upper part of Fig. 32. They are afterwards brought together as in the lower part of the figure, the lines being made to cross on the line of 1910. The estimate for Springfield is made by sweeping the curve forward as shown.

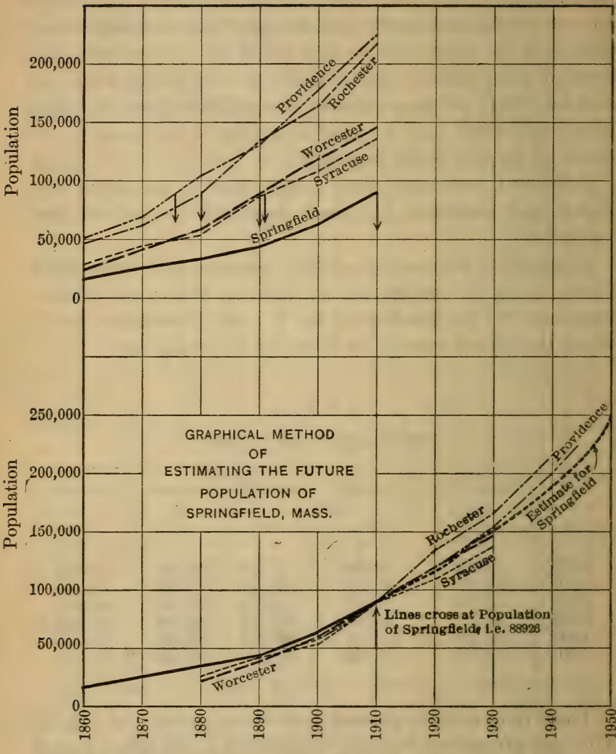


FIG. 32. — Example of Graphical Method of Estimating Future Population.

Accuracy of state censuses. — The U. S. Bureau of the Census does not recognize as generally acceptable the results obtained by those states which enumerate their populations in the years which end in 5, on the ground that it does not control such intermediate censuses and has no way of assuring itself of their accuracy. On the whole this position is probably sound. In Massachusetts, the last federal census in 1910 was made by the same state authority, namely the Director of the Massachusetts Bureau of Statistics, which has made the state census, and one census is presumably as accurate as the other.

In the past, however, the state censuses have evidently not been as accurate as the federal censuses. If the results of the federal censuses for Massachusetts are plotted the points fall on quite a smooth and regular curve from 1820 to 1910, the only important departure being during the decade of the Civil War. The figures for the state censuses do not all fall on this line, but rise and fall irregularly. This is presumptive, though not conclusive, proof of the inaccuracy of some of the figures (Fig. 33).

The Massachusetts state census was taken on May 1 from 1855 to 1905, inclusive; in 1915 it was taken on April 1.

The question often comes up for decision, shall the state censuses be used in estimating populations for the years in the last half of each decade? For the sake of uniformity it is best to use the federal figures only. But these figures should, of course, be modified if the state census reveals that there have been important changes in conditions. The state figures should be used, therefore, as a check on the estimates based on the federal results. Should glaring differences be noticed their cause should be investigated. If state figures are used this fact should be stated in connection with the estimate.

Urban and rural population. — It is common to classify the population of a country into "urban" and "rural." This is done for purposes of discussion, the idea being to separate the people living in sparsely settled regions and small villages from those living in cities, on the theory that

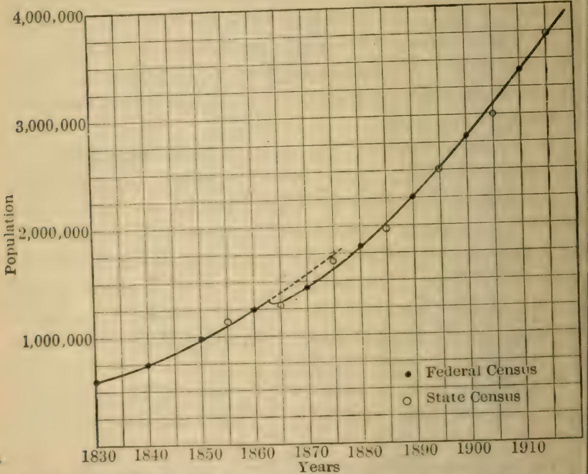


FIG. 33. — Population of Massachusetts according to Federal and State Censuses. (The effect of the Civil War should be noticed.)

the former lead a more individualistic life, while the latter lead a more communal life. In cities for example, water supplies, sewerage systems, food supplies, methods of transportation and various public utilities are used in common by all, while in the country each household has its own well, its own garden, its own cesspool, its own means of transportation. Thus urban and rural populations are supposed to live and work under different conditions.

Obviously the separation of the two classes must be an arbitrary one. In the United States prior to the Census of 1880 the limit of 8000 inhabitants was used. In 1880 it became recognized that many communities of less than 8000 inhabitants possessed "the distinctive features of urban life," and accordingly the limit was dropped to 4000, although the old limit was used in many of the tabulations of the census of that year, and also of the years 1890 and 1900. In 1900 some comparisons of the two limits were made. It was found, for example, that 32.9 per cent of the population of the United States would be classed as urban on the basis of the 8000 limit, and 37.3 per cent, on the basis of the 4000 limit.

In 1910 the limit was reduced by the Census Bureau to 2500. The reason for this probably lay in the extension of various public utilities, once existing only in the cities of larger size, to the smaller communities. In 1910, 46.3 per cent of the population were classed as urban on the basis of the 2500 limit, but only 38.8 were in cities larger than 8000.

One must be careful in drawing conclusions from "urban and rural statistics." In the first place, of necessity they relate to civil divisions. "Outside of New England," says the Census Report for 1900, "there is not much difficulty in distinguishing between the urban and rural elements of the population, as only dense bodies of population are chartered. But in New England a town, which is the usual division of the county, is chartered bodily as a city when certain conditions of population are fulfilled, so that a city may contain a considerable proportion of rural population, and, conversely, a town may contain a compact body of population of magnitude sufficient to be classed as urban." Evidently then, rural population does not necessarily mean people living in isolation, as on a farm. Almost every incorporated town or

borough has some center, and here people may live under communal conditions which may be quite as insanitary as those found in cities, with houses close together, with boarding houses, saloons, stables, numerous cesspools, and even sewers and public water supplies.

Attempts have been made by various writers to make a triple separation of the population into "rural," "village," and "urban," using populations of 1000 and 4000 as demarkations. These add but little to the value of the statistics and usually it is best to follow the practice of the Bureau of the Census. Populations of 3000 and 5000 have also been used as limits between rural and urban. Whatever limits are used the possible fallacies inherent in an arbitrary classification should be kept in mind.

In comparing conditions as shown by censuses ten years apart there is always likely to be some confusion caused by communities which have populations near the limit changing from one side to the other. The Bureau of the Census has followed this practice: "In order to contrast the *proportion* of the total population living in urban or rural territory the territory is classified according to the conditions *as they existed at each census;* but in order to contrast between the *rate of growth* of urban and rural communities it is necessary to consider the changes of population for the *same territory,* which have occurred between censuses, and the places included in the urban class are those which have populations above the limit at the last census, even though they were below the limit at the time of the previous census.

Since about 1820 the urban population of the country has been rapidly increasing, the rural population becoming relatively less. This is well shown by the following figures:

TABLE 25

TOTAL AND URBAN POPULATION AT EACH CENSUS: 1790–1910

(From U. S. Census, 1900, Vol. 1, Pt. 1, p. LXXXIII.)

Census year.	Total population.	Urban population.*	Number of places.	Per cent of urban of total population.
(1)	(2)	(3)	(4)	(5)
1790	3,929,214	131,472	6	3.3
1800	5,308,483	210,873	6	4.0
1810	7,239,881	356,920	11	4.9
1820	9,638,453	475,135	13	4.9
1830	12,866,020	864,509	26	6.7
1840	17,069,453	1,453,994	44	8.5
1850	23,191,876	2,897,586	85	12.5
1860	31,443,321	5,072,256	141	16.1
1870	38,558,371	8,071,875	226	20.9
1880	50,155,783	11,450,894	291	22.8
1890	62,947,714	18,327,987	449	29.1
1900	75,994,575	25,142,978	556	33.1
1910	91,972,266	35,726,720	778	38.8

* Population of places of 8000 or more at each census.

This is also shown by the increase in the number of large cities since 1860.

TABLE 26

TABLE SHOWING THE INCREASE IN NUMBER OF LARGE CITIES IN THE UNITED STATES BETWEEN 1860 AND 1910

Number of cities with population above.	1860.	1870.	1880.	1890.	1900.	1910.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
25,000	32	50	77	125	161	229
50,000	15	24	35	58	79	109
100,000	8	13	20	28	38	50
500,000	2	2	4	4	6	8
1,000,000	0	0	1	3	3	3

Density of population. — By the density of population we usually mean the number of persons dwelling upon a unit area of land, as a square mile or an acre. It is not to be supposed that the persons within this unit area are uniformly distributed over it. Usually they are not. The ratio is one of convenience, however, and variations of density within the area under consideration are tacitly assumed. On the catchment area of the Croton river (331 square miles) which supplied New York with unfiltered water the population in 1903 was on an average about 52 per square mile, while on the catchment areas of many German streams, where the water is filtered before being used the population per square mile is often 500 or 800. Thus the average density expressed in this way is a valuable means of comparing the relative liability of the water to be contaminated, even though both in Germany and on the Croton catchment area the population consists of villages and farms irregularly scattered.

The average density of the population of the United States is steadily increasing. In 1790 it was only 4.5 persons per square mile; in 1860 it was 10.6; in 1910 it was 30.9. The density varies greatly in the different states. Rhode Island has the greatest density. In 1910 it was 508.5 per square mile, Massachusetts came next with 418.8, then New Jersey with 337.7, Connecticut, 231.3, New York, 191.2, Pennsylvania, 171, Maryland, 130.3, Ohio, 117, Delaware, 103, and Illinois, 100.6. These were the only states above 100 per square mile. In Nevada the density was only 0.7 per square mile.

The density of population of the United States by counties is shown in Fig. 34.

When we need to know the variations in density more accurately we take a smaller unit of area. For the purpose of calculating the size of sewers required in a district, or for studying the congestion of population in a city the density

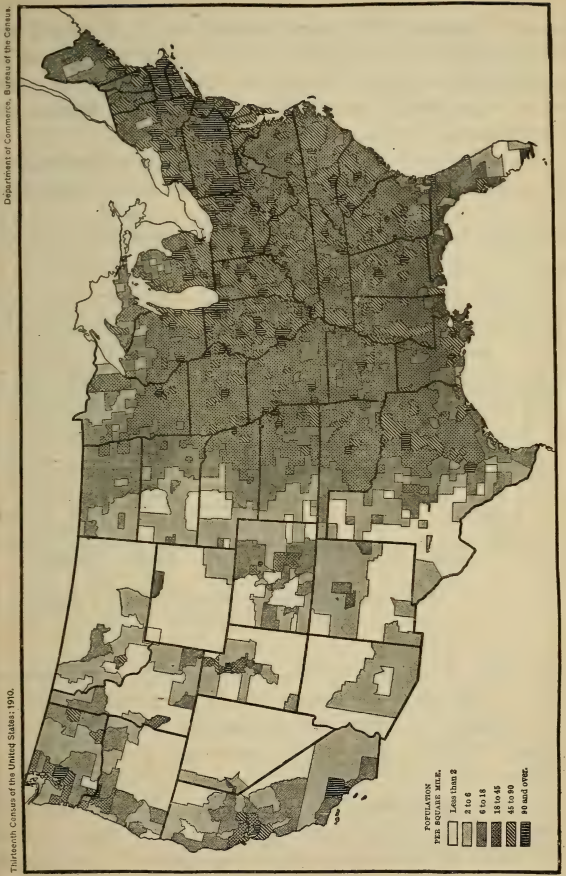


FIG. 34. — Population per Square Mile, by Counties: 1910.

per acre is computed. The population density in cities usually increases with their population. In the congested portions of cities the density may be several hundred per acre, sometimes over a thousand. Fig. 35 shows the densities of population in the different wards of Boston and Cambridge in the year 1910.

There are two ways in which the density of population of a city may be computed. The first and most com-

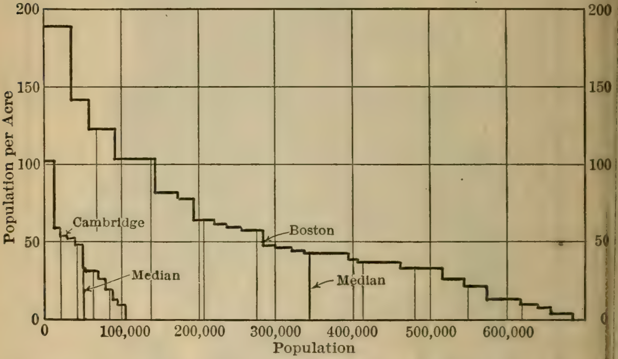


FIG. 35. — Density of Population by Wards in Boston and Cambridge, Mass.: 1910.

mon way is to divide the population by the area in acres. This gives the actual density per acre. In Boston, for example, in 1910 the population was 686,092, the acreage 27,674, and the number of persons on the average acre was 24.8. But if we look at the density from the standpoint of the people we find that the median person lives where the density is about 50 per acre and that 10 per cent of the population live where the density is 125 per acre, 5 per cent where it is 150. In

Cambridge, Mass., the average density per acre is 25.1, or practically the same as in Boston. The density for the median person is also about the same, *i.e.*, 50 per acre; but 10 per cent of the population live where it is less than 60 per acre, and 5 per cent where it is only 100. In no ward of Cambridge is the density as great as in five large populous wards of Boston.

For some purposes, as when we are providing a sewerage system, the density per acre is what is wanted, but when we are considering the crowded condition of the people it is the median density based on population which is needed, and the proportion of people living under conditions of different congestion. We need also to consider areas as small as single blocks.

Population of United States Cities.—The figures in Table 27 will be found useful in computing vital rates. They are based on published reports of the U. S. Bureau of the Census.

TABLE 27

POPULATION OF UNITED STATES CITIES HAVING, IN 1910, 25,000 INHABITANTS OR MORE

	1900.	1910.	1916 (estimate).
(1)	(2)	(3)	(4)
Alabama			
Birmingham	38,415	132,685	181,762
Mobile	38,469	51,521	58,221
Montgomery	30,346	38,136	43,285
Arkansas			
Little Rock	38,307	45,941	57,343
California			
Berkeley	13,214	40,434	57,653
Los Angeles	102,479	319,198	503,812
Oakland	66,960	150,174	198,604
Pasadena	9,117	30,291	46,450
Sacramento	29,282	44,696	66,895
San Diego	17,700	39,578	53,330
San Francisco	342,782	416,912	463,516
San José	21,500	28,946	38,902
Colorado			
Colorado Springs	21,085	29,078	32,971
Denver	133,859	213,381	260,800
Pueblo	28,157	44,395	54,462
Connecticut			
Bridgeport	70,996	102,054	121,579
Hartford	79,850	98,915	110,900
Meriden (town)	28,695	32,066	34,183
Meriden (city)	24,296	27,265	29,130
New Britain	25,998	43,916	53,794
New Haven	108,027	133,605	149,685
Norwich (town)	24,637	28,219	29,419
Stamford (town)	18,839	28,836	35,119
Stamford (city)	15,997	25,138	30,884
Waterbury	45,859	73,141	86,973
Delaware			
Wilmington	76,508	87,411	94,265
District of Columbia			
Washington	278,718	331,069	363,980
Florida			
Jacksonville	28,429	57,699	76,101
Tampa	15,839	37,782	53,886
Georgia			
Atlanta	89,872	154,839	190,558
Augusta	39,441	41,040	50,245

TABLE 27

POPULATION OF UNITED STATES CITIES HAVING, IN 1910, 25,000 INHABITANTS OR MORE — (*Continued*)

	1900.	1910.	1916 (estimate).
(1)	(2)	(3)	(4)
Georgia — (Continued)			
Macon	23,272	40,665	45,757
Savannah	54,244	65,064	68,805
Illinois			
Aurora	24,147	29,807	34,204
Bloomington	23,286	25,768	27 258
Chicago	1,698,575	2,185,283	2,497,722
Danville	16,354	27,871	32,261
Decatur	20,754	31,140	39,631
East St. Louis	29,655	58,547	74,708
Elgin	22,433	25,976	28,203
Joliet	29,353	34,670	38,010
Peoria	56,100	66,950	71,458
Quincy	36,252	36,587	36,798
Rockford	31,051	45,401	55,185
Springfield	34,159	51,678	61,120
Indiana			
Evansville	59,077	69,647	76,078
Fort Wayne	45,115	63,933	76,183
Indianapolis	169,164	233,650	271,708
South Bend	35,999	53,684	68,946
Terre Haute	36,673	58,157	66,093
Iowa			
Cedar Rapids	25,656	32,811	37,308
Clinton	22,698	25,577	27,386
Council Bluffs	25,802	29,292	31,484
Davenport	35,254	43,028	48,811
Des Moines	62,139	86,368	101,598
Dubuque	36,297	38,494	39,873
Sioux City	33,111	47,828	57,078
Waterloo	12,580	26,693	35,559
Kansas			
Kansas City	51,418	82,331	99,437
Topeka	33,608	43,684	48,726
Wichita	24,671	52,450	70,722
Kentucky			
Covington	42,938	53,270	57,144
Lexington	26,369	35,099	41,097
Louisville	204,731	223,928	238,910
Newport	28,301	30,309	31,927

TABLE 27

POPULATION OF UNITED STATES CITIES HAVING, IN 1910, 25,000 INHABITANTS OR MORE — (*Continued*)

	1900.	1910.	1916 (estimate).
(1)	(2)	(3)	(4)
Louisiana			
New Orleans	287,104	339,075	371,747
Shreveport	16,013	28,015	35,230
Maine			
Lewiston	23,761	26,247	27,809
Portland	50,145	58,571	63,867
Maryland			
Baltimore	508,957	558,485	589,621
Massachusetts			
Boston	560,892	670,585	756,476
Brockton	40,063	56,878	67,449
Brookline (town)	19,935	27,792	32,730
Cambridge	91,886	104,839	112,981
Chelsea	34,072	32,452	46,192
Chicopee	19,167	25,401	29,319
Everett	24,336	33,484	39,233
Fall River	104,863	119,295	128,366
Fitchburg	31,531	37,826	41,781
Haverhill	37,175	44,115	48,477
Holyoke	45,712	57,730	65,286
Lawrence	62,559	85,892	100,560
Lowell	94,969	106,294	113,245
Lynn	68,513	89,336	102,425
Malden	33,664	44,404	51,155
New Bedford	62,442	96,652	118,158
Newton	33,587	39,806	43,715
Pittsfield	21,766	32,121	38,629
Quincy	23,899	32,642	38,136
Salem	35,956	43,697	48,562
Somerville	61,643	77,236	87,039
Springfield	62,059	88,926	105,942
Taunton	31,036	34,259	36,283
Waltham	23,481	27,834	30,570
Worcester	118,421	145,986	163,314
Michigan			
Battle Creek	18,563	25,267	29,480
Bay City	27,628	45,166	47,942
Detroit	285,704	465,766	571,784
Flint	13,103	38,550	54,772
Grand Rapids	87,565	112,571	128,291

TABLE 27

POPULATION OF UNITED STATES CITIES HAVING, IN 1910, 25,000 INHABITANTS OR MORE — (*Continued*)

	1900.	1910.	1916 (estimate).
(1)	(2)	(3)	(4)
Michigan — (Continued)			
Jackson	25,180	31,433	35,363
Kalamazoo	24,404	39,437	48,886
Lansing	16,485	31,229	41,698
Saginaw	42,345	50,510	55,642
Minnesota			
Duluth	52,969	78,466	94,495
Minneapolis	202,718	301,408	363,454
St. Paul	163,065	214,744	247,232
Missouri			
Joplin	26,023	32,073	33,216
Kansas City	163,752	248,381	297,847
St. Joseph	102,979	77,403	85,236
St. Louis	575,238	687,029	757,309
Springfield	23,267	35,201	40,341
Montana			
Butte	30,470	39,165	43,425
Nebraska			
Lincoln	40,169	43,973	46,515
Omaha	102,555	124,096	165,470
South Omaha	26,001	26,259	....
New Hampshire			
Manchester	56,987	70,063	78,283
Nashua	23,898	26,005	27,327
New Jersey			
Atlantic City	27,838	46,150	57,660
Bayonne	32,722	55,545	69,893
Camden	75,935	94,538	106,233
East Orange	21,506	34,371	42,458
Elizabeth	52,130	73,409	86,690
Hoboken	59,364	70,324	77,214
Jersey City	206,433	267,779	306,345
Newark	246,070	347,469	408,894
Orange	24,141	29,630	33,080
Passaic	27,777	54,773	71,744
Paterson	105,171	125,600	138,443
Perth Amboy	17,699	32,121	41,185
Trenton	73,307	96,815	111,593
West Hoboken (town)	23,094	35,403	43,139

TABLE 27

POPULATION OF UNITED STATES CITIES HAVING, IN 1910, 25,000 INHABITANTS OR MORE — (*Continued*)

	1900.	1910.	1916 (estimate).
(1)	(2)	(3)	(4)
New York			
Albany	94,151	100,253	106,003
Amsterdam	20,929	31,267	37,103
Auburn	30,345	34,668	37,385
Binghamton	39,647	48,443	53,973
Buffalo	352,387	423,715	468,558
Elmira	35,672	37,176	38,120
Jamestown	22,892	31,297	36,580
Kingston	24,535	25,908	26,771
Mount Vernon	21,288	30,919	37,009
New Rochelle	14,720	28,867	37,759
New York	3,437,202	4,766,883	5,602,841
Manhattan Borough	1,850,093	2,331,542	575,876
Bronx Borough	200,507	430,980	1,928,734
Brooklyn Borough	1,166,582	1,634,351	2,634,224
Queen's Borough	152,999	284,041	366,126
Richmond Borough	67,021	85,969	97,881
Newburgh	24,943	27,805	29,603
Niagara Falls	19,457	30,445	37,353
Poughkeepsie	24,029	27,936	30,390
Rochester	162,608	218,149	256,417
Schenectady	31,682	72,826	99,519
Syracuse	108,374	137,249	155,624
Troy	60,651	76,813	77,916
Utica	56,383	74,419	87,401
Watertown	21,696	26,730	29,894
Yonkers	47,931	79,803	99,838
North Carolina			
Charlotte	18,091	34,014	39,823
Wilmington	20,976	25,748	29,892
Ohio			
Akron	42,728	69,067	85,625
Canton	30,667	50,217	60,852
Cincinnati	381,768	560,663	410,476
Cleveland	381,768	560,663	674,073
Columbus	125,560	181,511	214,878
Dayton	85,333	116,577	127,224
Hamilton	23,914	35,279	40,496
Lima	21,723	30,508	35,384
Lorain	16,028	28,883	36,964

TABLE 27

POPULATION OF UNITED STATES CITIES HAVING, IN 1910, 25,000 INHABITANTS OR MORE — (*Continued*)

	1900.	1910.	1916 (estimate).
(1)	(2)	(3)	(4)
Ohio — (Continued)			
Newark	18,157	25,404	29,635
Springfield	38,253	46,921	51,550
Toledo	131,822	168,497	191,554
Youngstown	44,885	79,066	108,385
Zanesville	23,538	28,026	30,863
Oklahoma			
Muskogee	4,254	25,278	44,218
Oklahoma City	10,037	64,205	92,943
Oregon			
Portland	90,426	207,214	295,463
Pennsylvania			
Allentown	35,416	51,913	63,505
Altoona	38,973	52,127	58,659
Chester	33,988	38,537	41,396
Easton	25,238	28,523	30,530
Erie	52,733	66,525	75,195
Harrisburg	50,167	64,186	72,015
Hazleton	14,230	25,452	28,491
Johnstown	35,936	55,482	68,529
Lancaster	41,459	47,227	50,853
McKeesport	34,227	42,694	47,521
Newcastle	28,339	36,280	41,133
Norristown (borough)	22,265	27,875	31,401
Philadelphia	1,293,697	1,549,008	1,709,518
Pittsburgh	451,512	533,905	579,090
Reading	78,961	96,071	109,381
Scranton	102,026	129,867	146,811
Shenandoah (borough)	20,321	25,774	29,201
Wilkes-Barre	51,721	67,105	76,776
Williamsport	28,757	31,860	33,809
York	33,708	44,750	51,656
Rhode Island			
Newport	22,441	27,149	30,108
Pawtucket	39,231	51,622	59,411
Providence	175,597	224,326	254,960
Warwick (town)	21,316	26,629	29,969
Woonsocket	28,204	38,125	44,360
South Carolina			
Charleston	55,807	58,833	60,734

TABLE 27

POPULATION OF UNITED STATES CITIES HAVING, IN 1910, 25,000 INHABITANTS OR MORE — (*Concluded*)

	1900.	1910.	1916 (estimate).
(1)	(2)	(3)	(4)
South Carolina—(Continued)			
Columbia	21,108	26,319	34,611
Tennessee			
Chattanooga	30,154	44,604	60,075
Knoxville	32,637	36,346	38,676
Memphis	102,320	131,105	148,995
Nashville	80,865	110,364	117,057
Texas			
Austin	22,258	29,860	34,814
Dallas	42,638	92,104	124,527
El Paso	15,906	39,279	63,705
Fort Worth	26,688	73,312	104,562
Galveston	37,789	36,981	41,863
Houston	44,633	78,800	112,307
San Antonio	53,321	96,614	123,831
Waco	20,686	26,425	33,385
Utah			
Ogden	16,313	25,580	31,404
Salt Lake City	53,531	92,777	117,399
Virginia			
Lynchburg	18,891	29,494	32,940
Norfolk	46,624	67,452	89,612
Portsmouth	17,427	33,190	39,651
Richmond	85,050	127,628	156,687
Roanoke	21,495	34,874	43,284
Washington			
Seattle	80,671	237,194	348,639
Spokane	36,848	104,402	150,323
Tacoma	37,714	83,743	112,770
West Virginia			
Huntington	11,923	31,161	45,629
Wheeling	38,878	41,641	43,377
Wisconsin			
Green Bay	18,684	25,236	29,353
La Crosse	28,895	30,417	31,677
Madison	19,164	25,531	30,699
Milwaukee	285,315	373,857	436,535
Oshkosh	28,284	33,062	36,065
Racine	29,102	38,002	46,486
Sheboygan	22,962	26,398	28,559
Superior	31,091	40,384	46,266

Metropolitan districts. — For some purposes the population of a city plus its adjacent suburbs is of more importance than that of the city itself. During recent years the growth of the suburbs has often been much greater than that of the city itself. This subject is discussed in U. S. Census, 1910, Population, Vol. I, p. 74.

In 1910, New York City had a population of 4,766,883, the adjacent territory, 1,863,716, or 39 per cent of the city's population. During the last decade the city increased 38.7 per cent and the adjacent territory 45.5 per cent.

In Boston the city's population was 560,892, that of the adjacent territory, 708,492, or 126 per cent of the city's population.

Classification of population. — One of the greatest mistakes which health officers make is failure to take into account the make-up of the population. Two places cannot be fairly compared as to death-rate or birth-rate, unless the composition of the population in the places is substantially the same. This point will be emphasized again in Chapter VII.

In many demographic studies it is necessary to take into account age, sex, and nationality as primary factors; and at times also such matters as marital condition, school attendance, illiteracy, ownership of homes, occupation, and so on.

It will not be possible in this volume to go into all of these classifications. They should be carefully studied, however, from the census reports themselves. Every health officer should know the composition of the people in the city or district under his jurisdiction.

Color or race, nativity and parentage. — The racial composition of the United States has changed materially in fifty years. This is well illustrated by Fig. 36. In 1850 about three-quarters of the people were native whites, now

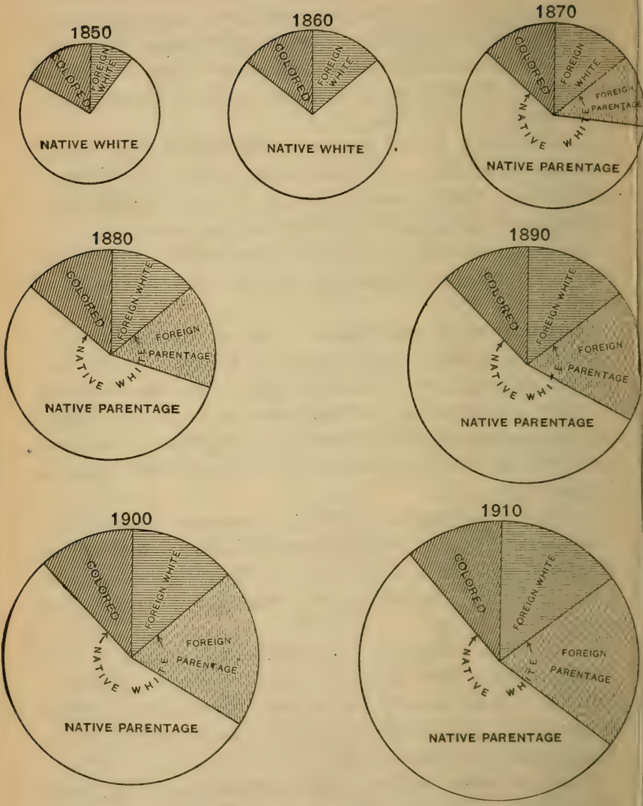


FIG. 36. — Racial Composition of Population of the United States.

only about one-half. There are great differences in different cities and states.

According to the U. S. Bureau of the Census the population is divided into six classes: (1) white, (2) negro, (3) Indian, (4) Chinese, (5) Japanese and (6) "all others." The white population is subdivided into:

a. Native, native parentage, having both parents born in the United States.

b. Native, foreign parentage, having both parents born in foreign countries.

c. Native, mixed parentage, having one native parent and the other foreign born.

d. Foreign born.

It is often desirable to subdivide the foreign born according to the country from which they came. This is true also of the parents.

Sex distribution. — There are two ways in which the sexes are compared, — one is to compute the percentage which the number of each sex is of the total population, the other is to compute the ratio of males to females. Thus, we have the following figures for 1910:

TABLE 28

COMPARISON OF SEXES IN THE UNITED STATES

United States.	Per cent.		Males to 100 females.
	Male.	Female.	
(1)	(2)	(3)	(4)
Total population	51.5	48.5	106.2
Native white, native parentage	51.0	49.0	104.1
Native white, mixed parentage	49.6	50.4	98.4
Native white, foreign parentage	50.0	50.0	100.0
Foreign born white	56.4	43.6	129.3

In most parts of the country males are in excess, and generally speaking the ratio of males to females increases from east to west. In only a few states do we find females in excess. One of these is Massachusetts, where in 1910 the ratio was only 96.6. In Nevada, on the other hand, the ratio was 181.5, not very far from two men to one woman.

Sex distribution ought to be studied in connection with age distribution.

Dwellings and families. — A knowledge of the number of persons in a dwelling or a family is of sociological interest, and it may be of practical use in estimating the population of an area the boundaries of which are not coincident with any civil division. Here we come again to the difficulty of definition. What is a dwelling? What is a family? A dwelling-house is considered to be "a place where one or more persons regularly sleep." A family is "a household or group of persons who live together, usually sharing the same table." This includes both private families, consisting of persons related by blood, and economic families.

The ideal family has been said to consist of a father and mother and three children with an occasional grandfather or grandmother, aunt or uncle. In the United States in 1910 the average number of persons to a family was only 4.5, — apparently much smaller than the ideal. The average number of persons to a dwelling was 5.2. Figures for different parts of the country are given in Table 29.

For housing problems it is not enough to know that the average number of persons per dwelling is 5.2. This extra two-tenths of a person is difficult to place. We need to know how many dwellings contain one person, how many two persons, and how many three, four, five, six, and so on. It is difficult to secure these data.

TABLE 29

SIZE OF FAMILIES AND HOUSEHOLDS

Place.	Persons per dwelling.	Persons per family.	Families per dwelling.
(1)	(2)	(3)	(4)
United States	5.2	4.5	1.15
Urban	5.9	4.5	1.31
Rural	4.7	4.6	1.02
New England	6.0	4.5	1.33
Urban	6.5	4.6	1.41
Rural	4.2	4.0	1.05
New York City	15.6	4.7	3.32
Borough of Manhattan	30.9	4.7	6.58
Boston	9.1	4.8	1.90
Cambridge, Mass.	7.2	4.6	1.57
Los Angeles, Cal.	4.6	4.1	1.12
Spokane, Wash.	5.1	4.6	1.11

Age distribution. — We now come to what is a most important division of the population, namely separation into age-groups. In connection with a study of death-rates and causes of death a knowledge of age distribution is fundamental. As a factor in vital statistics it is more important than sex or nationality or parentage or occupation or any other particular characteristic.

In taking a census it is impossible to find the exact age of every person in a community, and even if this could be done it would be impracticable to arrange the people in groups, varying by short intervals of time. Infants and young children may be grouped by their age in weeks or months, but older persons are seldom divided into groups for which the time interval is less than one year. Five-year and ten-year groups are even more commonly used. In this chapter we shall not consider smaller subdivisions than one year. The ages of infants will be taken up in the chapter which treats of infant mortality.

Census meaning of age. — If we wish to state a person's age in years, using a whole number, we may do so in one of two ways; we may give the age as that of the *last birthday* or as that of the *nearest birthday*. The difference is by no means insignificant in the case of children, for the difference of half a year would represent a large percentage of the age. In some parts of the world the *next birthday* is often stated as the age, an infant being regarded as one year of age even though he had been born only an hour. In the Orient age has to do also with the calendar year in which the child was born. A child born in November might in December be called a year old, but after January first might be called two years. These curious customs ought to be known by

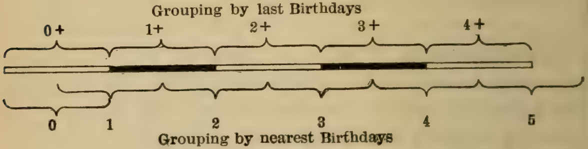


FIG. 37. — Age-Grouping by Years.

those enumerating the ages of persons in the foreign quarters of our cities and justify the check question asked by census enumerators, namely, the *date of birth*.

The *last birthday* method was used in the United States census of 1910, 1900 and 1880; it is the method used in England. In 1890, however, the *nearest birthday* was used. The effect of the definition of age on the age-grouping will be apparent from the following diagram: The nearest birthday method creates confusion in the ages of infants and children one year old. If infants include children up to the age of one year, then the "one-year" group must be limited to half a year or else there is duplication of those between six months

and one year. The discrepancies in the 1890 figures are plainly shown by the following table which gives the percentage distribution of the population under five years of age.

TABLE 30

PER CENT DISTRIBUTION OF POPULATION UNDER 5 YEARS OF AGE

Age in years.	"Nearest birthday."	"Last birthday."		
	1890.	1880.	1900.	1910.
(1)	(2)	(3)	(4)	(5)
	Per cent.	Per cent.	Per cent.	Per cent.
Under 1 year	20.5	20.9	20.9	20.9
1+	14.1	18 2	19.3	18 6
2+	22.7	20 6	20.0	20.4
3+	21.4	20.0	19.9	20.3
4+	21.3	20.3	20.0	19.9
Total under 5 years	100.0	100.0	100.0	100.0

The small number in the one-year group and the large number in the two-year group in 1890 should be noticed.

Errors in ages of children. — The above table shows that even by the last-birthday method the age distribution of children in one-year groups was unsatisfactory. Normally there are more children under one year of age than between one and two years, more between one and two than between two and three and more between three and four than between four and five. Yet in 1910 the one-year group contained fewer than the two-year group, and in 1900 the 3+ year group contained fewer than the 4+ year group.

These discrepancies are due to errors. They are greatest in populations where there is much illiteracy and where no attempt is made to check the age returns by asking the date of birth. Thus we may compare the data for Germany (1900) and the negro population of the United States (1910).

TABLE 31

PERCENTAGE DISTRIBUTION OF POPULATION UNDER 5 YEARS

Age.	Germany.	United States (Negro population.)
(1)	(2)	(3)
0+	20.6	20.0
1+	20.3	17.4
2+	20.2	20.6
3+	19.5	20.9
4+	19.3	21.1
Under 5	100.0	100.0

Errors due to use of round numbers. — An important source of error in age statistics is that of mixing round numbers with more accurate figures. In replying to the enumerator's questions concerning age most persons will state their age accurately, but some will give the nearest round number. An ignorant or careless person who may be 39 or 41 years old may give his age as 40, a figure which in his mind is near enough. This habit is encouraged by asking for the "nearest birthday" as was done in 1890. In most censuses there are enough instances of this sort to produce noticeable concentrations around the ages ending in 0 or 5. This is well illustrated by Fig. 38, which shows the population of Massachusetts males in 1905 distributed by groups. This error of round numbers is by no means confined to the subject of age. It is met with in all sorts of statistical work. Dates are often stated as "the first of the month," or the tenth or the fifteenth. These, mixed with more accurate statements, may produce abnormal concentrations. Methods of adjusting data troubled with these concentrations on the round numbers are used by statisticians and are referred to in Chapter XIV.

The U. S. Census Bureau in studying the error due to the abnormal use of round numbers has made use of a measure termed the "Index of Concentration." This was taken to be the "per cent which the number reported as multiples of

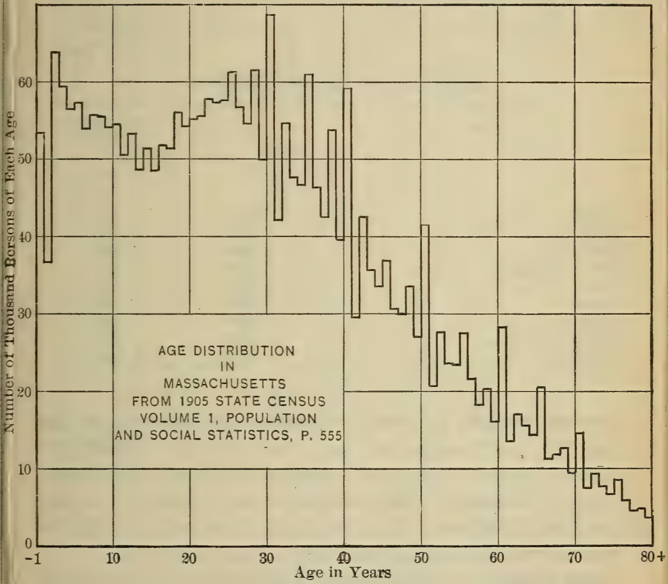


FIG. 38. — Age Distribution in Massachusetts.

5 forms of one-fifth of the total number between ages 23 to 62 years, inclusive." Thus in the U. S. there were 43 million persons aged 23 to 62 years. One-fifth of these would be 8.6 million. The total number of persons aged 23 to 62 whose age was reported as a multiple of 5 was 10.3 million. Hence the index of concentration was 10.3 ÷ 8.6, or 120.

It was found that the index of concentration increased directly with the ignorance and illiteracy. For the native white persons it was 112; for foreign born whites, 129; for colored persons, 153. It is interesting to compare these figures for those of certain other countries.

TABLE 32

ERRORS OF REPORTING AGE

Country.	Date.	Index of concentration.
(1)	(2)	(3)
Belgium	1900	100
England and Wales	1901	100
Sweden	1900	101
German Empire	1900	102
France	1901	106
Canada	1881	110
Hungary	1900	133
Russian Empire	1897	182
Bulgaria	1905	245

Other sources of error. — Besides ignorance as to age there are other sources of error. One of these is deliberate under-estimate of age, most conspicuous among middle-aged women. Another is over-estimate, most conspicuous among the aged. The latter is of relatively little weight, but the former tends to overload the early ages of adult life.

Age groups. — The primary tabulations of the census give the ages of the people by single years. For practical use and for application to particular localities it is necessary to combine them into groups of five, ten, or twenty years, or other groups suitable to particular needs. There appears to be no recognized standard of age grouping, and perhaps this is not desirable as there are many different uses to which the figures are put.

The U. S. Census states the boundaries of the groups in inclusive numbers, such as 0–4; 5–9; 10–14; etc., and not in round numbers, as 0–5; 5–10; 10–15. With the original age records given in years it is undoubtedly the most exact method.

A grouping largely used in the 1910 census was the following:

Under 5 Years
(Under 1 Year)
5–9
10–14
15–19
20–24
25–34
35–44
45–64
65 and over
Age unknown

In this arrangement we have one-year groups from ages one to five, 5-year groups from ages five to twenty-five, 10-year groups from twenty-five to forty-five, and above that twenty-year groups.

Persons of unknown age. — One of the puzzling things about age distribution is to know how to treat the "age unknown." Usually this number is not large, but in particular cases it may be. In 1910 only 0.18 per cent of the people of the United States were included in this group, and in 1900 only 0.26 per cent.

One way is to place them in a group by themselves, letting the size of the group stand as a sort of test of the accuracy of the investigation. On the whole this is probably the best thing to do.

Another way would be to distribute the unknowns pro rata through the other groups. But there is really no justi-

fication for doing so, because the persons of unknown age may be confined to certain selected ages, as the very old.

Redistribution of population. — If the ages of the people are tabulated by years it is of course easy to combine them into any desired age groups; but if the data are tabulated according to one age-grouping and it is desired to ascertain the numbers in other age-groups the problem is more difficult. Approximate results only can be expected and these can be obtained by graphical methods or by computation.

For this purpose the summation diagram is most convenient. In 1910 the population of Cambridge was as follows:

TABLE 33

POPULATION OF CAMBRIDGE, MASS., BY AGE-GROUPS

Age-group.	Number.	Age.	Persons less than stated age. Number.	Persons less than stated age. Per cent.
(1)	(2)	(3)	(4)	(5)
0–1	2,323	1	2,323	2.3
1–4	8,479	5	10,802	10.4
5–9	9,471	10	20,273	19.4
10–14	8,892	15	29,165	27.9
15–19	8,930	20	37,095	36.4
20–24	10,408	25	47,503	46.4
25–34	19,175	35	66,678	64.6
35–44	15,726	45	82,404	79.6
45–64	16,732	65	99,136	95.6
65–99	4,642	100	104,778	99.4
Unknown	61	....	61	0.6
Total	104,839	....	104,839	100.0

The figures in column (4) are plotted in Fig. 39. Let us suppose that we desire to obtain the number of persons in age-group 23–27 inclusive. The diagram shows that about 43,500 were less than 23 years old and about 53,500 less than

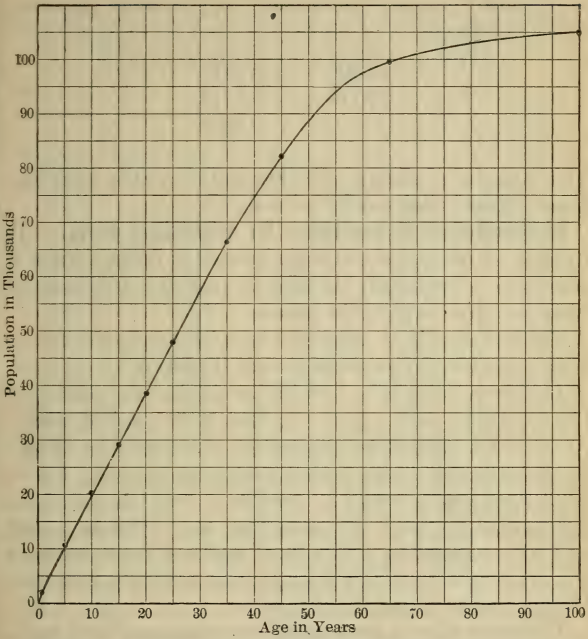


FIG. 39. — Age Distribution of Population shown by Summation Curve, Cambridge, Mass.: 1910.

28 years old. The group, therefore, contains 53,500 − 43,500, or 10,000.

In making a complete redistribution of the population in new age-groups it is well to check the results by adding them together to see that they equal the total. The accuracy of the result will depend upon the scale used for plotting and the smoothness of the curve.

We might compute the number of persons in age-group 23–27 as follows:

$$
\begin{array}{r}
10{,}408 \times \frac{2}{5} = 4163 \\
19{,}175 \times \frac{3}{10} = 5753 \\
\hline
9916
\end{array}
$$

This assumes a uniform age distribution within each age-group, which is not strictly correct.

Redistribution of population for non-censal years. — In the case of non-censal years the method of redistribution of population is essentially the same as that just described but there are three steps to the process.

The first step is to estimate the total population for the year in question by methods already described.

The second step is to find the percentage distribution of the population as it was at the time of the nearest census. As a rule the percentage composition of a population by age-groups does not change rapidly from year to year. For an intercensal year it would be possible to find the percentage distribution for both the preceding and following census and by interpolation obtain more accurate percentages for the intercensal years. The use of the summation curve is the most convenient method however.

The third step is to multiply the estimated total population by the percentage obtained in the second step. The feature of this problem obviously lies in the second step.

Let us try to find the age distribution of the population of Cambridge in the year 1906. In addition to the above figures for 1910 we have also from the census records the following figures for 1900:

TABLE 34

ESTIMATES OF POPULATION BY AGE-GROUPS FOR A NON-CENSAL YEAR: CAMBRIDGE, MASS.

Age-group.	Number.	Age.	Persons less than stated age.	
			Number.	Per cent.
(1)	(2)	(3)	(4)	(5)
0–1	2,123	1	2,123	2.3
1–4	7,519	5	9,642	10.5
5–9	8,343	10	17,985	19.6
10–14	7,331	15	25,316	27.5
15–19	7,781	20	33,097	36.0
20–24	10,588	25	43,685	47.5
25–29	9,973	30	53,658	58.4
30–34	8,157	35	61,815	67.3
35–44	12,377	45	74,192	78.5
45–54	8,561	55	82,753	90.0
55–64	5,028	65	87,781	95.5
65–99	3,652	100	91,433	99.4
Unknown	453	....	453	....
Total	91,886	....	91,886	....

The percentage distribution for 1900 and 1910 are both shown on Fig. 40. It will be noticed that the two curves coincide for the upper and lower ages, but not for the middle ages. For the year 1906 the percentages to be used would naturally lie somewhere between the two.

Progressive character of age distribution. — Among the causes of the variation of death-rates from year to year is the progressive change in age distribution. We often overlook this. We know that individuals grow old, but we forget that the 10 year old children of today will be 20 years old ten years hence, and 30 years old ten years later and so on. We are less wise than the motley fool who said:

"It is ten o'clock:
'Tis but an hour ago since it was nine,
And after one hour more 'twill be eleven;
And so from hour to hour, we ripe and ripe,
And then, from hour to hour, we rot and rot;
And thereby hangs a tale."

While the age distribution of a population does not change rapidly from year to year yet it does change. This is strikingly shown by the statistics of Sweden from 1750 to 1900.

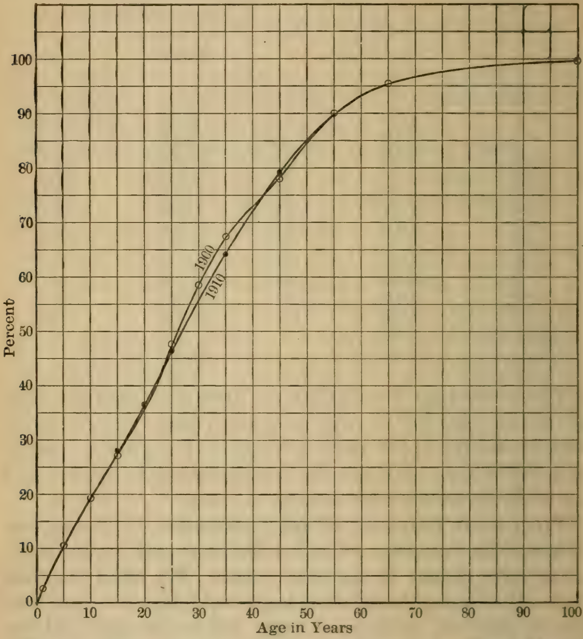


FIG. 40. — Percentage Age Distribution of Population, Cambridge, Mass., showing slight differences in ten years.

During this interval there was but little emigration or immigration, but the birth-rate varied considerably. In Fig. 41, the population data are plotted for five-year groups and for five-year intervals of time; consequently the persons who

appeared in the 0–4-group at one date would appear in the 5–9-group five years later, except as losses by death occurred. It is interesting to see how the influences which increase or decrease the numbers of children produce results which flow

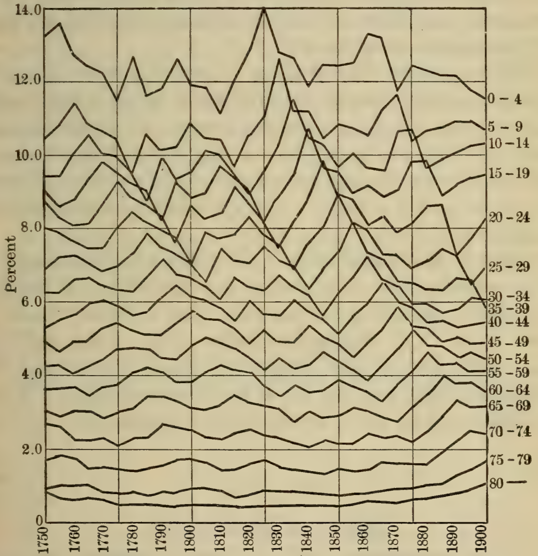


FIG. 41. — Age Distribution of the People of Sweden by Five-Year Groups: 1750–1900.

as waves throughout a long life-term. For example, the high birth-rate between 1820 and 1825, which caused a peak in the 0–4-group in 1825, caused a peak in the 5–9-group in 1830 and this could be traced for three-score years and ten. In the same way the trough in the 0–4 curve in 1810 can be followed for sixty years.

This same progressive change in age distribution can be observed in Massachusetts in spite of the fact that the curves are confused by accessions due to immigration. The peak in the 0–4-group in 1860 can be followed for fifteen years, but after that immigration appears to control. The immigration peak seen in the 20–24-group in 1880 can likewise be traced almost to 1910.

This progressive change of age is very important, for with constant specific death-rates for each age it would automatically control the general death-rate. It shows too that a loss of millions of young men in the present Great War will profoundly affect the age distribution of the nations of Europe for half a century to come. There is much food for reflection in this study.

Types of age distribution. — According to Sundbärg one of the striking features of normal age distribution is the fact that about one-half of the population are between 15 and 50 years of age. He distinguishes three types of age distribution. The first is the *Progressive Type*, the second the *Stationary Type*, and the third, the *Regressive Type*. These are illustrated by the following typical groupings:

TABLE 35

TYPES OF POPULATION

Age-group, years.	Per cent of population.		
	Progressive type.	Stationary type.	Regressive type.
(1)	(2)	(3)	(4)
0–14	40	33	20
15–49	50	50	50
50–	10	17	30

It will be noticed that in all cases, the proportion of middle-aged persons is the same, and that the classification depends upon the proportion of persons under 15 years of age to those more than 50 years of age.

To these classes might be added two more, one in which a population has lost many of its middle-aged persons by emigration and one in which a population has gained by accessions of middle-aged persons. If the percentage of persons between 15 and 50 years of age is much less than 50 it indicates that the place has lost by emigration and this may be termed the *secessive type;* while if the percentage of persons between 15 and 50 years of age is greater than 50 it may be termed the *accessive type.*

The following are examples of age distribution on the basis of this classification:

TABLE 36

TYPES OF POPULATION BASED ON AGE-GROUPING

	Per cent of population.		
	0–14 years.	15–49 years.	50 years and over.
(1)	(2)	(3)	(4)
Sweden (1751–1900)	33	50	17
United States (1910)	32	54	15
Massachusetts	27	57	16
Minnesota	32	54	14
New York State	27	58	15
Washington State	26	61	13
Maine	27	51	22
Mass., native white of native parentage	28	50	22
Mass., native white of foreign or mixed parentage	46	48	6
Mass., foreign-born white	6	74	20

It will be seen that Sweden has a normal stationary population, Massachusetts has an accessive population with 57 per cent between 15 and 50 years. Washington is even more accessive. Maine tends to be regressive, as it has an abnormally large number of persons over 50 years of age. This is also the case with the population of native-white parentage of Massachusetts. The native-white population of foreign or mixed parentage, however, is decidedly progressive.

Standards of age distribution. — For purposes of computation and comparison it is often convenient to have some standard of age distribution which can be used as a basis of reference. Several have been suggested.

A simple one was the actual population of Sweden in 1890. This was suggested because the country was not much influenced by emigration or immigration. This standard had only five groups. It was this:

TABLE 37

AGE DISTRIBUTION OF SWEDEN, 1890

Age-group.	Per cent.
(1)	(2)
0–1	2.55
1–19	39.80
20–39	26.96
40–59	19.23
60–	11.46
	100.00

The "Standard Million," namely the population of England and Wales in 1901, has been much used in adjusting birth-rates and death-rates. It is as follows:

TABLE 38

ENGLAND AND WALES STANDARD MILLION OF 1901

Age-group.	Males.	Females.	Persons.
(1)	(2)	(3)	(4)
0–5	57,039	57,223	114,262
5–9	53,462	53,747	107,209
10–14	51,370	51,365	102,735
15–19	49,420	50,376	99,796
20–24	45,273	50,673	95,946
25–34	76,425	85,154	161,579
35–44	59,394	63,455	122,849
45–54	42,924	46,298	89,222
55–64	27,913	31,828	59,741
65–74	14,691	18,389	33,080
75–	5,632	7,949	13,581

G. H. Knibbs and C. H. Wickens,[1] statisticians of the Commonwealth of Australia have worked out in a very elaborate way the probable normal age distribution of the people of Europe for the year 1900 or thereabouts. Eleven countries are considered. The results were as follows:

[1] The Determination and Uses of Population Norms representing the Constitution of Populations according to Age and Sex, and according to Age only, Transactions, 15th International Congress on Hygiene and Demography, Vol. VI, p. 352.

TABLE 39

PER CENT OF POPULATION AT EACH AGE

(Eleven Countries of Europe)

Age.	Per cent.	Age.	Per cent.	Age.	Per cent.	Age.	Per cent.	Age.	Per cent.
(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
0	2.46	19	1.90	38	1.25	57	0.67	76	0.20
1	2.43	20	1.86	39	1.21	58	0.64	77	0.18
2	2.41	21	1.83	40	1.18	59	0.62	78	0.16
3	2.38	22	1.80	41	1.15	60	0.59	79	0.13
4	2.35	23	1.76	42	1.11	61	0.57	80	0.11
5	2.33	24	1.73	43	1.08	62	0.54	81	0.10
6	2.30	25	1.69	44	1.05	63	0.51	82	0.08
7	2.27	26	1.66	45	1.02	64	0.49	83	0.07
8	2.24	27	1.62	46	0.99	65	0.46	84	0.05
9	2.21	28	1.59	47	0.96	66	0.44	85	0.04
10	2.19	29	1.56	48	0.93	67	0.42	86	0.03
11	2.15	30	1.52	49	0.89	68	0.39	87	0.02
12	2.12	31	1.49	50	0.86	69	0.37	88	0.02
13	2.09	32	1.45	51	0.84	70	0.34	89	0.01
14	2.06	33	1.41	52	0.81	71	0.32	90	
15	2.03	34	1.38	53	0.78	72	0.29	91	
16	2.00	35	1.35	54	0.75	73	0.27	92	0.02
17	1.96	36	1.31	55	0.73	74	0.24	93	
18	1.93	37	1.28	56	0.70	75	0.22	94	
								All ages	100.0

Age distribution of the population of the United States. — On account of the heterogeneous character of the people of the United States, due to immigration and to internal migrations, we find that states and cities vary widely in the age composition of their inhabitants. In the older parts of the country we find a more normal age distribution of the people, one that approaches that of Sweden and Switzerland, but in the newer sections, especially in the west, we find an abnormally large number of persons of middle-age. This is also true of cities to which persons of middle-age are drawn. On the other hand the rural districts are relatively low in the middle-age groups. There are also important differences

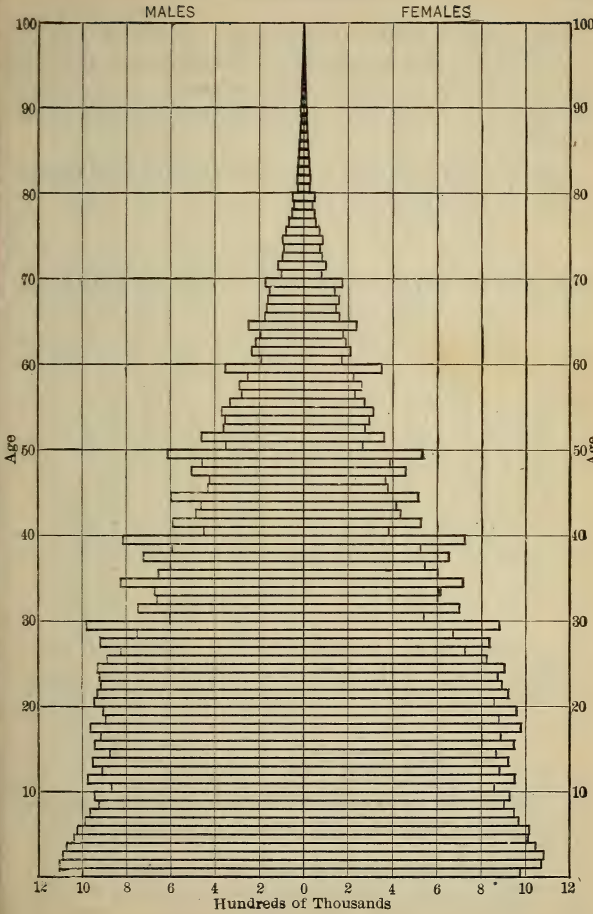


FIG. 42. — Distribution of Population by Age and Sex, United States, 1910.

between native whites, foreign born whites and negroes; and between males and females. The student is urged to study in the census reports these differences among different classes of populations and in different sections of the country.

The following table shows the percentages of total population in 1910 arranged by years:

TABLE 40

PER CENT OF TOTAL POPULATION, BY SINGLE YEARS, 1910

(United States)

Age	0	10	20	30	40	50	60	70	80	90
0	2.4	2.0	2.0	2.0	1.7	1.2	0.7	0.4		
1	2.1	1.9	1.9	1.2	0.9	0.7	0.4	0.2		
2	2.4	2.1	2.0	1.6	1.2	0.9	0.5	0.2	0.3	*
3	2.3	1.9	1.9	1.4	1.0	0.7	0.4	0.2		
4	2.3	2.0	1.9	1.4	0.9	0.7	0.4	0.2		
5	2.2	1.9	2.0	1.7	1.2	0.7	0.5	0.2		
6	2.2	2.0	1.8	1.4	0.9	0.7	0.3	0.2		
7	2.1	1.9	1.7	1.2	0.9	0.5	0.3	0.1	0.1	†
8	2.1	2.1	1.9	1.5	1.0	0.6	0.3	0.1		
9	2.0	1.9	1.5	1.2	0.9	0.5	0.3	0.1		

* Less than 0.1%. † Age unknown = 0.2%

The concentrations around the years ending in 0 and 5 should be noticed. The differences between the percentages for males and females in the whole population are relatively slight.

EXERCISES AND QUESTIONS

1. What were the points in the Washington controversy in regard to death-rates and population? [See Am. J. P. H., Apr., 1917, June, 1917, Feb., 1918.]

2. From the data given in Table 3, 13th Census, Population, Vol. I., p. 24, estimate by three methods the probable population of the United States in 1950.

3. What was the average annual percentage rate of increase of the population of the United States between 1790 and 1800, assuming a geometrical rate of increase? Between 1900 and 1910?

4. Under what temperature conditions do the people of the United States live? (See 11th Census, page ix.)

5. Under what rainfall conditions do the people of the United States live? (See 11th Census, page ix.)

6. From data given on page 314 of the 13th Census, Population, Vol. I, make a table giving the age distribution by single years of the entire population of the United States, the native white of native parentage and the foreign born white.

7. Make a plot of the last two.

8. Assuming the age distribution of the United States native white population of native parentage (both sexes) as given below, find by graphical methods the age distribution as indicated.

Given		Wanted	
Age	Per cent	Age	Per cent
6–4	13.2	0–4	?
5–9	11.8	5–14	?
10–29	21.1	15–24	?
20–29	17.7	25–34	?
30–39	13.1	35–44	?
40–49	9.2	45–64	?
50–59	6.9	65–84	?
60–69	4.4		
70–79	2.1		
80–89	0.5		

Check the result by computation from figures given in previous problems.

9. Look up the "Incremental Increase Method" of estimating future populations. (Jour. Am. Water Works Asso., March, 1915.)

10. What is meant by the "Center of Population?" Where was the centre of population in the United States in 1790? In 1910? [U. S. Census, 1910, Population, Vol. I, p. 45.]

11. What is the "median point"?

12. Which states have the largest per cents of urban population?

13. Describe Moore's "Expectancy Curve," for estimating future populations. (Engineering News, Nov. 2, 1916, p. 844.)

CHAPTER VI

GENERAL DEATH-RATES, BIRTH-RATES AND MARRIAGE-RATES

Gross death-rates (general death-rates.) — Stated simply, the death-rate is the rate at which a population dies. It is the ratio between the number of persons who die in a given interval of time and the median number of persons alive during the interval. Unless otherwise specified the interval of time is considered to be one year. For the sake of comparison the ratio mentioned is reduced to the basis of some round number of population, generally 1000. Not until such reduction is made may we consider this ratio as a " rate."

The computation is, of course, very simple. If in the year 1917 the number of deaths in a given city was 5710 and the population on July 1 of that year was 390,000, then the death-rate was:

$$5710 \div 390,000, \text{ or } 14.6 \text{ for each thousand.}$$

The death-rate for 1917 was therefore 14.6. We sometimes call this the " general " death-rate because it refers to the general population. Sometimes it is called the " crude " death-rate to distinguish it from rates corrected and adjusted in various ways. Or it may be called the " annual " death-rate. This is unnecessary, however, as death-rates are always assumed to refer to a year as the basis unless stated to the contrary. Perhaps the best term of all would be the " Gross death-rate," but this term is not as common.

Death-rates may be based on 10,000 or 100,000 or 1,000,000 of population, but 1000 is the common base for all general rates. The higher numbers, however, are often used for special rates, as described in the next chapter.

The method of estimating mid-year population was fully described in the preceding chapter.

Precision of death-rates. — The accuracy of a death-rate depends upon the accuracy of the number of deaths and the correctness of the estimated population. One or both of these may be in error. Only in a census year can the death-rate be computed from actual facts, because only in a census year is the population known by actual count. In other years, the population is *estimated*, and hence the death-rate based upon it must also be regarded as an *estimate*. Incorrect estimates of population obviously must produce incorrect death-rates. If this fact be kept in mind it will prevent one from drawing unwarranted conclusions in comparing rates which differ from each other by small amounts.

It is quite common to see the gross death-rate, referred to 1000 persons as a basis, expressed to the second place of decimals. This is warranted in the case of large populations for then the figures in the second decimal place have a significant value. It is not warranted for small populations. This, it will be remembered, was discussed in Chapter II, but the following figures will further illustrate the point.

In *A*, with a population of 1000, the number of deaths was 16 and, of course, the death-rate was 16. An error of one death, the smallest possible error, would have made the deaths 17 (or 15). In *B*, with a population of 10,000, an error of one death would have changed the rate from 16.0 to 16.1; in *C*, population 100,000, from 16.00 to 16.01, and in *D*, population 1,000,000, from 16.000 to 16.001. In a

TABLE 41

PRECISION OF DEATH-RATES

City.	Population.	Number of deaths.	Death-rate.
(1)	(2)	(3)	(4)
A	1,000 1,000	16 17	16.00 17.00
B	10,000 10,000	160 161	16.00 16.10
C	100,000 100,000	1,600 1,601	16.00 16.01
D	1,000,000 1,000,000	16,000 16,001	16.00 16.001

city of less than 10,000 population it would obviously be unreasonable to use two decimal places.

Similarly the following figures show the differences in population required to change the death-rate from 16.00 to 16.10 in cities of different size, the actual numbers of deaths remaining the same.

TABLE 42

PRECISION OF DEATH-RATES

City.	Death-rate.	Number of deaths.	Population.	Difference in population.
(1)	(2)	(3)	(4)	(5)
A	16.00 16.10	16 16	1,000 994	6
B	16.00 16.10	160 160	10,000 9,938	62
C	16.00 16.10	1,600 1,600	100,000 99,378	621
D	16.00 16.10	16,000 16,000	1,000,000 993,789	6211

It will be noticed that in all cases the percentage difference in population is the same, *i.e.*, 0.62 per cent. This percentage varies according to the death-rate. To alter the death-rate from 12.00 to 12.10, for example, if the number of deaths remained the same, would require a change of population of 0.83 per cent. The following figures show the percentage change in population required to alter the death-rate by 0.10 per 1000 from certain given death-rates.

TABLE 43

Change of rate from	Percentage change of population.
(1)	(2)
20.00 to 20.10	0.50
19.00 to 19.10	0.52
18.00 to 18.10	0.55
17.00 to 17.10	0.58
16.00 to 16.10	0.62
15.00 to 15.10	0.66
14.00 to 14.10	0.71
13.00 to 13.10	0.76
12.00 to 12.10	0.83
11.00 to 11.10	0.90
10.00 to 10.10	0.99

As a rough and ready rule we may therefore decide that for places smaller than 1000 the death-rate shall be stated in whole numbers; for places between 1000 and 100,000 one decimal shall be used; for places above 100,000 two decimal places shall be used.

Corrected death-rates. — What shall be taken as the number of deaths in a community? Shall non-residents who die within the geographical limits be included? Shall residents who die away from home be referred back to the place where they live? In other words shall the place for which the death-rate is computed be considered as a geo-

graphical area or as a community of persons? The answer must depend upon the use which is to be made of the facts.

The Bureau of the Census, looking at the matter in a broad way, takes the geographical point of view. It can hardly do otherwise. By recording deaths in the place where the deaths actually occur there is far less danger that all deaths will not be recorded and that no death will be counted twice than if a process of distribution by actual residence were attempted. It may be laid down as a general rule that in computing gross death-rates all deaths within the defined area shall be included and no others; that is, gross death-rates shall have a geographical basis.

This does not prevent the making of corrections to allow for local conditions. Often such corrections are desirable. If a hospital is located in a suburban town near a large city the deaths in that hospital should be included in the general death-rate of the town; but this figure could not be taken as an index of the hygienic or sanitary condition of the town. For such a purpose another rate — a corrected rate — should be computed, leaving out the hospital deaths. This might be called the *local death-rate*. This rate should not be used in place of the gross death-rate, but in addition to it.

If, besides the omission of non-resident deaths in institutions, the attempt is made to find and include the deaths of residents who have died away from home we might call the result the "*resident death-rate*."

The gross death-rate, or general rate, is best for purposes of national or state record. The local rate is best for environmental studies. The resident rate is useful for social and political studies.

In New York city the health department publishes a general death-rate and also a "corrected" death-rate in which the deaths are redistributed among the five boroughs

on the basis of residence. This is because so many persons residing in one borough are taken to hospitals in other boroughs. In some cases this makes an important difference. For the week ending Mar. 23, 1918, the death-rates for the five boroughs were as follows:

TABLE 44

DEATH-RATES IN NEW YORK CITY

Borough.	General death-rate.	Resident death-rate.
(1)	(2)	(3)
Manhattan	20.32	20.30
Bronx	20.20	17.68
Brooklyn	18.98	20.04
Queens	20.71	20.98
Richmond	25.13	18.98

On the basis of the gross death-rate Richmond is seen to have a death-rate much higher than Manhattan, but its resident, or "corrected," rate is lower than that of Manhattan.

At the end of a year a *preliminary* death-rate is often computed and published. Afterwards delayed reports of deaths are received and this necessitates a correction. The term "corrected" death-rate is sometimes applied to the new result. This of course is a proper use of the adjective, but a better term would be "*final*."

The term "corrected death-rate" has been used by some writers as synonymous with the "standardized death-rate," described on page 240. This use of the term is unfortunate and should be avoided.

Properly the word "corrected" should be applied only to death-rates in which changes are made in the number of deaths.

Revised death-rates. — Inasmuch as death-rates are based on *estimated* populations in post-censal years, and as these estimates are usually less accurate than intercensal estimates, it is always wise after each new census to recompute the death-rates for the preceding intercensal years if it is found that the new census is different from the estimated population. Sometimes the resulting changes are slight, but they may be considerable. The rates based on these revised estimates of population should be called "*revised death-rates.*"

Variations in death-rates in places of different size. — Wide fluctuations in the general death-rates from year to year are to be expected in small places. Having a small population a change of one death in a year may considerably alter the rate. In larger populations the fluctuations are less marked. This is well illustrated by the death-rates of three places in Massachusetts, — Boston (population 686,092 in 1910), Springfield (88,926) and Yarmouth (1420). Fig. 43 shows that the death-rate for Boston changed slowly, that of Springfield, although lower, fluctuated more, while that of Yarmouth varied through wide limits.

This very well illustrates what is sometimes called the principle of large numbers.

Errors in published death-rates. — It is necessary to use published death-rates and birth-rates with great caution. The old reports especially contain many unsuspected errors. For example, it was not at all uncommon ten or twenty years ago for the population of one census to be used year after year as the basis of death-rates, or until a new census was taken; that is, no intercensal estimates were made. Even the registration reports of Massachusetts are full of inconsistencies and cases of disagreements. In the following table the general death-rates are given in the second column as

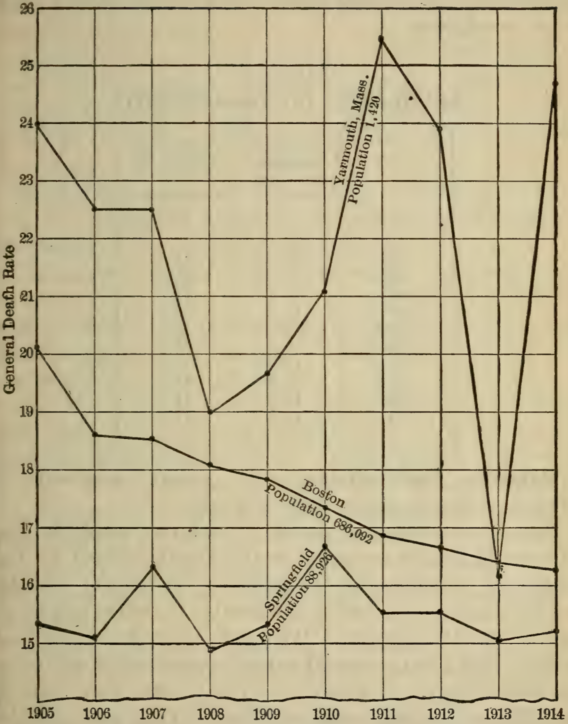


FIG. 43. — Comparison of Death-rates in a Large City, a City of Moderate Size and a Small Town.

they appeared originally in successive annual reports. In the third column the rates for the same years are given as published in the annual report for 1915, the rates having been recomputed.

TABLE 45

DEATH-RATES IN MASSACHUSETTS

Year.	As given originally in successive annual reports.	As given in report of 1915 (recomputed).
(1)	(2)	(3)
1905	16.8	16.7
1906	16.9	16.4
1907	18.1	17.2
1908	17.2	16.0
1909	17.0	15.5
1910	16.2	16.1
1911	15.8	15.8
1912	15.6	14.9
1913	15.9	14.9
1914	14.5	14.5

Rates for short periods. — The general death-rate is always computed on the basis of a year.

Strictly speaking the monthly death-rate would be the number of deaths occurring in the month divided by the estimated population for the middle of the month; and the weekly death-rate would be the number of deaths in a week divided by the estimated Wednesday population for that week. This practice would reduce population estimates to an absurd degree of precision. The months moreover do not all have the same number of days. On account of the varying estimates of population the sum of the monthly rates would not equal the annual rates.

It is much better for many reasons to reduce all rates for short periods to the basis of a year, and to use the popu-

lation estimated for July 1 for all months and weeks of the same year. Account must be taken, too, of the varying length of the months, and of the fact that a year is not exactly fifty-two weeks.

To find the death-rate for January we therefore multiply the number of deaths in January by $\frac{365}{31}$, and divide by the estimated population for July 1. For the months of thirty days the multiplier is $\frac{365}{30}$; for February it is $\frac{365}{28}$ in ordinary years, and $\frac{366}{29}$ in leap years.

To find the death-rate for any week in the year we multiply the number of deaths in the week by $\frac{365}{7}$ and divide by the population estimated for July 1st.

Birth-rates. — Birth-rates are computed in the same way as death-rates. We may have general rates, local rates, and resident rates; preliminary rates and final rates; corrected rates and revised rates. Weekly and monthly rates are reduced to a yearly basis.

One thing should be always remembered. If a child is born dead, that is if it is a "still-birth," it is not considered by statisticians as a birth. Births include only living births. Still-births are placed in a class by themselves. In some places, still-births have been included with the living births, and in comparing old birth-rates with present rates this must be kept in mind. It must be remembered also that birth registration is less complete than the registration of deaths.

Relations between birth-rates and death-rates. — The relations which exist between general birth-rates and general death-rates are very complicated. It is easy to say that because of a naturally high infant mortality, which until recently has seldom been less than 10 per cent and which in some countries is more than 25 per cent, the birth of many children means many deaths and hence a high birth-rate means a high death-rate. To a certain extent this is true.

It is true for a sudden increase in the birth-rate and its effect may last for five or ten years if the high birth-rate keeps up. But a high birth-rate adds to the population, and this increases the denominator of the birth-rate. Also most of the babies will within a few years become children and enter age groups where the specific death-rates are low. If a high birth-rate is long continued it may actually reduce the general death-rate. Fifty or sixty years after a high birth-rate there should be an excess of persons living within the advanced age groups when the specific death-rates are high and rapidly increasing.

Instead of becoming confused by trying to think out these puzzling relations, and especially so because wars and migrations upset all such reasonings, it is better to regard the birth-rate as something which together with deaths and migrations controls the age composition of the people. Conversely the age composition of the people influences both the given birth-rate and the general death-rate. One cannot think clearly on this subject without cutting loose from general rates and studying specific rates both for births and deaths.

Fecundity. — From a social standpoint the birth-rates computed in the usual way give an inadequate idea of some of the most important matters concerning the increase of population. They are ratios between births and total populations, and not all of the population included are child producers. If we are to follow the statistical principle of comparing things which are logically comparable we shall compute other ratios, that between births and women of child bearing age and that between births and married women of child bearing age, and we shall separate legitimate from illegitmate births, and take into account still births, though always keeping them separate from living, or true statistical births.

What are the chief factors which control the number of children born? The number of marriages; the effective duration of these marriages, that is the number of years between the age of the bride at marriage and the natural age when child-bearing ceases; and the frequency with which conception occurs. The number of marriages depends upon the age and sex composition of the population and upon economic and social conditions. The effective duration of marriage depends upon the age at marriage, especially the age of the bride. Obviously if marriage occurs late in life the effective duration of marriage is shortened. The frequency of conception depends to some extent upon the infant mortality as, a shortening of the period of suckling reduces the child-bearing interval; but to a considerable extent this is a matter which is, or may be, controlled by the husband and wife. The number of still births also has an influence on the intervals between living children.

Körösi[1] and others have attempted to compute tables of natality, similar to the life tables described in Chapter XIV. Statistics for Budapest indicated that the age of maximum fecundity for females reached its maximum between the eighteenth and nineteenth years, falling steadily to age fifty when it practically ceased. Males attain their maximum fecundity at the age of about twenty-five, after which there is a steady decline to age sixty-five or thereabouts. It is understood that these figures are not physiological limits necessarily, but include social and economical considerations. Late marriages therefore reduce the number of resulting children. Combinations of brides and grooms of different ages results in different probabilities of births. The following figures given by Körösi illustrate this. The percentages refer to the probability of a birth occurring in a year.

[1] 1899, Newsholme, Vital Statistics, p. 667.

TABLE 46

RELATION OF AGE TO FECUNDITY

Fecundity of mothers.				Fecundity of fathers.				
Age of father.	Age of mother.			Age of mother.	Age of father.			
	25 yrs.	30 yrs.	35 yrs.		25 yrs.	35 yrs.	45 yrs.	55 yrs.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(7)	(9)
	Per cent	Per cent	Per cent		Per cent	Per cent	Per cent	Per cent
25–29	36	25	21	20	49	......	......	......
30–34	31	24	20	20–24	43	31	16	......
35–39	27	22	19	25–29	31	27	18	......
40–44	......	17	14	30–34	33	24	14	8
45–49	......	14	11	35–39	......	19	12	7
50–54	......	......	11	40–44	......	7	6	3

Nationalities differ considerably in the number of children per marriage. For example, in Russia, the number of children per marriage in 1894 averaged as high as 5.7, while in France it was only 3.0. During recent years in most countries the birth-rates have fallen considerably. In studying this subject in its social relations, these natural conditions of fecundity as influenced by the age composition of the people, the age of marriage and the influence of nationality must be taken into account.

Illegitimate births. — Children born to unmarried women are called illegitimate. In computing general birth-rates they are included, but in the study of social problems they should be considered by themselves. The illegitimate birth-rate is the ratio between illegitimate births and the total population expressed in thousands. The percentage of illegitimacy is sometimes computed, that is the ratio between illegitimate and total births, but this ratio may be misleading as the total number of births depends on the marriage-

rate, which fluctuates more or less according to economic conditions. As a measure of morality a more useful ratio is that between illegitimate births and unmarried women of child-bearing age. It is just as important to consider the age and sex composition of a population in studying illegitimate births as in studying all births.

Newsholme has given the following interesting comparisons between two sections of London, Kensington, an aristocratic fashionable district, and Whitechapel, a poor industrial parish.

TABLE 47

BIRTH-RATES IN KENSINGTON AND WHITECHHAPEL, 1891

Birth-rate.	Legitimate.			Illegitimate.		
	Kensington.	Whitechapel.	Excess in Whitechapel.	Kensington.	Whitechapel.	Excess in Whitechapel.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
			Per cent.			Per cent.
A Per thousand of population	21.8	39.9	83	1.2	1.3	6
B Per thousand of women, aged 15–44 years	61.6	172.1	177	3.4	5.4	62
C Per thousand married women, aged 15–44 years	215.4	328.3	53			
D Per thousand, unmarried women, aged 15–44 years.	......	......	...	4.7	11.4	136

We see from this table that on the basis of married women of child-bearing age the birth-rate in the industrial district of Whitechapel was only 53 per cent greater than in the fashionable district of Kensington. On the basis of the general birth-rate or the rate computed for all women of child-bearing age the difference between the two districts

would have been said to be much greater. In Kensington there were many unmarried servants. The illegitimate birth-rate computed on the basis of total population was only 6 per cent greater in Whitechapel than in Kensington, but on the basis of unmarried women of child-bearing age it was 136 per cent greater. This is an excellent example of the necessity of considering specific rates in the study of illegitimacy. Fallacious conclusions in regard to the relative morality of different nationalities, of urban and rural districts, of different states and cities have resulted from failure to take the proper ratios as a basis of study.

Marriage-rates. — The marriage-rate is found by dividing the *number of persons* married in a year by the estimated mid-year population, expressed in thousands. The wedding-rate would be one-half of the marriage-rate. In some places this wedding-rate is called the marriage-rate, but this is not according to present-day practice. To prevent misunderstanding it is a good plan to use the expression "persons married per 1000 population."

Divorce-rates. — Similarly the divorce-rate is found by dividing the number of persons divorced in a year by the mid-year population.

Divorce in the United States is becoming more and more important as a social problem. The conditions are different in different states. In Massachusetts the data, obtained originally from court records, are published in the State Registration Report. The following figures are from the report of 1914.

The divorce-rate, based on an average for five years of which the census year was the median, has increased as follows.

TABLE 48

DIVORCE-RATE, MASSACHUSETTS

Median year.	Average rate per 100,000 population.	Average per 100,000 of married population.
(1)	(2)	(3)
1880	30	....
1890	32	86
1900	46	123
1905	58	153
1910	56	146
1914	60	156

The relative number of divorces granted to wives is larger than the number granted to husbands. At present the ratio is in round numbers 7:3.

The percentage distribution of divorces according to cause has been as follows:

TABLE 49

CAUSES OF DIVORCE, MASSACHUSETTS, 1860 to 1914

Cause.	Percentage.	
	Granted to husband.	Granted to wife.
(1)	(2)	(3)
	Per cent.	Per cent.
Desertion	56.7	41.5
Adultery	34.8	14.8
Intoxication	5.8	14.4
Cruel and abusive treatment	1.5	19.8
Nullity of marriage	0.7	0.4
Extreme cruelty	0.2	4.1
Impotency	0.2	0.2
Neglect to provide	..[1]	4.2
Imprisonment	..[1]	0.5
Total	100.0%	100.0%

[1] Less than 0.1 per cent.

About three out of every four applications for divorce in Massachusetts are granted. About nine out of ten are not contested.

The distribution of divorces according to the duration of marriage is interesting. In 1914 the average duration of the marriage at the time application for divorce was made 10.9 years. The 2963 applications were distributed as follows:

TABLE 59

DURATION OF MARRIAGES ENDING IN DIVORCE

Duration of marriage.	Per cent of applications.
(1)	(2)
0– 6 months	0.7
6–11 "	....
1–4 years	27.4
5– 9 "	30.7
10–19	28.7
20–29	10.3
30	2.2
Total......	100.0

In discussions of the divorce problem comparison is sometimes made between marriage-rates and divorce-rates. This is not a logical comparison. Why? The student must begin to answer such questions as this on the basis of his own reasoning.

In 1910 in Massachusetts divorce was dissolving each year about 3 marriages out of every 1000 in existence, or, more exactly, one out of every 342; in 1897 one out of every 580.

The U. S. Bureau of the Census has estimated the probability of divorce[1] as not less than 1 in 16, and probably 1 in 12. This figure was based on the statistics of 1900 and

[1] Marriage and Divorce, 1867–1906, Vol. I, pp. 23, 24.

means that one marriage in every 16 would probably be ended by divorce instead of continuing until "death do us part."

This general figure must not be taken too seriously, as it includes all classes of people living under many different conditions and represents past rather than present conditions.

Divorce statistics ought to be studied *specifically*, just as much as births and deaths.

Natural rate of increase. — The difference between the birth-rate and the death-rate gives the natural rate of increase (or decrease) in population per 1000 inhabitants. In the absence of immigration and emigration, and if the data are correct, the excess of births over deaths will correspond with the increase of population as revealed by the census counts. This may be illustrated by the statistics of Sweden from 1750 to 1900.

TABLE 51

INCREASE OF POPULATION IN SWEDEN

Year.	Population at end of given year (thousands).	Per 1000 of population at middle of period.		
		Increase as shown by census.	Excess of births over deaths.	Emigration (computed from last two columns).
(1)	(2)	(3)	(4)	(5)
1750	1781	8.89	8.89	0.00
1760	1925	7.76	8.43	0.67
1770	2043	5.92	6.60	0.68
1780	2118	3.71	4.14	0.43
1790	2188	3.22	4.03	0.81
1800	2347	6.99	7.96	0.97
1810	2396	2.04	2.63	0.59
1820	2584	7.60	7.52	−0.08
1830	2888	11.02	11.00	−0.02
1840	3139	8.32	8.69	0.37
1850	3482	10.39	10.51	0.12
1860	3860	10.36	11.10	0.74
1870	4169	7.57	11.24	3.67
1880	4566	9.05	12.21	3.16
1890	4785	4.69	12.12	7.43
1900	5136	7.13	10.78	3.65

The figures in the last column show that there was very little emigration before 1870, but that since then the losses by emigration have been considerable. It is quite likely, however, that some of the early birth-rates were not as accurate as the more recent ones.

Comparison of general rates. — The object of computing gross death-rates is to enable us to compare the general mortality in places of different population and of different years in the same place; and yet, as will be demonstrated in the next chapter, such comparisons are very apt to be misleading unless the percentage composition of the population remains substantially constant in all the places and in all the years which are compared. One naturally asks, "Why, if such is the case, should we compute it at all?" The answer is that in a general and crude way the gross rate does show differences in the mortality of different places, and that in any given place the composition of the population changes slowly from year to year. Large differences in general death-rates may be significant, but small differences are usually not significant.

What is true of general death-rates is also true of birth-rates and marriage-rates. Far too much attention in studies of vital statistics is given to comparisons of general rates. Such comparisons are likely to be superficial and sterile of results. Nevertheless one should have a general appreciation of the changes which have taken place in birth-rates and death-rates throughout the world during the last fifty years. A few examples will be given, but the reader should consult more extended works on the subject and compile for himself tables of rates taken from official reports.

Marriage-rates, birth-rates and death-rates in Sweden. — One of the longest records of birth-rates, marriage-rates and death-rates is that of Sweden. Table 48 shows these

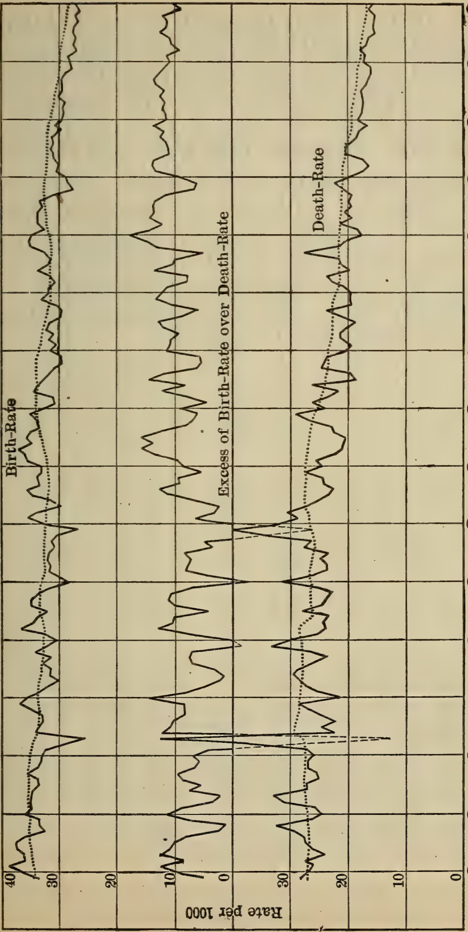


FIG. 44. — Vital Statistics of Sweden: 1750 to 1900.

rates from 1749 to 1900. The death-rates and birth-rates are also shown in Fig. 44. It will be seen that the birth-rate has had a general downward trend for a long time, but especially during the last fifty years. The death-rate has fallen more than the birth-rate so that the natural increase has risen. Of course, there have been fluctuations and some very abnormal rates will be found. As one would naturally expect the birth-rate has fluctuated synchronously with the marriage-rate. At intervals great epidemics have occurred which carried the death-rate far above the birth-rate. As a statistical series this diagram is deserving of careful study. The dotted line shows the "moving average" referred to in Chapter II.

TABLE 48

MARRIAGE-RATES, BIRTH-RATES, AND DEATH-RATES

Sweden, 1749-1900 (After Sundbärg)

Year.	Marriage-rate.	Birth-rate.	Death-rate.	Natural increase.	Year.	Marriage-rate.	Birth-rate.	Death-rate.	Natural increase.
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
1749	17.10	33.82	28.13	5.69					
1750	18.48	36.40	28.83	9.57	1780	17.06	35.70	21.74	13.96
1751	18.54	38.63	26.18	12.45	1781	14.66	33.46	25.55	7.91
1752	18.52	35.91	27.34	8.57	1782	15.36	32.05	27.26	4.79
1753	17.42	36.12	24.03	12.09	1783	15.98	30.33	28.11	2.22
1754	18.90	37.22	26.33	10.89	1784	14.96	31.53	29.75	1.78
1755	18.32	37.52	27.38	10.14	1785	15.64	31.43	28.30	3.13
1756	17.00	36.12	27.66	8.46	1786	16.04	32.89	25.94	6.95
1757	15.94	32.61	29.92	2.69	1787	15.90	31.47	23.95	7.52
1758	16.14	33.42	32.37	1.05	1788	15.78	33.87	26.68	7.19
1759	19.50	33.62	26.27	7.35	1789	15.86	32.01	33.13	-1.12
1760	19.52	35.70	24.78	10.92	1790	16.50	30.48	30.43	0.05
1761	18.88	34.82	25.80	9.02	1791	21.68	32.63	25.49	7.14
1762	17.92	35.08	31.22	3.86	1792	20.02	36.58	23.90	12.68
1763	17.28	34.98	32.90	2.08	1793	17.80	34.39	24.27	10.12
1764	17.58	34.70	27.24	7.46	1794	16.36	33.79	23.60	10.19
1765	16.30	33.41	27.68	5.73	1795	15.18	32.04	27.94	4.10
1766	16.54	35.36	25.06	10.30	1796	17.24	34.68	24.65	10.03
1767	16.54	35.36	25.63	9.73	1797	16.88	34.77	23.81	10.96
1768	16.92	33.61	27.17	6.44	1798	16.58	33.68	23.08	10.60
1769	16.26	33.06	27.15	5.91	1799	14.70	32.02	25.18	6.84
1770	16.24	32.98	26.06	6.92	1800	14.90	28.72	31.43	0.9
1771	15.52	32.24	27.77	4.47	1801	14.50	30.04	26.08	3.96
1772	13.64	28.89	37.41	-8.52	1802	15.66	31.72	23.71	8.01
1773	15.52	25.52	52.45	-26.93	1803	16.38	31.36	23.77	7.59
1774	8.77	34.45	22.36	12.09	1804	16.14	31.90	24.87	7.03
1775	18.90	35.63	24.84	10.79	1805	16.74	31.73	23.48	8.25
1776	18.02	32.92	22.50	10.42	1806	16.08	30.75	27.51	3.24
1777	18.14	33.03	24.93	8.12	1807	16.40	31.16	26.22	5.94
1778	18.10	34.82	26.65	8.17	1808	16.24	30.39	34.85	5.54
1779	17.34	36.70	28.50	8.20	1809	15.62	26.67	40.04	13.37

TABLE 48

MARRIAGE-RATES, BIRTH-RATES, AND DEATH-RATES

Sweden, 1749-1900 (After Sundbärg)

Year.	Marriage-rate.	Birth-rate.	Death-rate.	Natural increase.	Year.	Marriage-rate.	Birth-rate.	Death-rate.	Natural increase.
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
1810	21.52	32.95	31.57	1.38	1840	14.14	31.43	20.35	11.08
1811	21.32	35.30	28.81	6.49	1841	14.34	30.33	19.42	10.91
1812	18.26	33.57	30.27	3.30	1842	14.22	31.65	21.06	10.59
1813	15.48	29.74	27.37	2.37	1843	14.38	30.78	21.45	9.33
1814	15.04	31.19	25.07	6.12	1844	14.88	32.15	20.27	11.88
1815	19.22	34.77	23.59	11.18	1845	14.58	31.45	18.83	12.62
1816	19.60	35.32	22.66	12.66	1846	13.80	29.94	21.83	8.11
1817	16.68	33.40	24.25	9.15	1847	13.64	29.58	23.69	5.89
1818	16.92	33.83	24.37	9.46	1848	14.64	30.33	19.68	10.65
1819	16.28	32.99	27.36	5.63	1849	15.66	32.84	19.84	13.00
1820	16.88	32.97	24.46	8.51	1850	15.18	31.89	19.79	12.10
1821	17.62	35.44	25.57	9.87	1851	14.72	31.74	20.72	11.02
1822	18.58	35.88	22.59	13.29	1852	13.68	30.69	22.70	7.99
1823	17.98	36.83	21.02	15.81	1853	14.40	31.37	23.66	7.71
1824	17.66	34.56	20.77	13.79	1854	15.38	33.50	19.76	13.74
1825	17.20	36.49	20.54	15.95	1855	15.04	31.75	21.45	10.30
1826	16.16	34.84	22.61	12.23	1856	14.88	31.47	21.77	9.70
1827	14.44	31.30	23.05	8.25	1857	15.50	32.43	27.58	4.85
1828	15.82	33.61	26.74	6.87	1858	16.22	34.77	21.69	13.08
1829	15.82	34.85	28.97	5.88	1859	16.56	34.99	20.13	14.86
1830	15.46	32.91	24.08	8.83	1860	15.60	34.83	17.65	17.18
1831	13.80	30.49	26.00	4.49	1861	14.54	32.57	18.47	14.10
1832	14.38	30.86	23.38	7.48	1862	14.52	33.38	21.40	11.98
1833	15.66	34.11	21.74	12.37	1863	14.52	33.62	19.33	14.29
1834	16.02	33.74	25.68	8.06	1864	13.96	33.61	20.25	13.36
1835	15.00	32.67	18.55	14.12	1865	14.14	32.81	19.36	13.45
1836	14.34	31.84	19.97	11.87	1866	13.44	33.11	19.98	13.13
1837	13.80	30.84	24.65	6.19	1867	12.18	30.83	19.64	11.19
1838	12.18	29.37	24.10	5.27	1868	10.92	27.47	20.98	6.49
1839	13.54	29.49	23.56	5.93	1869	11.28	28.25	22.27	5.98

TABLE 48

MARRIAGE-RATES, BIRTH-RATES, AND DEATH-RATES

Sweden, 1749-1900 (After Sundbärg)

Year.	Marriage-rate.	Birth-rate.	Death-rate.	Natural increase.	Year.	Marriage rate.	Birth-rate.	Death-rate.	Natural increase.
(1)	(2)	(3)	(3)	(5)	(1)	(2)	(3)	(4)	(5)
1870	12.04	28.78	19.80	8.98					
1871	12.98	30.42	17.21	13.21					
1872	13.86	30.04	16.28	13.76					
1873	14.62	30.80	17.20	13.60					
1874	14.54	30.85	20.32	10.53					
1875	14.10	31.17	20.27	10.90					
1876	14.16	30.84	19.59	11.25					
1877	13.66	31.07	18.66	12.41					
1878	12.94	29.83	18.06	11.77					
1879	12.58	30.52	16.94	13.58					
1880	12.64	29.36	18.10	11.26					
1881	12.38	29.07	17.68	11.39					
1882	12.66	29.35	17.35	12.00					
1883	12.86	28.94	17.31	11.63					
1884	12.06	30.01	17.53	12.48					
1885	13.26	29.44	17.75	11.69					
1886	12.82	29.76	16.61	13.15					
1887	12.50	29.66	16.13	13.53					
1888	11.84	28.78	15.99	12.79					
1889	11.98	27.74	15.99	11.75					
1890	11.98	27.95	17.12	10.83					
1891	11.66	28.27	16.81	11.46					
1892	11.38	26.98	17.88	9.10					
1893	11.30	27.36	16.83	10.53					
1894	11.48	27.10	16.38	10.72					
1895	11.74	27.49	15.19	12.30					
1896	11.90	27.18	15.64	11.54					
1897	12.12	26.67	15.35	11.32					
1898	12.28	27.11	15.08	12.03					
1899	12.48	26.35	17.65	8.70					
1900	12.30	27.00	16.84	10.16					

Downward trend in birth-rates and death-rates.—For nearly half a century there has been a general downward trend in the birth-rates and death-rates of almost all civilized countries. There is space here for only a few figures which represent averages for quinquennial periods. They are taken from the reports of the Registrar-General of England.

TABLE 49

CHRONOLOGICAL CHANGES IN VITAL RATES

Country.	Quinquennial averages.						
	1881–5	1886–90	1891–5	1896–00	1901–5	1906–10	1911–15
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Birth-rates.							
England and Wales	33.5	31.4	30.5	29.3	28.1	26.3	23.6
Germany	37.0	36.5	36.3	36.0	34.3	32.7	....
France	24.7	23.1	22.3	21.9	21.2	19.9	....
Hungary	44.6	43.7	41.7	39.4	37.2	37.0	....
Death-rates.							
England and Wales	19.4	18.9	18.7	17.7	16.0	14.7	14.3
Germany	25.3	24.4	23.3	21.2	19.9	17.5	....
France	22.2	22.0	22.3	20.7	19.6	19.2	....
Hungary	33.1	32.1	31.8	27.9	26.2	25.0	....
Rates of natural increase.							
England and Wales	14.1	12.5	11.8	11.6	12.1	11.6	9.3
Germany	11.7	12.1	13.0	14.8	14.4	15.2	....
France	2.5	1.1	0.0	1.2	1.6	0.7	....
Hungary	11.5	11.6	9.9	11.5	11.0	12.0	....

In most countries the natural rate of increase tends to lie between the limits of 8 and 14 per 1000, *i.e.*, between 0.8 and 1.4 per cent, but sometimes it runs above 1.4 per cent or below 0.8 per cent per year. France is an example of

an extremely low rate of natural increase. In Germany both the birth-rates and death-rates have been higher than in England. In Hungary both rates have been much higher than in Germany, yet the rate of natural increase has been lower. The student should seek to explain all of these facts.

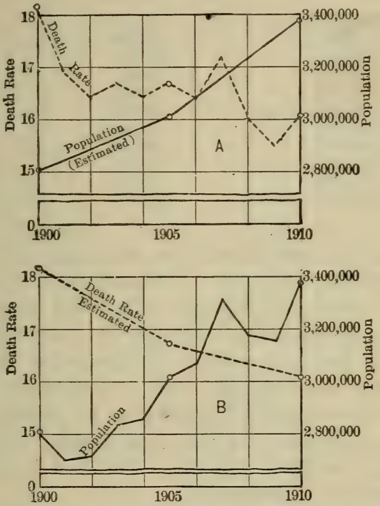


FIG. 45. — Estimated Death-rates and Populations, Massachusetts, 1900–1910.

Variations due to population estimates. — Some of the variations in the general death-rates from year to year are due to the use of incorrect population estimates. The following comparison is interesting.

Fig. 45 shows the populations and death-rates for the state of Massachusetts from 1900 to 1910, based on the following data:[1]

[1] Registration Report, 1914, p. 176.

TABLE 50

DEATH-RATES: MASSACHUSETTS

Year.	Population.	Deaths.	Death-rates.
(1)	(2)	(3)	(4)
1900	**2,805,346** (census)	51,156	18.2
1901	2,849,047	48,275	16.9
1902	2,889,386	47,491	16.4
1903	2,929,725	49,054	16.7
1904	2,970,064	48,482	16.3
1905	**3,015,872** (census)	50,486	16.7
1906	3,089,029	50,624	16.4
1907	3,162,186	54,234	17.2
1908	3,235,343	51,788	16.0
1909	3,308,500	51,236	15.5
1910	**3,380,151** (census)	54,407	16.1

The upper diagram shows the estimated population as a uniform change from 1900 to 1905 and again from 1905 to 1910. The death-rates computed from the actual deaths and estimated populations are seen to vary irregularly. But suppose we assume that the changes in death-rates between 1900 and 1905 and 1905 and 1910 are uniform. Then we can compute the changes in population from these estimated rates and the actual deaths. The results are shown in the lower diagram. Do these irregular fluctuations seem to be reasonable?

There is no way of telling exactly how much of the increase or decrease in the general death-rate is due to actual increase in mortality and how much to error in the estimated population. Both factors are involved.

Birth-rates and death-rates in Massachusetts. — Fig. 46 shows the annual variations in birth-rates and death-rates from 1850 to 1915. The stars indicate the so-called panic years, or years of business depression. The tendency has been for the marriage-rate and the birth-rate to fall for a number of years after a period of depression. Since

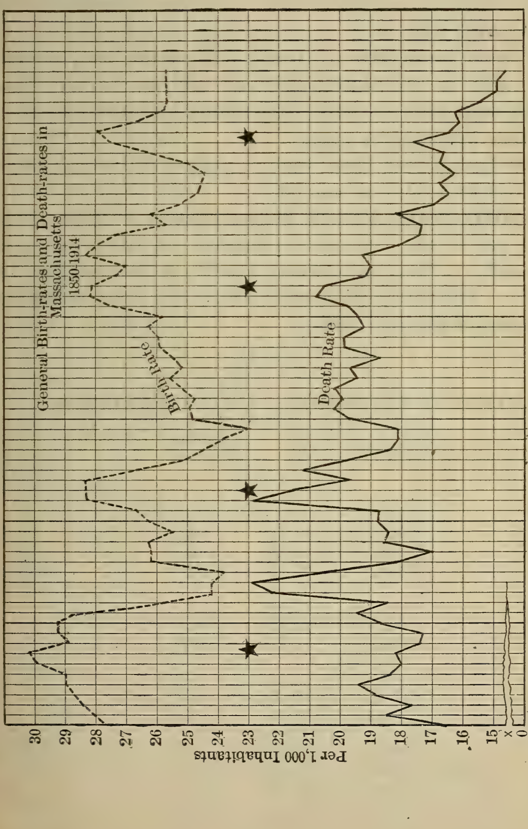


FIG. 46. — Birth-rates and Death-rates in Massachusetts. The stars show the "Panic Years."

1892 the death-rate has decreased considerably. The general synchronism between the birth-rate and the death-rate should be noticed, — and also the numerous exceptions to the rule.

In recent years there has been a marked tendency towards uniformity in the death-rate, not only in the state as a whole from year to year, but among the subdivisions of the state in any given year.

The fluctuations from year to year are due in part to incorrect estimates of population.

Monthly death-rates in Massachusetts. — The general death-rate is not constant throughout the year, but varies seasonally. There are several ways in which this may be shown. The following figures are for the state of Massachusetts for the year 1915.

TABLE 51

MONTHLY DEATH-RATES: MASSACHUSETTS, 1915

Month.	Rate.	Per cent of annual rate.
(1)	(2)	(3)
Jan.	14.65	102
Feb.	13.60	94
Mar.	16.80	117
Apr.	17.45	121
May	13.95	97
June	12.35	86
July	12.55	87
Aug.	13.10	91
Sept.	13.75	96
Oct.	13.40	93
Nov.	13.10	91
Dec.	16.00	111
Year	14.40	100

Monthly rates vary considerably from year to year in the same place and are different for different places. Climatic conditions have an influence on these short term rates. The chance occurrence of communicable diseases also has its effect. Weekly rates fluctuate even more widely than monthly rates, and daily rates *à fortiori*.

The ten year average for Massachusetts for 1905–14 gave the highest winter rate for February and not for April as in 1915; and the highest summer rate was in August, not September as in 1915.

The student should compute and study monthly rates for places where the climatic conditions are different.

Marriage-rates in Massachusetts. — Marriage-rates rise and fall periodically. The rate is influenced by social and economic conditions, by the age distribution of the population, the ratio of the sexes at marriageable age, by nationality, and by many causes. There has been no steady downward trend in the marriage-rate as in the case of the death-rate and birth-rate. There is, however, a seasonal variation. June and October are the most popular months for weddings.

In 1915 the Massachusetts marriage-rate for the year was 17.0; in June it was 27.5, in October, 23.9, but in March only 6.6. There were four times as many weddings in June as in March.

Since 1870 the median marriage-rate in Massachusetts has been 18.0. After the panic of 1873 the annual rates for several years ranged between 15 and 16.5, and after other periods of business depression they have been below 17. The highest annual rate in nearly fifty years was during 1871 and 1872, when it was 21.1 per thousand.

The published statistics of marriages generally include tables classified according to the age and nativity of the bride and groom; the number of the marriage (whether first,

second, third, etc.); and the previous state of the persons wed (whether bachelors, maids, widowers, or widows).

Divorce-rate in Massachusetts. — In 1915 the number of persons divorced was 1.22 per 1000 population; in 1914 the rate was 1.21; in 1910, 1.15; in 1890, 0.58; in 1870, 0.52.

Limited use of general death-rates (gross rates). — General death-rates are composite figures. They cover the entire population, both sexes, all ages and nationalities, all occupations, all causes of death, while the estimates of population are often inaccurate. Fluctuations from year to year depend in part on the size of the population, in part upon the composition of the population, as well as upon causes of death. Under these conditions it is evident that they cannot be safely used as an index of mortality conditions in different places and for long periods of time in any one place.

A general death-rate, or gross death-rate, is of little use until it has been analyzed.

The "total solids" in a water analysis gives the chemist almost no idea of the quality of the water: it is necessary to separate the "solids" into their constituent parts. In the same way a general death-rate must be broken up into its constituent parts. At the present time the analysis of death-rates is practiced but little. Death-rate analysis today is in about the same condition that water analysis was in fifty years ago.

The necessary analysis cannot be made until the important subject of specific death-rates has been considered in the next chapter.

The ideal death-rate. — Is there such a thing as an ideal death-rate? At present our general death-rates are falling. They cannot continue to fall forever, for man is mortal and all must die? A large part of the decrease in the

death-rate can be traced to sanitary, hygienic and medical improvements. Another part may be due to a lowering birth-rate following a relatively high birth-rate, or in other words to an increasing ratio of persons in the young and middle-aged groups. This condition will not continue permanently. In due course the young will become middle aged and the middle aged will become old, the excess of population will enter those age-groups where the specific death-rates are high and this will cause the general death-rate to rise. Or the birth-rate will rise and temporarily this will raise the general death-rate.

Unless public health officials learn how to view general death-rates in a proper light — a good way being not to view them at all — they may be surprised and discouraged some day to find that the death-rate is rising.

The Great War in a most horrible and pitiful way cut out a large number of males in the middle-aged groups in many countries. Temporarily this will increase the general death-rate. On the other hand these young men will not live to enter the old-age groups where the specific death-rates are high. What effect will this have on the future trend of the death-rate? What effect will it have on the birth-rate?

Perhaps it may be for the best interest of the race that the general death-rate be higher than it now is. This would be the case if there should be more babies and more grandfathers and grandmothers. To answer the question as to what is the lowest practicable death-rate we must first decide what is an ideal distribution of population as to age and sex, and then consider what diseases at the different ages we can reasonably expect to eliminate. It is an interesting problem for thought and discussion.

EXERCISES AND QUESTIONS

1. Plot the general death-rates of Massachusetts by years from 1850 to date. Connect the points with straight lines. Then draw straight lines connecting the death-rates for the years divisible by ten. Why is the resulting curve so regular? Connect the points for years ending in 9. Why is the resulting curve so irregular?

2. Compare the published statistics for tuberculosis as given by local, state and federal authorities. Explain the differences. [See Am. J. P. H., May 1913, p. 431.]

3. Compute the following death-rates, carrying the results only as far as accuracy warrants.

Population	Deaths per year
5,461,200	70,210
261,500	2,913
35,000	421
5,260	98
897	17

4. How does the marriage-rate ordinarily compare with the ratio of marriages to persons eligible to marriage (bachelors, spinsters, widowers, widows and divorced persons, all of marriageable age)? [Newsholme's "Vital Statistics," p. 58.]

5. How do the marriage-rates in cities compare with those in rural districts?

6. Is the marriage-rate a reliable "barometer of prosperity," as Dr. Farr called it?

7. What effect has war on the marriage-rate?

8. What proportion of marriages are remarriages?

9. Are remarriages more common among widowers or widows?

10. Prepare a table showing the marriage state (single, married, widowed, divorced) of the population of some civil division for each sex and for different age-groups above age fifteen. [Consult census reports.]

11. At what ages do people in different social positions marry?

12. What changes, if any, have taken place in the age of marriage among people of different social position during recent years?

13. How do the general birth-rates for urban and rural districts compare with each other?

14. How do the birth-rates for urban and rural districts compare with each other if based on the number of married women of child-bearing age?

15. What relation is there between birth-rates based on married women of child-bearing age and the social position of these women?

16. What relation is there between the birth-rate thus computed and the age of marriage?

17. What influence has war on the general birth-rate?

18. What influence has national prosperity on fecundity?

19. How do the general birth-rates compare for different political countries, such as England, Ireland, France, Germany, Austria, Belgium, etc.?

20. How do the birth-rates for different nationalities in the United States compare with each other?

21. How does the birth-rate for the Irish in Ireland compare with that of the Irish in Massachusetts?

22. How do the birth-rates among Catholics compare with that among Protestants? Consult the statistics of Canada, especially the provinces of Ontario and Quebec.

23. What is the ratio of males to females among births?

24. What is the ratio of males to females among still-births?

25. What is the ratio of males to females among illegitimate births?

CHAPTER VII

SPECIFIC DEATH-RATES

Although general death-rates have their uses, something more is needed if statistics of mortality are to be used to their best advantage. The tendencies of human beings to die are not constant; diseases differ in their fatality; persons of different age differ in susceptibility to disease; sex, nationality, connubial condition are likewise variable factors. One cannot properly use mortality statistics in public health work without taking these factors into account, at least without considering the most important of them. This brings us to a consideration of *specific death-rates.* General death-rates are ratios between the entire population of a given place and all deaths which occur in a year. We may restrict these rates in several ways.

Restrictions of death-rates. — We may consider a shorter period than a year, and compute the rate for a month or a week and thus obtain a partial rate or a short-term rate as described in the previous chapter. This, however, is not usually classed as a specific rate.

We may restrict the computation to a special class or group of the population; that is, we may take into account only males or only females and compute the death-rate for them alone. These would be specific death-rates by sex. We may consider each age-group by itself and find the death-rate for it alone. This would be to compute specific death-rates by age-groups. Or we may take only persons of the same nationality or occupation and compute specific death-rates for them.

Again we may consider separately the different causes of death, and compute specific death-rates for tuberculosis, for scarlet fever, or for cancer.

Finally we may consider particular diseases and at the same time restrict the computation to certain classes or groups of people; thus we may compute the "typhoid fever death-rate for males in age group 15–19 years."

It has been suggested that these various modes of restriction might be designated by such expressions as "special death-rates," "particular" rates, "limited" rates, etc., but apparently the common expression "specific" death-rate serves every useful purpose.

It is the purpose of the present chapter to describe the methods of computing specific rates and to call attention to their importance. It is not too much to say that *an understanding of specific rates is the key to the interpretation of vital statistics.* Failure to appreciate the important influences of age is alone responsible for scores of fallacious conclusions derived from tables of vital statistics.

Age. — The span of human life has been divided into age periods in many different ways. Shakespeare[1] vividly describes the seven ages of man.

> *Jaques:* All the world's a stage,
> And all the men and women merely players:
> They have their exits and their entrances;
> And each man in his time plays many parts,
> His acts being seven ages. At first the infant,
> Mewling and puking in the nurse's arms;
> Then the whining school-boy, with his satchel
> And shining morning face, creeping like snail
> Unwillingly to school; and then the lover,
> Sighing like furnace, with a woeful ballad
> Made to his mistress' eyebrow; then a soldier,
> Full of strange oaths and bearded like the pard,

[1] Jaques in *As You Like It*, Act II, Scene VII.

Jealous in honour, sudden and quick in quarrel,
Seeking the bubble reputation
Even in the cannon's mouth; and then the justice,
In fair round belly with good capon lin'd,
With eyes severe and beard of formal cut,
Full of wise saws and modern instances;
And so he plays his part; the sixth age shifts
Into the lean and slipper'd pantaloon,
With spectacles on nose and pouch on side,
His youthful hose well sav'd, a world too wide
For his shrunk shank; and his big manly voice,
Turning again toward childish treble, pipes
And whistles in his sound; last scene of all,
That ends this strange eventful history,
Is second childishness and mere oblivion,
Sans teeth, sans eyes, sans taste, sans every thing.

Just where to draw the age lines between Shakespeare's seven ages is a most difficult matter and it would be hard to get any two people to agree. The divisions suggested in Fig. 47 are merely for provoking discussion.

Physiologically seven fairly distinct states may be recognized — the pre-natal state, infancy, childhood, youth (maidenhood), early manhood and manhood (child-bearing age and maturity), and finally old age, or senility. The age limits of the early groups are fairly well marked. The later groups are more indistinct. He would be a bold person who would undertake to establish an age limit for senility. Every one knows what was said about Dr. Osler when he attempted to do something of that sort. In Fig. 47 the biblical limit of "three score years and ten" has been used. The author believes that he may safely hide behind that. The division between childhood and youth in boys is perhaps not quite the same as the division between childhood and maidenhood in girls.

From the standpoint of environment there are several fairly distinct age periods. Infancy in this case means the

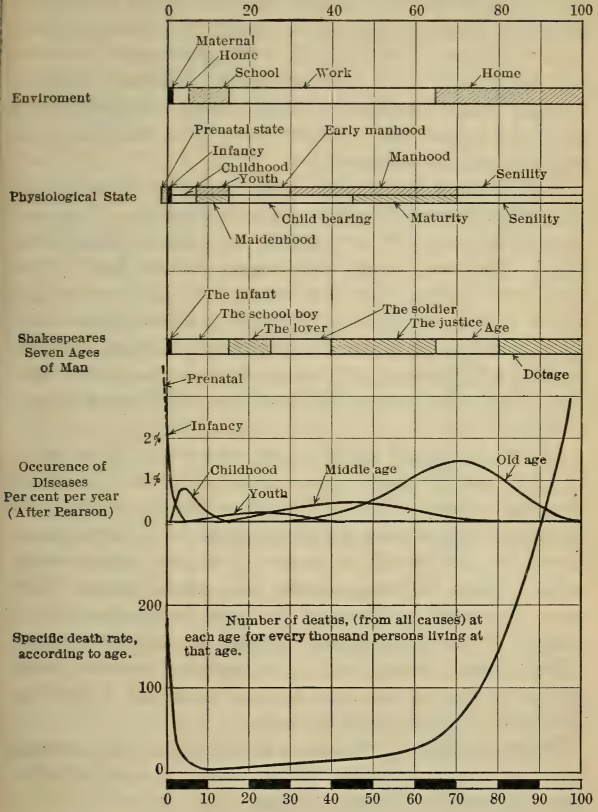


FIG. 47. — Ages of Man.

earliest period, in which the environment is maternal. It terminates when the child is weaned. Then follows the period of home environment. Later the school environment controls. After that the work place comes in as an important factor. Of course the home influence continues through life, and in the case of most women it predominates after the school age. Indeed after the school age the environment becomes complex.

Karl Pearson has analyzed the curve which shows the age distribution of deaths in an interesting way. He concludes that there are five groups of diseases, those of infancy, childhood, youth, middle age and old age. All of these extend over wide limits, but culminate at the ages shown in Fig. 47. One may die of an old age disease at thirty, or one may have a children's disease at forty. Endless complications exist in special cases, yet in the main the distinctions between the five classes of diseases are well known.

At the bottom of the diagram we see the curve which shows the specific death-rate in its characteristic variations through the span of life. This curve in its general form is the same for both sexes, for all nationalities, for all climates. There are differences, of course, but over all the other factors which influence death, age predominates. This curve, it should be observed, is based on deaths from all causes. It would not necessarily apply to particular diseases.

The student should study this curve of specific death-rates according to age until he can reproduce it with approximate accuracy from memory.

Vision of Mirza. — Those who do not enjoy studying statistics may appreciate the following paragraph taken from Addison's "Vision of Mirza."

"The bridge thou seest, said he, is *Human Life;* consider it attentively. Upon a more leisurely survey of it,

I found that it consisted of *threescore and ten entire arches*, with several broken arches, which, added to those that were entire, made up the number *about an hundred*. As I was counting the arches, the Genius told me that this bridge consisted at first of a *thousand arches;* but that a great flood swept away the rest, and left the bridge in the ruinous condition I now beheld it. But tell me further, said he, what thou discoverest on it. I see multitudes of people passing over it, said I, and a black cloud hanging on each end of it. As I looked more attentively, I saw several of the passengers dropping through the bridge into the great tide that flowed underneath it: and upon further examination perceived that there were innumerable trap-doors that lay concealed in the bridge which the passengers no sooner trod upon, but they fell through them into the tide, and immediately disappeared. These hidden pit-falls were set very thick at the entrance of the bridge, so that throngs of people no sooner break through the cloud, but many of them fell into them. They grew *thinner towards the middle*, but multiplied and laid closer together towards the end of the arches that were entire. There were, indeed, persons, but their number was very small, that continued a kind of hobbling march of the broken arches, but fell through one after another, being quite tired and spent with so long a walk."

How to compute specific death-rates. — The specific death-rate for any age-group is found by dividing the number of deaths of persons whose ages lie within the group limits by the number of thousands of persons in the same group alive at mid-year. The computation is precisely the same as that for the general death-rate except that both deaths and population are confined to specific age-groups. If both quantities are known the process is merely arithmetical.

Example: — Given the following data for New South Wales, 1901 (Columns 1, 2, 3).

TABLE 52

Age-group.	Population.	Deaths.	Specific death-rate.
(1)	(2)	(3)	(4)
0–1	40,500	3,234	79.9
1–19	704,000	1,960	2.8
20–39	514,900	2,251	4.4
40–59	256,600	2,965	11.6
60–	89,800	5,400	60.1
Total	1,605,800	15,810	9.85

To find the specific death-rate for the age-group 1–19 years, divide the number of deaths in that group, *i.e.*, 1960 by 704, the number of thousands of population. The result is 2.8 per 1000. Similarly the specific death-rate for age-group 20–39 is 2251 ÷ 515 = 4.4 per 1000. The figures in Column 4 were thus computed. The total deaths divided by the total population, in thousands, gives the general death-rate, *i.e.*, 15810 ÷ 1605.8 = 9.85 per 1000.

If the number of deaths within the age-group is known but the population is unknown, it is necessary to estimate the population in the group. This can usually be done with sufficient accuracy from the data provided by the censuses. The methods of making these estimates both for censal and non-censal years has been already described. This may involve a redistribution of the population from those given in the census to those corresponding to the death statistics.

If the population in the group is known but the number of deaths is unknown the computation cannot be made with accuracy. It might be possible to redistribute the deaths into age-groups corresponding to the population, but

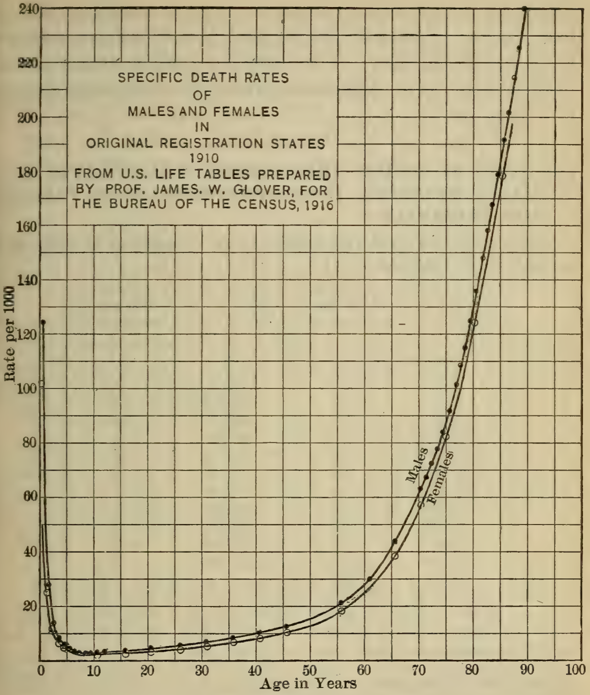


Fig. 48. — Specific Death-rates.

specific death-rates obtained in this way would in most cases be unreliable.

Specific death-rates by ages for males and females. — Looking at the two curves for the specific death-rates of males and females shown in Fig. 48 one would say at first that they were much alike, but that the rates at all ages were

higher for males than for females. In a general way this is true, but a closer study shows that the differences are not the same for all ages. The table from which these curves were plotted gave data from which the following figures were obtained.

TABLE 53

PER CENT BY WHICH THE SPECIFIC DEATH-RATES FOR MALES EXCEEDED THOSE FOR FEMALES IN VARIOUS AGE INTERVALS

(Based on the original registration states; population in 1910, and deaths in 1909, 1910 and 1911).

Age interval.	Per cent (approximate).	Age interval.	Per cent (approximate).
(1)	(2)	(3)	(4)
yr.		yr.	
0-1	20	55–56	14
5–6	5	60-61	19
10–11	15	65–66	14
15–16	5	70–71	10
20–21	15	75–76	12
25–26	6	80–81	8
30–31	10	85–86	7
35–36	20	90–91	3
40–41	35	95–100	−2
45–46	27	100–101	3
50–51	23		

In infancy the death-rate for males exceeds that for females by 20 per cent. Between five and twenty-five years of age the differences vary considerably in successive years but average about 10 per cent.[1] Above age twenty-five the male death-rate begins to exceed the female death-rate by considerable amounts and this continues to the age of forty, when the excess is 35 per cent. After that it steadily decreases. In old age the two rates are much alike. It must be remembered that these figures are for a certain

[1] The error of population due to concentration on round numbers probably accounts for some of these differences.

limited area and for a short interval of time and for a particular composition of people with respect to nationality, birth-rate, and so on. They are to be regarded merely as illustrative of the differences between males and females. What are the reasons for the differences here shown?

Effect of marital condition on specific death-rates. — Students will find it interesting to compute specific death-rates for males and females according to their marital condition. It will be found that the rates for single men are considerably higher than for married men. Between thirty and forty years of age they may be nearly twice as high; at higher ages the percentage differences become less. The death-rates of single females are higher than those of married females except that during part of the child-bearing period, — say from twenty to forty-five, — the rates are higher for married women.

Professor Walter F. Willcox, of Cornell University, has computed the following specific death-rates for New York State, the cities of New York and Buffalo excluded, for 1909–1911, arranged by age-groups and by classes corresponding to marital condition, as follows:

TABLE 54

SPECIFIC DEATH-RATES ACCORDING TO AGE AND MARITAL CONDITIONS, NEW YORK, 1909-11

Age-group.	Males.			Females.		
	Single.	Married.	Widowed or divorced.	Single.	Married.	Widowed or divorced.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
20–29	6.6	4.2	12.0	4.7	5.7	9.4
30–39	12.9	5.9	14.1	7.4	6.3	9.5
40–49	19.5	9.5	17.3	10.0	8.2	12.1
50–59	28.7	17.0	30.5	19.9	14.5	18.8
60–69	51.0	31.9	48.6	37.1	28.1	38.2
70–79	101.4	72.7	96.0	82.2	61.4	87.2
80–	204.2	205.1	315.7	279.8	194.8	269.8

Among the theories suggested in explanation of these differences is the effect of leading a better supervised and more restrained life among married persons, the better economic conditions of the married, the effect of marriage selection and the effect of the marriage relation itself.

Nationality and specific death-rates. — Specific death-rates for different ages and sexes are not the same for all nationalities. It is very difficult, however, to say how much of this is due to racial difference and how much is due to environmental conditions; that is, it is hard to separate the physiological from the social and economic factors. Practically, however, these factors must be considered together in discussing nationalities in the United States. We see these differences well marked between the negro and the white populations of the original registration states. The figures in Table 55, taken from Professor Glover's report, will show this.

The figures in this table are carried to an unnecessary degree of precision so far as this particular point is concerned and in the case of the advanced ages for negroes probably not even the whole numbers are accurate. The rate for male negroes is almost double that for male whites up to the age of sixty or thereabouts; above eighty the rate for negroes is lower than for whites. Substantially the same relations hold for white and colored females.

It should be noticed that in these various comparisons the effect of age is a factor which must never be left out of account.

TABLE 55

SPECIFIC DEATH-RATES FOR WHITE AND NEGRO MALES

United States, Original Registration States, 1910

Age interval.	Rates per 1000.	
	White.	Negro.
(1)	(2)	(3)
0–1	123.26	219.35
5–6	4.71	8.56
10–11	2.38	5.02
15–16	2.83	7.87
20–21	4.89	11.96
25–26	5.54	12.28
30–31	6.60	14.96
35–36	8.52	17.28
40–41	10.22	21.03
45–46	12.64	23.99
50–51	15.53	31.42
55–56	21.50	39.50
60–61	30.75	50.79
65–66	43.79	64.33
70–71	62.14	83.98
75–76	92.53	112.77
80–81	135.75	131.27
85–86	191.11	179.82
90–91	255.17	201.01
95–96	324.86	227.76
100–101	427.46	336.29

Effect of the age composition of a population on the death-rate. — It is evident also, from our acquired knowledge of specific death-rates, that the general death-rates of two places cannot be reasonably compared unless the age composition of the population is substantially the same in the two places. The following simple example will make this plain:

Two places, A and B, have the same total population, *i.e.*, 50,000; and they have the same specific death-rates at

different ages. The ages of the people, however, differ as shown in the table. From these figures we may compute the general death-rate for each place.

TABLE 56

EFFECT OF AGE COMPOSITION OF POPULATION ON THE GENERAL DEATH-RATE

Age.	Population.		Specific death-rate per 1000.	Computed deaths.		Computed death-rates per 1000.	
	A	*B*		*A*	*B*	*A*	*B*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0–4	10,000	20,000	25	250	500		
5–59	35,000	20,000	10	350	200		
60–79	5,000	10,000	60	300	600		
Total	50,000	50,000		900	1300	18	26

In *B*, a place with a large number of children and old people, the rate is 26 per 1000, while in *A*, a place with a large middle-aged population, the rate is only 18. This is, of course, an exaggerated case, but slight differences in age distribution make a greater difference in the general death-rate than one would suppose.

In 1899, according to a report of the U. S. Secretary of War, the annual death-rate of soldiers in the Philippines was 17.20, while the death-rate of Boston was 20.09, of Washington 20.74 and of San Francisco 19.41. The obvious inference was that the mortality in the army compared favorably with the mortalities of the cities mentioned. The facts unstated were that soldiers are picked men in a limited age-group while the cities contain a conglomerate population. A better comparison would have been one between the soldiers and males between 20 and 40 years of age in the United States, the usual death-rate for which is

less than 10 per 1000. Hence the mortality among the troops in the Philippines was nearly twice as high as that of males of similar age in the United States.

In 1911 the general death-rate of Chicago was 14.5 and that of Cambridge, Mass., was 15.2. Was Chicago the healthier city? No, indeed! The following figures show that the specific death-rates were lower in Cambridge for all ages except the age-intervals of 10–19 years and 65 years and over. The reason for Chicago's lower rate was because there were relatively more people in Chicago at those middle ages where the specific death-rates are naturally low.

TABLE 57

COMPARISON OF DEATH-RATES IN CAMBRIDGE, MASS., AND CHICAGO

Age in years.	Per cent of population in age-groups. Both sexes. 1910.		Specific death-rates per 1000, in 1911.	
	Cambridge.	Chicago.	Cambridge.	Chicago.
(1)	(2)	(3)	(4)	(5)
Under 5	10.3	10.2	39.1	39.5
5–9	9.1	8.8	4.3	4.7
10–14	8.5	8.6	3.5	2.6
15–19	8.5	9.6	4.6	3.7
20–24	9.9	11.5	3.8	5.3
25–34	18.2	19.7	5.5	6.9
35–44	15.0	14.5	9.0	11.4
45–54	16.0	9.6	16.0	19.3
55–64	....	4.5	29.4	35.1
65–74	4.5	2.0	64.0	63.6
75 and over (including unknown)	....	1.0	148.8	144.2
Total	100.0	100.0	15.2	14.5

Obviously the two general death-rates tell us very little that we want to know — that is, not until they have been analyzed.

Effect of race composition on death-rates. — If different races have different specific death-rates then the general death-rates of two places which have different percentages of various races cannot be fairly compared. The general death-rates of southern cities cannot be fairly compared with those of northern cities. In 1911 the general death-rate in New York City was 15.2; in Washington it was 18.7; in New Orleans, 20.4. The death-rate for the white population in Washington, however, was only 15.5 and in New Orleans only 16.6. Even these figures are not strictly comparable as they do not take into account age distribution.

Changes in specific death-rates through long periods. — We have seen that the general, or gross, death-rates have been falling for a long time. Are the same changes occurring in the specific death-rates at different ages and for different classes of the population? This is a most important question. If we can answer it we shall have come close to measuring the effect of our sanitary, hygienic and medical improvements during recent years. Far too little effort has been made to compile statistics of this sort. Let us see what we can learn from Massachusetts records.

In 1830 Lemuel Shattuck computed specific death-rates for Boston. It will be interesting to compare these with figures for the year 1911, published in the U. S. Mortality Statistics by the Bureau of the Census and recast to make the age-groups correspond.

TABLE 58

SPECIFIC DEATH-RATES, BOTH SEXES, FOR BOSTON

Age interval.	Rate per 1000.	
	1830.	1911.
(1)	(2)	(3)
		(approximate)
0–1	....	161
1–5	....	17
0–5	59.6	....
5–9	8.1	4
10–14	5.5	2.4
15–19	4.9	4
20–29	10.4	6
30–39	20.1	10
40–49	22.4	15
50–59	29.3	27
60–69	45.8	52
70–79	92.4	102
80–89	162.1	....
90–	321.4	....

During the 81 years there has been a marked reduction in the specific death-rates at all ages below sixty. In the case of children and youths the reduction was as much as one half. In 1898 Dr. Samuel W. Abbott, then Secretary of the Massachusetts State Board of Health, computed a life table for the State[1] for the years 1893–7 in which the specific death-rates were given for certain age-groups. It is interesting to compare these with the figures given for Massachusetts in the U. S. Life Tables for 1910.

[1] Ann. Rept. 1898, p. 810.

TABLE 59

SPECIFIC DEATH-RATES FOR MASSACHUSETTS

Age-group.	Rate per 1000 1893–7		Age-group.	Rate per 1000 1910	
	Males.	Females.		Males.	Females.
(1)	(2)	(3)	(4)	(5)	(6)
0–4	60.12	52.22			
5–9	5.69	5.82	7–8	3.37	3.13
10–14	3.11	3.40	12–13	2.27	2.05
15–19	5.29	5.68	17–18	3.43	3.17
20–24	7.48	7.32	22–23	5.16	4.30
25–34	9.33	8.78	30–31	6.60	5.97
35–44	11.19	10.74	40–41	10.00	8.14
45–54	16.67	14.88	50–51	16.05	12.58
55–64	30.42	26.00	60–61	33.15	27.03
65–74	59.67	51.37	70–71	67.91	56.47
75–84	116.20	99.88	80–81	137.43	123.49
85–94	223.50	184.81	90–91	251.53	244.90
95–	429.20	367.07	100–101	483.90	392.91

Here we see the specific death-rates still falling up to age sixty. For the later ages there has been a slight tendency to increase. It should be noticed, however, that the age-groups are not quite the same for the two periods.

In 1830 and also in 1893–7 the specific rates at ages five to twenty, or thereabouts, were higher for females than for males, but in 1910 the opposite was true.

If we should make similar comparisons of specific death-rates for other places and for different periods we should almost always find that in recent years the rates have been falling for all ages below fifty or sixty.

What have been the reasons for this reduction? Undoubtedly improved sanitary and hygienic conditions, advances in medical and surgical science and the arts of preventive medicine have tended to reduce the number of

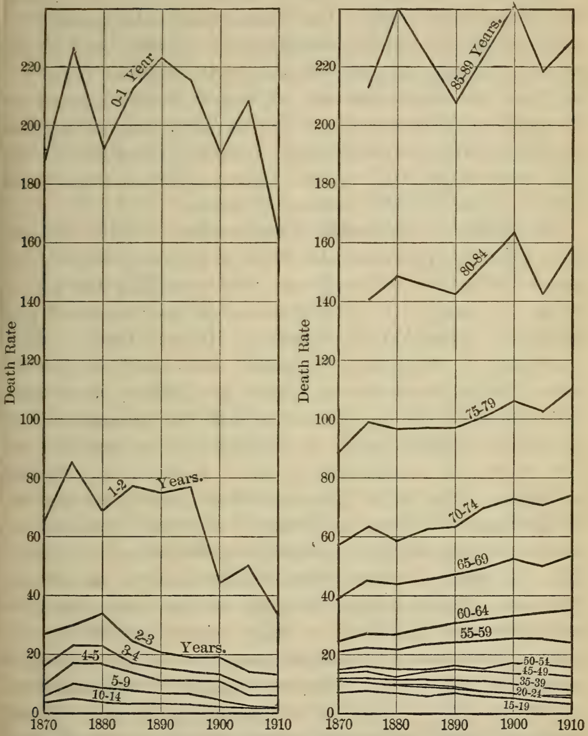


FIG. 49. — Specific Death-rates by Age-Groups, Massachusetts, 1870–1910.

cases of sickness and to increase the percentage of recoveries of those who are taken sick. This has been especially true in the earlier ages. But it must not be forgotten that changes in the relative numbers of married and single persons in each sex, and of persons of different nationality, have also their influence. A reason for the increase in the specific death-rates above fifty or sixty years of age has been frequently discussed of late, namely an increase in certain degenerative and organic diseases. This is important, if true, but it is a difficult thing to prove.

The fallacy of concealed classification. — Now that we have come to appreciate the effect of age, sex, nationality, and such factors on death-rates, but especially the factor of age, we can better understand what may be called the fallacy of concealed classification. If we classify males according to occupation we might find that the death-rate of bank presidents was higher than that of newsboys; but this would not be because of different occupation but because of different ages. In classifying by occupation we have concealed a grouping by age. If, in classifying the employees of the city of Boston or New York by occupation we distinguish between policemen and street cleaners, we might find that we had concealed a classification by nationality, the street cleaners being Italians and the policemen Irishmen. Similarly in classifying railroad employees into conductors and brakemen, we might conceal age differences, and under the class of Pullman porters we might conceal a nationality difference. When we consider stenographers as a separate class we conceal a classification by sex. These concealed classifications and groupings are sometimes very illusive; they creep into our statistics unawares and upset what might otherwise be sound reasoning. Illustrations may be found on every hand. Every one who uses statistics should be continually on the watch for them.

Use of specific death-rates. — It must be evident from what has been said that in order to compare the mortality conditions in various places the best way is to compare age with age, sex with sex, nationality with nationality, or in other words to compare the various places, classes and groups on the basis of their specific death-rates. To do this in great detail involves labor and the use of many figures. Hence there has always been a fascination in combining these figures so as to obtain a single figure which may be regarded as an index of mortality. There are at least two ways of doing this. If there were such a thing as a standard population — and several such standards have been suggested, notably the Standard Million (see page 181) — and if we knew the specific death-rates by ages and sex for any place, we could apply these rates to the standard population and find what the general death-rate would have been in the given place if the population had been standard. And we might do the same for another place and thus obtain figures for the death-rates which could be compared with some degree of justice.

When general death-rates are adjusted to a standard population in this way the results are called "*Standardized death-rates*." Sometimes they have been referred to as "corrected" death-rates, but this is a poor use of the word, for the process is not one of correcting errors or mistakes and the final result is not "correct," for it does not take into account all differences in population. Nor is the expression "standardized" a good one, because it is not the death-rate which is standardized, but only the population. A better term is "Death-rates adjusted to a Standard Population."

In the annual report of the Massachusetts State Board of Health for 1902 may be found another method of "correcting" death-rates, used by Dr. Samuel W. Abbott. He

took as a standard the specific death-rates of Massachusetts by age and sex. He then applied these to the age and sex groups of the cities of the state, to obtain what he called the standard death-rate for each place. Then he found the ratio between the "standard death-rate" for each place and the general death-rate of the state, and called this the "factor of correction." Finally he multiplied the actual general death-rate of each place by this factor to obtain his "corrected death-rate." The advantage of this method was that he did not need to use the age distribution of deaths for each place. The method is interesting, but is not one for general adoption, because it would be hard to decide on a standard of specific death-rates.

Another way of using specific death-rates is that of constructing what are called life tables. These will be described in Chapter XIV.

But the best way of using specific death-rates is to use them directly. To be sure it means that one must carry more figures in one's mind. Instead of having to think of one figure for the general death-rate it is necessary to think of figures for infant deaths, for the deaths of children, of adults and of the aged — but, after all, are not these the really important figures? Statistics are worthless unless they can be used. If specific death-rates are more usable than general death-rates, we should make the specific rates more prominent and educate people to think in terms of them.

Death-rates adjusted to a standard population. — A few examples will now be given to show how general rates may be adjusted to a standard population. For the sake of simplicity age differences only will be considered. The data required are (*a*) the number of deaths by age-groups in the given place; (*b*) the number of persons living at mid-year in the corresponding age-groups; (*c*) an assumed

standard population for the same age-grouping. First of all, therefore, some system of age-grouping must be decided upon. Let us take first a simple case, that is, one where there are only a few groups.

On page 226 were given data for New South Wales, from which the specific death-rates were computed. Let us apply these specific death-rates to the population of Sweden in 1890 which we will take as a standard. This is given in column (5) of Table 60. For age-group 1–19 years the specific rate was 2.78 per 1000; hence, among 398 persons the number of deaths would be 0.398×2.78 or 1.11 as given in column (6). And so for the other age-groups. The figures in column (6), therefore, give the number of deaths in each group of the standard thousand of population, and their sum is the total number of deaths in the standard thousand. Hence the death-rate of New South Wales adjusted to the standard population was 13.44. This is much higher than the general death-rate, which was only 9.85.

TABLE 60

ADJUSTED DEATH-RATE FOR NEW SOUTH WALES, 1901

Age-group in years.	Population.	Number of deaths in one year.	Specific death-rate per 1000.	Standard age distribution per 1000.	Computed deaths per 1000 of total population.
(1)	(2)	(3)	(4)	(5)	(6)
0–1	40,500	3,234	79.88	25.5	2.04
1–19	704,000	1,960	2.78	398.0	1.11
20–39	514,900	2,251	4.37	269.6	1.18
40–59	256,600	2,965	11.56	192.3	2.22
60 and over	89,800	5,400	60.13	114.6	6.89
Total	1,605,800	15,810	9.85	1000.0	13.44

Why this difference? The answer is found by comparing the age distribution of the people of New South Wales with the assumed standard population.

TABLE 61

COMPARISON OF POPULATION DISTRIBUTION OF NEW SOUTH WALES WITH THAT OF SWEDEN IN 1890

Age-group.	New South Wales.		Sweden.
	Number.	Per thousand.	Per thousand.
(1)	(2)	(3)	(4)
0–1	40,500	24.9	25.5
1–19	704,000	439.0	398.0
30–39	514,900	320.0	269.6
40–59	256,600	160.5	192.3
60–	89,800	55.6	114.6
All Ages	1,605,800	1000.0	1000.0

It will be seen that in New South Wales there were fewer old persons, for whom the specific death-rates are naturally high, but more persons in middle life, for whom the specific death-rates are naturally low. This is an extreme case, but characteristic of a new population built up by immigration.

Let us now take a more complicated situation.

In 1914 there were 1452 deaths in Cambridge,[1] Mass., distributed by age as follows:

TABLE 62

DISTRIBUTION OF DEATHS: CAMBRIDGE, MASS., 1914

Age.	Number.	Age.	Number.	Age.	Number.	Age.	Number.
(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
[0–1]	[243]	20–24	43	45–49	90	70–74	110
0–5	340	25–29	49	50–54	83	75–79	72
5–9	20	30–34	62	55–59	73	80–84	66
10–14	26	35–39	58	60–64	107	85–89	33
15–19	33	40–44	56	65–69	109	90–94	18
						95–99	4

[1] U. S. Mortality Statistics, 1914, p. 264.

The Standard Million[1] will be taken as the standard of population. It is necessary to take an age-grouping which will correspond to this, and find the number of persons in Cambridge in 1914 in each of these groups. There was no census in Cambridge in 1914, but in 1910 the population was 104,839, in 1900 it was 91,886. The estimated population July 1, 1914, was 110,357. In 1910 the age distribution of the people of Cambridge was given by the census. It was as follows (columns 1, 2 and 3):

TABLE 63

PERSONS LESS THAN STATED AGE: CAMBRIDGE, MASS.

Age.	Actual number of persons in 1910	Per cent.	Computed number of persons in 1914.
(1)	(2)	(3)	(4)
1	2,323	2.3	2,430
5	10,802	10.4	11,500
10	20,273	19.4	21,400
15	29,165	27.9	30,800
20	37,095	36.4	40,200
25	47,503	46.4	51,200
35	66,678	64.6	70,500
45	82,404	79.6	88,000
65	99,136	95.6	105,700
100	104,778	99.4	110,280
Unknown	61	0.6	77
Total	104,839	100.0	110,357

It may be fairly assumed that the percentages of column 3 for 1910 apply also with no great change to 1914. By multiplying 110,357, therefore, by these percentages we get the following numbers of persons in each group for 1914 (column 4).

The figures in column (4) may be redistributed in any

[1] See p. 181.

desired age-grouping as described on p. 172. In this way the figures in column (2) of the following table were obtained:

TABLE 64

ADJUSTED DEATH-RATES FOR CAMBRIDGE, MASS.

Age-group.	Estimated population in 1914 (approximate).	Number of deaths in 1914.	Specific death-rate per 1000.	Standard age distribution per 1000.	Computed deaths.
(1)	(2)	(3)	(4)	(5)	(6)
0–4	11,500	340	29.5	114.262	0.380
5–9	9,900	20	2.02	107.209	0.217
10–14	9,400	26	2.76	102.735	0.284
15–19	9,400	33	3.51	99.796	0.350
20–24	11,000	43	3.91	95.946	0.374
25–34	19,200	111	5.78	161.579	0.935
35–44	17,400	114	6.53	122.849	0.803
45–54	11,600	173	14.9	89.222	1.330
55–64	5,800	180	31.1	59.741	1.856
65–74	3,100	219	70.6	33.080	2.330
75–	2,100	193	91.8	13.581	1.246
Total	110,400	1452	13.15	1000.000	13.105

From columns (2) and (3) the specific death-rates are obtained (column 4), and these applied to the standard age distribution (column 5) give the number of computed deaths in each age-group (column 6). Their sum gives 13.1, which is the death-rate adjusted to the standard age distribution. We have done all this work to get a result which differs but fractionally from the general, or crude death-rate, *i.e.*, 13.15. Not worth while? Yes, it is if we are to use a death-rate at all. It was only because the age distribution of the Cambridge population happened to be so near that of the standard million that the two death-rates came so close together. In another case the result might be very different.

A fair criticism of this last computation would be that the age-groupings below age five and above middle age are too wide, for it is in these groups where the specific death-rates are highest. Dr. Wm. L. Holt, C.P.H. (School of Public Health, Harvard University and Mass. Inst. of Tech.), investigated this subject of grouping and concluded that seven properly selected groups would give results which compared well with those obtained by using the eleven groups of the Standard Million. The author believes that even five well-chosen groups would suffice, but the matter is one which needs free discussion. Certainly something more convenient than the Standard Million is possible.

TABLE 65

COMPUTED DEATH-RATES IN BOSTON AND CAMBRIDGE, 1905

(Computations by Dr. Wm. L. Holt)

Boston.

Age-group.	Population.	Deaths.	Specific death-rate.	Adjusted rates per 1000. Eleven groups.	Nine groups.	Seven groups.	Six groups.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0–4	52,152	3128	60.1	6.850	6.850	6.850	6.850
5–9	54,091	253	4.68	0.501	0.501	1.963	1.963
10–14	48,694	157	3.22	0.331	0.785		
15–19	47,608	218	4.58	0.457			
20–24	57,421	380	6.61	0.634	0.634		
25–34	119,632	1070	8.95	1.441	1.441	2.840	2.840
35–44	95,946	1081	11.28	1.388	1.388		
45–54	58,810	1255	21.4	1.910	1.910	1.910	4.135
55–64	33,602	1308	38.9	2.325	2.325	2.325	
65–74	16,711	1213	72.6	2.403	4.802	2.403	2.403
75–	6,413	937	146.0	1.980		1.980	1.980
Total				20.220	20.636	20.271	20.171

(Continued on next page.)

Cambridge.

Age-group.	Population.	Deaths.	Specific death-rate.	Adjusted rates per 1000. Eleven groups.	Adjusted rates per 1000. Seven groups.
(1)	(2)	(3)	(4)	(5)	(6)
0–4	9,088	412	45.3	5.176	5.176
5–9	9,096	27	2.96	0.317	
10–14	8,078	20	2.48	0.255	1.468
15–19	8,512	34	4.00	0.399	
20–24	10,789	51	4.72	0.452	
25–34	18,671	130	6.97	1.125	2.253
35–44	14,148	130	9.19	1.127	
45–54	9,267	133	14.35	1.280	1.280
55–64	5,628	165	29.4	1.755	1.755
65–74	2,864	155	54.1	1.790	1.790
75–	1,251	179	143.2	1.950	1.950
Total				15.626	15.672

Examples of death-rates adjusted to a standard population. — The U. S. Mortality Statistics give numerous examples of death-rates adjusted to the Standard Million.

Let us first of all compare Cambridge, Mass., and Chicago, Ill.

TABLE 66

City.	Death-rate, 1911. Gross.	Death-rate, 1911. Adjusted.
(1)	(2)	(3)
Cambridge, Mass.	15.2	15.4
Chicago, Ill.	14.5	16.4

Here again we see that adjustment of the Cambridge rate changes it but little,[1] while that of Chicago was increased by 1.9, making the adjusted rate higher than that of Cambridge. Why?

[1] In this computation sex as well as age was considered.

In every instance in the following table the adjusted death-rate exceeded the gross death-rate, the excesses ranging from 1 to 18 per cent and averaging 8.4 per cent. As would naturally be expected the differences were less in the older cities of the East than in the newer cities of the West, but New York, Pittsburgh and a few others with large numbers of recent immigrants were exceptions to this rule. The following figures illustrate this:

TABLE 67

COMPARISON OF GROSS AND ADJUSTED DEATH-RATES FOR CERTAIN CITIES

City.	Death-rates per 1000.		
	Gross.	Adjusted.	Difference.
(1)	(2)	(3)	(4)
New Haven, Conn.	16.7	17.7	1.0
Boston, Mass.	17.1	17.9	0.8
New York, N. Y.	15.2	17.2	2.0
Pittsburgh, Pa.	14.9	16.9	2.0
Cleveland, Ohio	13.8	15.3	1.5
Chicago, Ill.	14.5	16.4	1.9
Spokane, Wash.	11.6	13.7	2.1
Seattle, Wash.	8.8	10.4	1.6

Adjustment to a standard population tends to equalize the death-rates in different places. The rural districts of New England contain a large percentage of persons of advanced age. This tends to cause the adjusted rate to be lower than the gross rate. Taking figures for entire states we find this to be true, as the following figures show. In the western states this difference is not as marked, as they have not suffered by emigration as have the New England States.

TABLE 68

COMPARISON OF GROSS AND ADJUSTED DEATH-RATES FOR CERTAIN STATES

State.	Death-rates per 1000.		
	Gross.	Adjusted.	Difference.
(1)	(2)	(3)	(4)
Massachusetts	15.3	15.0	−0.3
New Hampshire	17.1	14.2	−2.9
Maine	16.1	13.0	−3.1
Connecticut	15.4	14.8	−0.6
Indiana	12.9	12.3	−0.6
Kentucky	13.2	13.4	0.2
Michigan	13.2	12.4	−0.8
Minnesota	10.5	10.8	0.3
Missouri	13.1	13.1	0.0
Montana	10.2	11.6	1.4

The following figures show the relation between the crude and adjusted death-rates for various countries:

TABLE 69

COMPARISON OF GROSS AND ADJUSTED DEATH-RATES FOR CERTAIN COUNTRIES

Country.	Year.	Death-rates per 1000.			Ratio of adjusted rate to that of England and Wales.
		Gross.	Adjusted.	Difference.	
(1)	(2)	(3)	(4)	(5)	(6)
Russia	1896-1898	32.80	28.61	−4.19	166.7
Spain	1900–1902	27.63	26.53	−1.10	154.6
Austria	1899–1901	24.83	23.12	−1.71	134.7
Italy	1900–1902	22.72	20.23	−2.49	117.9
Germany	1901	20.84	19.52	−1.32	113.8
U. S. (Registration area)	1900	17.55	18.05	0.50	105.2
Scotland	1900–1902	17.91	17.61	−0.30	102.6
France	1900-1902	20.80	17.50	−3.30	102.0
England & Wales	1900–1902	17.16	17.16	0.00	100.0
Switzerland	1899-1901	18.22	16.86	−1.36	98.3
Belgium	1899–1901	18.53	16.78	−1.75	97.8
Ireland	1900–1902	18.27	16.59	−1.68	96.7
Netherlands	1898–1900	17.32	15.40	−1.92	89.7
Sweden	1899–1901	16.78	13.88	−2.90	80.9
New South Wales	1900–1902	11.72	13.10	1.38	76.3
Victoria	1900–1902	13.12	13.08	−0.04	76.2
South Australia	1900–1902	11.02	11.73	0.71	68.4

Adjustment for racial differences. — In certain parts of the United States, especially in the South, the crude death-rates are absolutely useless for purposes of comparison unless allowance is made for the number of colored persons at different ages. The specific death-rates for colored persons are higher at all ages than for white persons, as the following figures for the U. S. registration states in 1900 show:

TABLE 70

COMPARISON OF SPECIFIC DEATH-RATES FOR WHITE AND COLORED PERSONS

Age-group (both sexes).	Death-rates per 1000 (exclusive of still-births).		Ratio of colored to white death-rate.
	Native white.	Colored.	
(1)	(2)	(3)	(4)
0–4	49.1	106.4	2.17
5–9	4.5	8.9	1.98
10–14	2.9	9.0	3.10
15–19	4.7	11.4	2.43
20–24	6.8	11.6	1.71
25–34	8.2	12.2	1.49
35–44	9.6	15.0	1.56
45–54	12.7	24.5	1.93
55–64	22.6	42.5	1.88
65–74	50.4	69.5	1.38
75	138.5	143.3	1.03
Crude death-rate	16.5	25.0	1.52

The following figures for the cities of Washington, Baltimore and New Orleans show the necessity of taking into account these striking differences between the white and colored people:

TABLE 71

ADJUSTED DEATH-RATES FOR CITIES HAVING LARGE COLORED POPULATIONS

	Death-rates per 1000 (both sexes), 1911.		
	Gross.	Adjusted.	Difference.
(1)	(2)	(3)	(4)
Washington, D. C.			
White	15.5	14.6	−0.9
Colored	26.6	30.5	3.9
Total	18.7	18.9	0.2
Baltimore, Md.			
White	16.2	16.7	0.5
Colored	30.9	35.4	4.5
Total	18.4	19.4	1.0
New Orleans, La.			
White	16.6	17.5	0.9
Colored	31.2	34.0	2.8
Total	20.4	21.8	1.4

Death-rates for particular diseases. — Death-rates for particular diseases are computed in the same way as other specific death-rates. The numerator of the ratio is limited to the disease in question. The denominator may be the entire population, or it may be confined to some specific part of it. In order to avoid the use of too many decimals it is well to express the death-rates for particular diseases as so many per 100,000 instead of so many per 1000. This practice is becoming universal. The use of 10,000 as a base should be avoided.

If all of the deaths from typhoid fever be compared with the total mid-year population, we have the general typhoid fever death-rate of the place. General rates for particular diseases are much used and have practical value. Specific

rates in which deaths from typhoid fever in a given age-group are compared with the population in the same age-group are sometimes computed, but are useful only when the numbers involved are large.

Special death-rates. — In epidemiological studies it is necessary to compute death-rates in all sorts of ways, to separate the people into classes according to where and how they live, according to their occupation or their exposure to certain risks. This causes us to deal with many special rates.

In studying birth statistics we may find the general birth-rate, by taking the ratio between the number of births and the total population. But we may also desire to find the ratio between births and women of child-bearing age, or between births and married women of child-bearing age.

In interpreting all of these many sorts of rates and ratios the principles already outlined hold good. We must see that the data compared are logically comparable, that there are no concealed classifications and that the rules of precision are not violated.

EXERCISES AND QUESTIONS

1. Are the changes in age-composition from decade to decade in Massachusetts sufficient to explain a considerable part of the falling general death-rate of the state, assuming the specific death-rates by ages to remain constant?

2. Compute the specific death-rates by sex and age-groups for three Massachusetts cities for 1910, obtaining data from the census and registration reports.

3. Compare the specific death-rates by age-groups for white and colored persons in some southern city for some selected year.

4. Adjust the death-rate of some western city in 1910 to the Swedish standard of population.

5. Repeat this computation using the standard million as a basis of adjustment.

6. Select from the Mortality Reports examples of the need of adjustment of death-rates of cities for purposes of comparison.

7. Adjust the death-rates of some selected city to the basis of the Standard Million for 1915, 1910, 1905, 1900 and as far back as record can be obtained.

CHAPTER VIII

CAUSES OF DEATH

Nosography. — The description and systematic classification of disease is called nosography. The word is derived from the Greek word *nosos*, which means sickness, or disease. (The word is pronounced noss'ography, not noze-ography.)

Nosology. — The science of classifying disease is similarly called nosology.

The purpose of nosology. — At one time it was thought that a knowledge of nosology was necessary for the practical treatment of disease. Many systems were proposed and abandoned. Today the idea has few, if any, supporters.

Nosology is of great importance as one of the foundation stones of our modern structure of vital statistics. Without uniform definitions of disease which furnish us with adequate statistical units our statistics would be worthless. It is because of changes in our definitions of disease that we fall into so many errors in comparing past conditions with those of the present day. Such changes are inevitable as medical science advances, but they ought to be universally recognized when they are made.

Dr. William Farr was one of the first to recognize the importance of "statistical nosology."

History of nosography. — Nosography emerged from its former chaotic condition in 1893 when the use of the International Classification of Diseases and Causes of Death was begun. This was due chiefly to the labors of Dr. Jacques Bertillon of France.

In 1853 Dr. William Farr and Dr. Marc d'Espine, of Geneva, had been selected by the First Statistical Congress, which met at Brussels, to present a report on the subject. The list of diseases reported by them was adopted in Paris in 1855, in Vienna in 1857, and was translated into six languages. It was revised several times between 1864 and 1886. In 1893 the International Statistical Institute, the successor of the Statistical Congress, met in Chicago and adopted this list with some changes. Provision was made for decennial revisions by an International Commission, and such revisions were made in Paris in 1900 and again in 1909, the latter a year earlier in order that the new list might be used in the censuses of 1910. The present list is intended to stand unchanged until 1919. In 1898 the International List was endorsed by the American Public Health Association. England adopted the list in 1911. It is used by all English and Spanish speaking countries, but it is not yet universal. A few of our own states do not follow it exactly, namely: Alabama, New Hampshire, New Mexico, Rhode Island and West Virginia.

International list of the causes of death. — In 1911 the U. S. Bureau of the Census published a Manual of 297 pages, being a revision of a former manual published in 1902. This list is the present standard for the United States and has come to be almost universally used. This manual is very complete. It gives the standard list of the causes of death, with synonyms, and is indexed alphabetically as well as according to the chosen classification.

The Bureau of the Census also publishes a Physician's Pocket Reference to the International List of the Causes of Death, which can be obtained without charge by anyone who makes request of the Director of the Census, Washington, D. C. This is a small pamphlet of 28 pages, vest pocket size.

Classification of diseases in 1850. — Dr. Farr classified diseases as follows:

Class I. Epidemic, Endemic and Contagious diseases (*Zymotici*).
- Order 1. *Miasmatic* diseases, — small-pox, ague, etc.
- Order 2. *Enthenic* diseases, — syphilis, glanders.
- Order 3. *Dietetic* diseases, — scurvy, ergotism.
- Order 4. *Parasitic* diseases.

Class II. Constitutional Diseases (*Cachectici*).
- Order 1. *Diathetic* diseases, — gout, dropsy, cancer, etc.
- Order 2. *Tubercular* diseases, — scrofula, consumption.

Class III. Local Diseases (*Monorganici*).
- Order 1. Diseases of the brain.
- Order 2. Diseases of the circulation.
- Order 3. Diseases of respiration.
- Order 4. Diseases of digestion.
- Order 5. Diseases of the urinary system.
- Order 6. Diseases of reproduction.
- Order 7. Diseases of locomotive system.
- Order 8. Diseases of integumentary system.

Class IV. Developmental Diseases (*Metamorphici*).

Class V. Violent Deaths or Diseases (*Thanatici*).

It is extremely interesting to study this list in detail as given in the 16th Annual Report of the Registrar General of England, Appendix, pp. 71–79.

Present classification. — The list recognizes 189 causes of death, which are divided into fourteen classes. It is not claimed that these are all of the possible causes. For convenience of reference and tabulation each of these diseases is given a number. The following is the list as given in the Physician's Pocket Reference. It is recommended that only the names printed in heavy type be used. The terms in italics are indefinite or otherwise undesirable. An abridged list of causes of death useful for annual reports of health departments may be found on page .

INTERNATIONAL LIST OF CAUSES OF DEATH

(I. — General Diseases)

1. **Typhoid fever.**
2. **Typhus fever.**
3. **Relapsing fever.** [Insert "(**spirillum**)."]
4. **Malaria.**
5. **Smallpox.**
6. **Measles.**
7. **Scarlet fever.**
8. **Whooping cough.**
9. **Diphtheria** and *croup.*
10. **Influenza.**
11. **Miliary fever.** [True **Febris miliaris** only.]
12. **Asiatic cholera.**
13. *Cholera nostras.*
14. **Dysentery.** [**Amebic? Bacillary?** Do not report ordinary diarrhea and enteritis (104, 105) as dysentery.]
15. **Plague.**
16. **Yellow fever.**
17. **Leprosy.**
18. **Erysipelas.** [State also cause; see Class XIII.]
19. Other epidemic diseases:
    Mumps,
    German measles,
    Chicken-pox,
    Rocky Mountain spotted (tick) fever,
    Glandular fever, etc.
20. Purulent infection and **septicemia.** [State also cause; see Classes VII and XIII especially.]
21. **Glanders.**
22. **Anthrax.**
23. **Rabies.**
24. **Tetanus.** [State also cause; see Class XIII.]
25. Mycoses. [Specify, as **Actinomycosis of lung, etc.**]
26. **Pellagra.**
27. **Beriberi.**
28. **Tuberculosis of the lungs.**
29. **Acute miliary tuberculosis.**
30. **Tuberculous meningitis.**

31. **Abdominal tuberculosis.**
32. **Pott's disease.** [Preferably **Tuberculosis of spine.**]
33. *White swellings.* [Preferably **Tuberculosis of —— joint.**]
34. **Tuberculosis** of other organs. [Specify organ.]
35. Disseminated **tuberculosis.** [Specify organs affected.]
36. **Rickets.**
37. **Syphilis.**
38. **Gonococcus infection.**
39. **Cancer** [1] **of** *the buccal cavity.* [State part.]
40. **Cancer** [1] **of the stomach, liver.**
41. **Cancer** [1] **of the peritoneum, intestines, rectum.**
42. **Cancer** [1] **of** *the female genital organs.* [State organ.]
43. **Cancer** [1] **of the breast.**
44. **Cancer** [1] **of** *the skin.* [State part.]
45. **Cancer** [1] **of** *other or unspecified organs.* [State organ.]
46. Other *tumors* (tumors of the female genital organs excepted.) [Name kind of *tumor* and organ affected. **Malignant?**]
47. **Acute articular rheumatism.** [Always state "rheumatism" as acute or **chronic.**]
48. **Chronic rheumatism** [preferably **Arthritis deformans**] and **gout.**
49. **Scurvy.**
50. **Diabetes.** [**Diabetes mellitus.**]
51. **Exophthalmic goiter.**
52. **Addison's disease.**
53. **Leukemia.**
54. **Anemia, chlorosis.** [State form or cause. **Pernicious?**]
55. Other general diseases:

    Diabetes insipidus,
    Purpura haemorrhagica, etc.

56. **Alcoholism (acute** or **chronic).**
57. **Chronic lead poisoning.** [State cause. Occupational?]
58. Other chronic occupational poisonings. [State exact **name of poison,** whether the poisoning was **chronic and due to occupation,** and also please be particularly careful to see that the **Special Occupation and Industry** are fully stated. If

[1] "Cancer and other malignant tumors." Preferably reported as **Carcinoma of ——, Sarcoma of ——, Epithelioma of ——,** etc., stating the exact nature of the neoplasm and the **organ or part of the body first affected.**

the occupation stated on the certificate is not that in which the poisoning occurred, add the latter in connection with the statement of cause of death, *e.g.*, "**Chronic occupational phosphorus necrosis (dipper, match factory, white phosphorus).**" Give full details, including pathologic conditions contributory to death. Following is a List of Industrial Poisons (*Bull. Bureau of Labor*, May, 1912) to which the attention of physicians practicing in industrial communities should be especially directed:

Acetaldehyde,
Acridine,
Acrolein,
Ammonia,
Amyl acetate,
Amyl alcohol,
Aniline,
Aniline dyestuffs [name],
Antimony compounds [name],
Arsenic compounds [name],
Arseniureted hydrogen,
Benzine,
Benzol,
Carbon dioxide,
Carbon disulphide,
Carbon monoxide (coal vapor, illuminating water gas, producer gas),
Chloride of lime,
Chlorine,
Chlorodinitrobenzol,
Chloronitrobenzol,
Chromium compounds [name],
Cyanogen compounds [name],
Diazomethane,
Dimethyl sulphate,
Dinitrobenzol,
Formaldehyde,
Hydrochloric acid,
Hydrofluoric acid,
Lead (57),
Manganese dioxide,
Mercury,
Methyl alcohol,
Methyl bromide,
Nitraniline,
Nitrobenzol,
Nitroglycerin,
Nitronaphthalene,
Nitrous gases,
Oxalic acid,
Petroleum,
Phenol,
Phenylhydrazine,
Phosgene,
Phosphorus (yellow or white),
Phosphorus sesquisulphide,
Phosphureted hydrogen,
Picric acid,
Pyridine,
Sulphur chloride,
Sulphur dioxide,
Sulphureted hydrogen,
Sulphuric acid,
Tar,
Turpentine oil.

Not all substances in the preceding list are likely to be reported as causes of death, but the physician should be familiar with it in order to recognize, and to report, if required, cases of illness, and

should also be on the alert to discover new forms of industrial poisoning not heretofore recognized. In the Bulletin cited full details may be found as to the branches of industry in which the poisoning occurs, mode of entrance into the body, and the symptoms of poisoning. Attention should also be called to industrial infection, *e.g.*, Anthrax (22), and the influence of gases and vapors, dust, or unhygienic industrial environment.

59. Other chronic poisonings:

    Chronic morphinism,
    Chronic cocainism, etc.

(II. — Diseases of the Nervous System and of the Organs of Special Sense)

60. *Encephalitis.*
61. *Meningitis:*
    Cerebrospinal fever or **Epidemic cerebrospinal meningitis,**
    Simple meningitis. [State cause.]
62. **Locomotor ataxia.**
63. Other diseases of the spinal cord:
    Acute anterior poliomyelitis,
    Paralysis agitans,
    Chronic spinal muscular atrophy,
    Primary lateral sclerosis of spinal cord,
    Syringomyelia, etc.
64. **Cerebral hemorrhage, apoplexy.**
65. *Softening of the brain.* [State cause.]
66. *Paralysis without specified cause.* [State form or cause.]
67. **General paralysis of the insane.**
68. Other forms of mental alienation. [Name **disease causing death.** Form of insanity should be named as CONTRIBUTORY CAUSE only, *unless* it is actually the disease causing death.]
69. **Epilepsy.**
70. *Convulsions (nonpuerperal).* [State cause.]
71. *Convulsions of infants.* [State cause.]
72. **Chorea.**
73. **Neuralgia** and **neuritis.** [State cause.]
74. Other diseases of the nervous system. [Name the disease.]
75. Diseases of the eyes and their annexa. [Name the disease.]
76. Diseases of the ears. [Name the disease.]

(III. — Diseases of the Circulatory System)

77. **Pericarditis.** [**Acute** or **chronic**; **rheumatic** (47), etc.]
78. **Acute endocarditis.** [Cause? Always report "endocarditis" or "myocarditis" as **acute** or **chronic.** Do not report when mere terminal condition.]
    Acute myocarditis.
79. Organic diseases of the heart: [Name the disease.]
    Chronic valvular disease, [Name the disease.]
    Aortic insufficiency,
    Chronic endocarditis, [See note on (78).]
    Chronic myocarditis, [See note on (78).]
    Fatty degeneration of heart, etc.
80. **Angina pectoris.**
81. Diseases of the arteries, **atheroma, aneurism,** etc.
82. **Embolism** and **thrombosis.** [State organ. Puerperal (139)?]
83. Diseases of the veins (**varices, hemorrhoids, phlebitis,** etc.).
84. Diseases of the lymphatic system (**lymphangitis,** etc.). [Cause? Puerperal?]
85. *Hemorrhage;* other diseases of the circulatory system. [Cause? *Pulmonary hemorrhage* from **Tuberculosis of lungs** (28)? Puerperal?]

(IV. — Diseases of the Respiratory System)

86. Diseases of the nasal fossae. [Name disease.]
87. Diseases of the larynx. [Name disease. Diphtheritic?]
88. Diseases of the thyroid body. [Name disease.]
89. **Acute bronchitis.** } [Always state as **acute** or **chronic.** Was it
90. **Chronic bronchitis.** } **tuberculous?**]
91. **Bronchopneumonia.** [If secondary, give *primary* cause.]
92. *Pneumonia.* [If lobar, report as **Lobar pneumonia.**]
93. **Pleurisy.** [Cause? If tuberculous, so report (28).]
94. *Pulmonary congestion, pulmonary apoplexy.* [Cause?]
95. **Gangrene of the lung.**
96. **Asthma.** [Tuberculosis?]
97. **Pulmonary emphysema.**
98. Other diseases of the respiratory system (tuberculosis excepted). [Such indefinite returns as "*Lung trouble,*" "*Pulmonary hemorrhage,*" etc., compiled here, vitiate statistics. **Tuberculosis of lungs** (28)? Name the disease.]

(V. — Diseases of the Digestive System)

99. Diseases of the mouth and annexa. [Name disease.]
100. Diseases of the pharynx. [Name disease. Diphtheritic?] **Streptococcus sore throat.**
101. Diseases of the esophagus. [Name disease.]
102. **Ulcer of the stomach.**
103. Other diseases of the stomach (cancer excepted). [Name disease. Avoid such indefinite terms as "*Stomach trouble,*" "*Dyspepsia,*" "*Indigestion,*" "*Gastritis,*" etc., when used vaguely.]
104. **Diarrhea and enteritis** (under 2 years).
105. **Diarrhea and enteritis** (2 years and over).
106. **Ankylostomiasis.** [Better, for the United States, **Hookworm disease** or **Uncinariasis.**]
107. Intestinal parasites. [Name species.]
108. **Appendicitis** and typhlitis.
109. **Hernia,** intestinal obstruction. [State form and whether **strangulated.**]
    Strangulated inguinal hernia (**operation**),
    Intussusception,
    Volvulus, etc.
110. Other diseases of the intestines. [Name disease.]
111. **Acute yellow atrophy of the liver.**
112. **Hydatid tumor of the liver.**
113. **Cirrhosis of the liver.**
114. **Biliary calculi.**
115. Other diseases of the liver. ["*Liver complaint*" is not a satisfactory return.]
116. Diseases of the spleen. [Name disease.]
117. *Simple* **peritonitis (nonpuerperal).** [Give cause.]
118. Other diseases of the digestive system (cancer and tuberculosis excepted). [Name disease.]

(VI. — Nonvenereal Diseases of the Genito-Urinary System and Annexa)

119. **Acute nephritis.** [State primary cause, especially **Scarlet fever,** etc. Always state "nephritis" as **acute** or **chronic.**]
120. Bright's disease. [Better, **Chronic interstitial nephritis, Chronic parenchymatous nephritis,** etc. Never report mere names of symptoms, as "*Uremia,*" "*Uremic coma,*" etc. See also note on (119).]

121. **Chyluria.**
122. Other diseases of the kidneys and annexa. [Name disease.]
123. **Calculi of** the urinary passages. [Name **bladder, kidney**.]
124. Diseases of the bladder. [Name disease.]
    Cystitis. [Cause?]
125. Diseases of the urethra, **urinary abscess,** etc. [Name disease. Gonorrheal (38)?]
126. Diseases of the prostate. [Name disease.]
127. Nonvenereal diseases of the male genital organs. [Name disease.]
128. **Uterine hemorrhage (nonpuerperal).** [Cause?]
129. **Uterine tumor (noncancerous).** [State kind.]
130. Other diseases of the uterus. [Name disease.]
    Endometritis. [Cause? Puerperal (137)?]
131. **Cysts** and other tumors **of the ovary.** [State kind.]
132. **Salpingitis** and other diseases of the female genital organs. [Name disease. Gonorrheal (38)? Puerperal (137)?]
133. **Nonpuerperal** diseases of the **breast** (cancer excepted). [Name disease.]

(VII. — THE PUERPERAL STATE)

NOTE. — The term **puerperal** is intended to include pregnancy, parturition, and lactation. Whenever parturition or miscarriage has occurred within one month before the death of the patient, the fact should be certified, even though childbirth may not have contributed to the fatal issue. Whenever a woman of childbearing age, especially if married, is reported to have died from a disease which might have been puerperal, the local registrar should require an explicit statement from the reporting physician as to whether the disease was or was not puerperal in character. The following diseases and symptoms are of this class:

Abscess of the breast,
Albuminuria,
Cellulitis,
Coma,
Convulsions,
Eclampsia,
Embolism,
Endometritis,
Gastritis,
Hemmorrhage (uterine or unqualified),
Lymphangitis,
Metritis,
Metroperitonitis,
Metrorrhagia,
Nephritis,
Pelviperitonitis,
Peritonitis,
Phlegmasia alba dolens,
Phlebitis,
Pyemia,
Septicemia,
Sudden death,
Tetanus,
Thrombosis,
Uremia.

Physicians are requested always to write **Puerperal** before the above terms and others that might be puerperal in character, or to add in parentheses (**Not puerperal**), so that there may be no possibility of error in the compilation of the mortality statistics; also to respond to the requests of the local registrars for additional information when, inadvertently, the desired data are omitted. The value of such statistics can be greatly improved by cordial coöperation between the medical profession and the registration officials. If a physician will not write the true statement of puerperal character on the certificate, he may privately communicate that fact to the local or state registrar, or write the number of the International List under which the death should be compiled, *e.g.*, "Peritonitis (137)."

134. Accidents[1] of pregnancy: [Name the condition.]
 Abortion, [Term not used in invidious sense; **Criminal abortion** should be so specified (184).]
 Miscarriage.
 Ectopic gestation.
 Tubal pregnancy, etc.

135. **Puerperal hemorrhage.**
136. Other accidents[2] of labor: [Name the condition.]
 Caesarean section,
 Forceps application,
 Breech presentation,
 Symphyseotomy,
 Difficult labor,
 Rupture of uterus in labor, etc.

137. **Puerperal septicemia.**
138. **Puerperal albuminuria** and **convulsions.**
139. **Puerperal phlegmasia alba dolens, embolus, sudden death.**
140. *Following childbirth* (*not otherwise defined*). [Define.]
141. **Puerperal** *diseases* **of the breast.** [Name disease.]

(VIII. — Diseases of the Skin and Cellular Tissue)

142. **Gangrene.** [State part affected, **Diabetic** (50), etc.]
143. **Furuncle.**
144. *Acute abscess.* [Name part affected, nature, or cause.]
145. Other diseases of the skin and annexa. [Name disease.]

[1] In the sense of *conditions or operations dependent upon* pregnancy or labor, not "accidents" from external causes.

[2] In the sense of *conditions or operations dependent upon* pregnancy or labor, not "accidents" from external causes.

(IX. — DISEASES OF THE BONES AND OF THE ORGANS OF LOCOMOTION)

146. Diseases of the bones (tuberculosis excepted): [Name disease.]
Osteoperiostitis, [Give cause.]
Osteomyelitis,
Necrosis, [Give cause.]
Mastoiditis, etc. [Following **Otitis media** (76)?]

147. Diseases of the joints (tuberculosis and rheumatism excepted). [Name disease; always specify **Acute articular rheumatism** (47), **Arthritis deformans** (48), **Tuberculosis of — joint** (33), etc., when cause is known.]

148. *Amputations.* [Name disease or injury requiring amputation, thus permitting proper assignment elsewhere.]

149. Other diseases of the organs of locomotion. [Name disease.]

(X. — MALFORMATIONS)

150. **Congenital malformations** (stillbirths not included): [Do not include **Acquired hydrocephalus** (74) or **Tuberculous hydrocephalus (Tuberculous meningitis)** (30) under this head.)
Congenital hydrocephalus,
Congenital malformation of heart,
Spina bifida, etc.

(XI. — DISEASES OF EARLY INFANCY)

151. *Congenital debility,* **icterus,** and **sclerema**: [Give cause of *debility.*]
Premature birth,
Atrophy, [Give cause.]
Marasmus, [Give cause.]
Inanition, etc. [Give cause.]

152. Other diseases peculiar to early infancy:
Umbilical hemorrhage,
Atelectasis,
Injury by forceps at birth, etc.

153. **Lack of care.**

(XII. — OLD AGE)

154. *Senility.* [Name the **disease** causing the death of the old person.]

(XIII. — Affections Produced by External Causes)

Note. — Coroners, medical examiners, and physicians who certify to deaths from violent causes, should always clearly indicate the fundamental distinction of whether a death was due to **Accident, Suicide,** or **Homicide**; and then state the **Means** or **instrument of death.** The qualification "*probably*" may be added when necessary.

155. **Suicide by poison.** [Name poison.]
156. **Suicide by asphyxia.** [Name means of death.]
157. **Suicide by hanging** or strangulation. [Name means of strangulation.]
158. **Suicide by drowning.**
159. **Suicide by firearms.**
160. **Suicide by** cutting or piercing instruments. [Name instrument.]
161. **Suicide by jumping from** high places. [Name place.]
162. **Suicide by** crushing. [Name means.]
163. Other **suicides.** [Name means.]
164. **Poisoning by food.** [Name kind of food.]
165. Other **acute poisonings.** [Name poison; specify **Accidental.**]
166. **Conflagration.** [State fully, as **Jumped from Window of burning dwelling, Smothered — burning of theater, Forest fire,** etc.]
167. **Burns** (conflagration excepted). [Includes **Scalding.**]
168. Absorption of deleterious gases (conflagration excepted):

    Asphyxia by illuminating gas (accidental),
    Inhalation of ——— (accidental), [Name gas.]
    Asphyxia **(accidental),** [Name gas.]
    Suffocation **(accidental),** etc. [Name gas.]

169. **Accidental drowning.**
170. Traumatism by **firearms.** [Specify **Accidental.**]
171. Traumatism by **cutting** or **piercing instruments.** [Name instrument. Specify **Accidental.**]
172. Traumatism by **fall.** [For example, **Accidental fall from window.**]
173. Traumatism in mines and quarries:

    Fall of rock in coal mine,
    Injury by blasting, slate quarry, etc.

174. Traumatism by **machines.** [Specify kind of machine, and if the Occupation is not fully given under that head, add sufficient to show the exact industrial character of the fatal injury. Thus, **Crushed by passenger elevator; Struck by piece of emery wheel (knife grinder); Elevator accident (pile driver),** etc.]

175. Traumatism by other crushing:

> **Railway collision,**
> **Struck by street car,**
> **Automobile accident,**
> **Run ove by dray,**
> **Crushed by earth in sewer excavation, etc.**

176. Injuries by animals. [Name animal.]
177. **Starvation.** [Not " inanition " from disease.]
178. Excessive cold. **[Freezing.]**
179. Excessive heat. **[Sunstroke.]**
180. **Lightning.**
181. **Electricity** (lightning excepted). [How? Occupational?]
182. **Homicide by firearms.**
183. **Homicide by cutting** or **piercing instruments.** [Name instrument.]
184. **Homicide by** other means. [Name means.]
185. *Fractures* (*cause not specified*). [State means of injury. The nature of the lesion is necessary for hospital statistics but not for general mortality statistics.]
186. Other external causes:

> **Legal hanging,**
> **Legal electrocution,**
> *Accident*, *injury*, or *traumatism* (unqualified). [State **Means of injury.**]

(XIV. — Ill-Defined Diseases)

Note. — If physicians will familiarize themselves with the nature and purposes of the International List, and will coöperate with the registration authorities in giving additional information so that returns can be properly classified, the number of deaths compiled under this group will rapidly diminish, and the statistics will be more creditable to the office that compiles them and more useful to the medical profession and for sanitary purposes.

187. Ill-defined organic disease:

> *Dropsy*, *Ascites*, etc. [Name the disease of the heart, liver, or kidneys in which the dropsy occurred.]

188. *Sudden death.* [Give cause. Puerperal?]

189. Cause of death not specified or ill-defined. [It may be extremely difficult or impossible to determine definitely the cause of death in some cases, even if a post-mortem be granted. If the physician is absolutely unable to satisfy himself in this respect, it is better for him to write **Unknown** than merely to guess at the cause. It will be helpful if he can specify a little further, as **Unknown disease** (which excludes external causes), or **Unknown chronic disease** (which excludes the acute infective diseases), etc. Even the ill-defined causes included under this head are at least useful to a limited degree, and are preferable to no attempt at statement. Some of the old "chronics," which well-informed physicians are coming less and less to use, are the following: *Asphyxia; Asthenia; Bilious fever; Cachexia; Catarrhal fever; Collapse; Coma; Congestion; Cyanosis; Debility; Delirium; Dentition; Dyspnea; Exhaustion; Fever; Gastric fever; HEART FAILURE; Laparotomy; Marasmus; Paralysis of the heart; Surgical shock;* and *Teething.* In many cases so reported the physician could state the **disease** (not mere symptom or condition) **causing death.**]

LIST OF UNDESIRABLE TERMS

UNDESIRABLE TERM. (It is understood that the term criticised is in the *exact form* given below, *without further explanation or qualification.*)	REASON WHY UNDESIRABLE, AND SUGGESTION FOR MORE DEFINITE STATEMENT OF CAUSE OF DEATH.
(1)	(2)
"*Abscess*," "*Abscess of brain*," "*Abscess of lung*," etc.	Was it tuberculous or due to other infection? Traumatic? The return of "Abscess," unqualified, is worthless. State cause (in which case the fact of "abscess" may be quite unimportant) and location.
"*Accident*," "*Injury*," "*External causes*," "*Violence*." Also more specific terms, as "*Drowning*," "*Gun-shot*," which might be either accidental, suicidal, or homicidal.	Impossible to classify satisfactorily. Always state (1) whether **Accidental**, **Suicidal**, or **Homicidal**; and (2) **Means of injury** (*e.g.*, Railroad accident). The lesion (*e.g.*, Fracture of skull) may be added, but is of secondary importance for general mortality statistics.
"*Anasarca*," "*Ascites*."	See "*Dropsy*."
"*Atrophy*," "*Asthenia*," "*Debility*," "*Decline*," "*Exhaustion*," "*Inanition*," "*Weakness*," and other vague terms.	Frequently cover tuberculosis and other definite causes. **Name the disease causing the condition.**
"*Blood poisoning*"	See "*Septicemia*." Syphilis?
"*Cancer*," "*Carcinoma*," "*Sarcoma*," etc.	In all cases **the organ or part first affected by cancer should be specified.**
"*Catarrh*"	Term best avoided, if possible.
"*Cardiac insufficiency*," "*Cardiac degeneration*," "*Cardiac weakness*," etc.	See "*Heart disease*" and "*Heart failure.*"
"*Cardiac dilatation*"	Do not report when a mere terminal condition. State cause.
"*Cellulitis*"	See "*Abscess*," "*Septicemia*."
"*Cerebrospinal meningitis*."	See "*Meningitis*."
"*Congestion*," "*Congestion of bowels*," "*Congestion of the brain*," "*Congestion of kidneys*," "*Congestion of lungs*," etc.	Alone, the word "*congestion*" is worthless, and in combination it is almost equally undesirable. If the disease amounted to *inflammation*, use the proper term (lobar pneumonia, chronic nephritis, enteritis, etc.); merely passive congestion should not be reported as a cause of death. State the primary cause.

LIST OF UNDESIRABLE TERMS (Continued)

UNDESIRABLE TERM.	REASON WHY UNDESIRABLE, AND SUGGESTION FOR MORE DEFINITE STATEMENT OF CAUSE OF DEATH.
(1)	(2)
"*Convulsions,*" "*Eclampsia,*" "*Fit,*" or "*Fits.*"	"It is hoped that this indefinite term ["*Convulsions*"] will henceforth be restricted to those cases in which the true cause of that *symptom* can not be ascertained. At present more than eleven per cent of the total deaths of infants under one year old are referred to '*convulsions*' merely." — *Registrar-General.* "*Fit.* — This is an objectionable term; it is indiscriminately applied to epilepsy, convulsions, and apoplexy in different parts of the country." — *Dr. Farr, in First Rep. Reg.-Gen.*, 1839.
"*Croup*"	"*Croup*" is a most pernicious term from a public health point of view, is not contained in any form in the London or Bellevue Nomenclatures, and should be entirely disused. Write **Diphtheria** when this disease is the cause of death.
"*Dentition,*" "*Teething*"	**State disease causing death.**
"*Disease,*" "*Trouble,*" or "*Complaint*" of [*any organ*], *e.g.*, "*Lung trouble,*" "*Kidney complaint,*" "*Disease of brain,*" etc.	**Name the disease**, *e.g.*, **Lobar pneumonia, Tuberculosis of lungs, Chronic interstitial nephritis, Syphilitic gumma of brain, etc.**
"*Dropsy*"	"'*Dropsy*' should never be returned as the cause of death without particulars as to its probable origin, *e.g.*, in disease of the heart, liver, kidneys, etc." — *Registrar-General.* **Name the disease causing** (the *dropsy* and) **death.**
"*Edema of lungs*"	Usually terminal. **Name the disease causing the condition.**
"*Fever*"	**Name the disease**, as **Typhoid fever, Lobar pneumonia, Malaria, etc.**, in which the "**fever**" occurs.
"*Fracture,*" "*Fracture of skull,*" etc.	Indefinite; the principle of classification for general mortality statistics is *not* the lesion but (1) the nature of the violence that produced it (**Accidental, Suicidal, Homicidal**), and (2) the **Means** of injury.
"*Gastritis,*" "*Gastric catarrh,*" "*Acute indigestion.*"	Frequently worthless as a statement of the actual cause of death; the terms should not be loosely used to cover almost any fatal affection with irritation of stomach. Gastroenteritis? Acute or chronic, and cause?

LIST OF UNDESIRABLE TERMS (Continued)

UNDESIRABLE TERM	REASON WHY UNDESIRABLE, AND SUGGESTION FOR MORE DEFINITE STATEMENT OF CAUSE OF DEATH.
(1)	(2)
"*General decay*, etc.	See "*Old age*."
"*Heart disease*," "*Heart trouble*," even "*Organic heart trouble*."	The exact form of the cardiac affection, as **Mitral regurgitation**, **Aortic stenosis**, or, less precisely, as **Valvular heart disease**, should be stated.
"*Heart failure*," "*Cardiac weakness*," "*Cardiac asthenia*," "*Cardiac exhaustion*," "*Paralysis of the heart*," etc.	"*Heart failure*" is a recognized synonym, even among the laity, for ignorance of the cause of death on the part of the physician. Such a return is forbidden by law in Connecticut. If the physician can make no more definite statement, it must be compiled among the class of ill-defined diseases (*not* under Organic heart disease).
"*Hemorrhage*," "*Hemoptysis*," "*Hemorrhage of lungs*."	Frequently mask tuberculosis or deaths from injuries (traumatic hemorrhage), **Puerperal hemorrhage**, or hemorrhage after operation for various conditions. What was the **cause** and **location** of the hemorrhage? If from violence, state fully (p. 11).
"*Hydrocephalus*"	"It is desirable that deaths from hydrocephalus of tuberculous origin should be definitely assigned in the certificate to **Tuberculous meningitis**, so as to distinguish them from deaths caused by simple inflammation or other disease of the brain or its membranes. **Congenital hydrocephalus** should always be returned as such." — *Registrar-General.*
"*Hysterectomy*"	See "*Operation*."
"*Infantile asthenia*," "*Infantile atrophy*," "*Infantile debility*," "*Infantile marasmus*," etc.	See "*Atrophy*."
"*Infantile paralysis*"	This term is sometimes used for paralysis of infants caused by i strumental delivery, etc. The importance of the disease in its recent endemic and epidemic prevalence in the United States makes the exact and unmistakable expressions **Acute anterior poliomyelitis** or **Infantile paralysis (acute anterior poliomyelitis)** desirable.
"*Inflammation*"	Of what organ or part of the body? Cause?
"*Laparotomy*"	See "*Operation*."

LIST OF UNDESIRABLE TERMS (Continued)

Undesirable Term.	Reason Why Undesirable, and Suggestion for More Definite Statement of Cause of Death.
(1)	(2)
"*Malignant*," "*Malignant disease*."	Should be restricted to use as qualification for neoplasms; see Tumor.
"*Malnutrition*"	See "*Atrophy*."
"*Marasmus*"	This term covers a multitude of worthless returns, many of which could be made definite and useful by giving the name of the disease causing the "marasmus" or wasting. It has been dropped from the English Nomenclature since 1885 ("Marasmus, term no longer used"). The Bellevue Hospital Nomenclature also omits this term.
"*Meningitis*," "*Cerebral meningitis*," "*Cerebrospinal meningitis*," "*Spinal meningitis*."	Only two terms should ever be used to report deaths from **Cerebrospinal fever,** *synonym*, **Epidemic cerebrospinal meningitis,** and they should be written as above and in no other way. It matters not in the use of the latter term whether the disease be actually *epidemic* or not in the locality. A single sporadic case should be so reported. The first term (Cerebrospinal fever) is preferable because there is no apparent objection to its use for any number of cases. No one can intelligently classify such returns as are given in the margin. Mere terminal or symptomatic meningitis should not be entered at all as a cause of death; name the disease in which it occurred. **Tuberculous meningitis should be reported as such.**
"*Natural causes*"	This statement eliminates external causes, but is otherwise of little value. **What disease (probably) caused death?**
"*Old age*," "*Senility*," etc.	Too often used for deaths of elderly persons who succumbed to a definite disease. **Name the disease causing death.**
"*Operation*," "*Surgical operation*," "*Surgical shock*," "*Amputation*," "*Hysterectomy*," "*Laparotomy*," etc.	All these are entirely indefinite and unsatisfactory — unless the surgeon desires his work to be held primarily responsible for the death. **Name the disease, abnormal condition, or form of external violence (Means of death; accidental, suicidal, or homicidal?), for which the operation was performed. If death was due to an anesthetic (chloroform, ether, etc.), state that fact and the name of the anesthetic.**

LIST OF UNDESIRABLE TERMS (Continued)

Undesirable Term.	Reason Why Undesirable, and Suggestion for More Definite Statement of Cause of Death.
(1)	(2)
"*Paralysis*," "*General paralysis*," "*Paresis*," "*General paresis*," "*Palsy*," etc.	The vague use of these terms should be avoided, and the precise form stated, as **Acute ascending paralysis, Paralysis agitans, Bulbar paralysis**, etc. Write **General paralysis of the insane** in full, not omitting any part of the name; this is essential for satisfactory compilation of this cause. Distinguish **Paraplegia** and **Hemiplegia**; and in the latter, when a sequel of **Apoplexy** or **Cerebral hemorrhage**, report the primary cause.
"*Peritonitis*"	"Whenever this condition occurs — either as a consequence of **Hernia, Perforating ulcer of the stomach or bowel [Typhoid fever?], Appendicitis**, or **Metritis** (puerperal or otherwise), or else as an extension of morbid processes from other organs [Name the disease], the fact should be mentioned in the certificate." — *Registrar-General.* Always specify **Puerperal peritonitis** in cases resulting from abortion, miscarriage, or labor at full term. Always state if due to **tuberculosis** or **cancer.** When traumatic, report **means of injury** and whether **accidental, suicidal, or homicidal.**
"*Pneumonia*," "*Typhoid pneumonia.*"	"*Pneumonia*," without qualification, is indefinite; it should be clearly stated either as **Bronchopneumonia** or **Lobar pneumonia**. The term **Croupous pneumonia** is also clear. "The term '*Typhoid pneumonia*' should never be employed, as it may mean either Enteric fever [**Typhoid fever**] with pulmonary complications, on the one hand or Pneumonia with so-called typhoid symptoms on the other." — *Registrar-General.* When lobar pneumonia or bronchopneumonia occurs in the course of or following a disease the primary cause should be entered first, with duration, and the lobar pneumonia or bronchopneumonia be entered beneath as the contributory cause, with duration. Do not report "*Hypostatic pneumonia*" or other mere terminal conditions as causes of death when the disease causing death can be ascertained.

LIST OF UNDESIRABLE TERMS (Continued)

Undesirable Term.	Reason Why Undesirable, and Suggestion for More Definite Statement of Cause of Death.
(1)	(2)
"*Ptomain poisoning*," "*Autointoxication*," "*Toxemia*," etc.	These terms are used very loosely and it is impossible to compile statistics of value unless greater precision can be obtained. They should not be used when merely descriptive of symptoms or conditions arising in the course of diseases, but the disease causing death should alone be named. "Ptomain poisoning" should be restricted to deaths resulting from the development of putrefactive alkaloids or other poisons in food, and the food should be named, as **Ptomain poisoning (mussels)**, etc.
"*Pulmonary congestion*," "*Pulmonary hemorrhage*."	See "*Congestion*," "*Hemorrhage*."
"*Pyemia*"	See "*Septicemia*."
"*Septicemia*," "*Sepsis*," "*Septic infection*," etc.	Always state cause of this condition, and, if localized, part affected. Puerperal? Traumatic (see p. 11)?
"*Shock*" (post-operative)	See "*Operation*."
"*Specific*"	The word *specific* should never be used without further explanation. It may signify *syphilitic*, *tuberculous*, *gonorrheal*, *diphtheritic*, etc. **Name the disease.**
"*Tabes mesenterica*," "*Tabes*"	"The use of this term ["*Tabes mesenterica*"] to describe tuberculous disease of the peritoneum or intestines should be discontinued, as it is frequently used to denote various other wasting diseases which are not tuberculous. **Tuberculous peritonitis** is the better term to employ when the condition is due to tubercle." — *Registrar-General*. **Tabes dorsalis** should not be abbreviated to "*Tabes*."
"*Teething*"	See "*Dentition*."
"*Toxemia*"	See "*Ptomain poisoning*."
"*Tuberculosis*"	The organ or part of the body affected should always be stated, as **Tuberculosis of the lungs, Tuberculosis of the spine, Tuberculous meningitis, Acute general miliary tuberculosis**, etc.

LIST OF UNDESIRABLE TERMS (Concluded)

Undesirable Term.	Reason Why Undesirable, and Suggestion for More Definite Statement of Cause of Death.
(1)	(2)
"*Tumor*," "*Neoplasm*," "*New growth*."	These terms should never be used without the qualfying words **Malignant, Nonmalignant,** or **Benign.** If malignant, they belong under **Cancer,** and should preferably be so reported, or under the more exact terms **Carcinoma, Sarcoma,** etc. In all cases the organ or part affected should be specified.
"*Uremia*"	Name the disease causing death, *i.e.*, the primary cause, not the mere terminal conditions or symptoms, and state the duration of the primary cause.
"*Uterine hemorrhage*"	See "*Hemorrhage.*"

Some of the synonyms used for "typhoid fever." — The following is a partial list of terms which have been used to describe typhoid fever:

Abdominal fever,
Abdominal typhoid,
Abdominal typhus,
Abortive typhoid,
Ambulant typhoid,
Cerebral typhoid,
Cerebral typhus
Continued fever,
Enteric fever,
Enterica,
Gastroenteric fever
Hæmorrhagic typhoid fever,
Ileotyphus,
Intermittent typhoid fever,
Malignant typhoid fever,
Mountain fever,
Paratyphoid fever.
Paratyphus,
Posttyphoid abscess
Rheumatic typhoid fever,
Typhobilious fever,
Typhoenteritis,
Typhogastric fever,
Typhoid fever,
Typhoid malaria,
Typhoid meningitis,
Typhoid stupor,
Typhoid ulcer
Typhomalaria,
Typhomalarial fever,
Typhoperitonitis,
Typhus
Typhus abdominalis.

This shows the great need of standardization.

Joint causes of death. — The Bureau of the Census in 1914 published an "Index of Joint Causes of Death," which shows the proper method of assignment to the preferred title of causes of death when two causes are simultaneously reported. This index, alphabetically arranged, is very useful. Physicians sometimes report two or more causes of death upon the death-certificate. This may be historically true as one disease may be a complication of the other. For statistical purposes, however, only one cause can be tabulated for each death. Out of the two or more causes given one must be selected, and it is a matter of great importance how this is done. For some years the attempt has been made to separate the diseases reported into the primary cause and secondary cause. As this gave rise to some uncertainty as to which was which, the form of the Revised U. S. Standard Certificate of Death asks for "The Cause of Death" and for "The Contributory Cause (Secondary)." The English, French and Germans have laid down certain rules for making the proper selections. In general, it may be said that the primary cause is the real, or underlying, cause of death (the primary affection with respect to time and causation). The following are the American instructions as printed on the back of the standard death-certificate.

STANDARD CERTIFICATE OF DEATH

Statement of occupation. — Precise statement of occupation is very important, so that the relative healthfulness of various pursuits can be known. The question applies to each and every person, irrespective of age. For many occupations a single word or term on the first line will be sufficient, *e.g.*, *Farmer* or *Planter*, *Physician*, *Compositor*, *Architect*, *Locomotive engineer*, *Civil engineer*, *stationary fireman*, etc. But in many cases, especially in industrial employments, it is necessary to know (*a*) the kind of work and also (*b*) the nature of the business or industry, and therefore an additional line is provided for the latter statement; it should be used only when needed. As examples: (*a*)

Spinner, (*b*) *Cotton mill,* (*a*) *Salesman,* (*b*) *Grocery,* (*a*) *Foreman,* (*b*) *Automobile factory.* The material worked on may form part of the second statement. Never return "Laborer," "Foreman," "Manager," "Dealer," etc., without more precise specification, as *Day laborer, Farm laborer, Laborer — Coal mine,* etc. Women at home who are engaged in the duties of the household only (not paid *Housekeepers* who receive a definite salary) may be entered as *Housewife, Housework,* or *At home,* and children, not gainfully employed, as *At school* or *At home.* Care should be taken to report specifically the occupations of persons engaged in domestic service for wages, as *Servant, Cook, Housemaid,* etc. If the occupation has been changed or given up on account of the DISEASE CAUSING DEATH, state occupation at beginning of illness. If retired from business, that fact may be indicated thus: *Farmer* (*retired,* 6 *yrs.*). For persons who have no occupation whatever, write *None.*

Statement of cause of death. — Name, **first**, the DISEASE CAUSING DEATH (the primary affection with respect to time and causation), using always the same accepted term for the same disease. Examples: *Cerebro-spinal fever* (the only definite synonym is "Epidemic cerebro-spinal meningitis"); *Diphtheria* (avoid use of "Croup"); *Typhoid fever* (never report "Typhoid pneumonia"); *Lobar pneumonia; Bronchopneumonia* ("Pneumonia," unqualified, is indefinite); *Tuberculosis of lungs, meninges, peritoneum,* etc., *Carcinoma, Sarcoma,* etc., of ———————— (name origin: "Cancer" is less definite; avoid use of "Tumor" for malignant neoplasms); *Measles; Whooping cough; Chronic valvular heart disease; Chronic interstitial nephritis,* etc. The contributory (secondary or intercurrent) affection need not be stated unless important. Example: *Measles* (disease causing death), 29 *ds.; Broncho-pneumonia* (secondary), 10 *ds.* Never report mere symptoms or terminal conditions, such as "Asthenia," "Anemia," (merely symptomatic), "Atrophy," "Collapse," "Coma," "Convulsions," "Debility" ("Congenital," "Senile," etc.), "Dropsy," "Exhaustion," "Heart failure," "Hemorrhage," "Inanition," "Marasmus," "Old age," "Shock," "Uremia," "Weakness," etc., when a definite disease can be ascertained as the cause. Always qualify all diseases resulting from childbirth or miscarriage, as "Puerperal *septicemia,*" "Puerperal *peritonitis,*" etc. State cause for which surgical operation was undertaken.

For VIOLENT DEATHS state MEANS OF INJURY and qualify as ACCIDENTAL, SUICIDAL, or HOMICIDAL, or as *probably* such, if impossible to determine definitely. Examples: *Accidental drowning; Struck by railway*

train — accident; Revolver wound of head — homicide; Poisoned by carbolic acid — probably suicide. The nature of the injury, as fracture of skull, and consequences (*e. g., sepsis tetanus*) may be stated under the head of "Contributory." (Recommendations on statement of cause of death approved by Committee on Nomenclature of the American Medical Association.)

Cases for the Medical Examiners. — Under the provisions of chapter 24 of the Revised Laws deaths under the following conditions must be referred to the Medical Examiners:

1. Deaths following injury or violence, as *Burns, Falls, Drowning, Gas poisoning, Suicide, Homicide,* etc.
2. Deaths supposedly caused by violence, as *Criminal abortion, Poisoning, Starvation, Suffocation, Exposure,* etc.
3. Sudden deaths of persons not disabled by recognized disease, as *A death upon the street, or one supposed to be due to Alcoholism,* etc.
4. Deaths under circumstances unknown, as *A person found dead,* etc.

The following supplementary suggestions are also useful.[1]

1. Select the primary cause, that is, the real or underlying *cause of death.* This is usually —
   - (*a*) The cause first in order.
   - (*b*) The cause of longer duration. If the physician writes the cause of shorter duration first, inquiry may be made whether it is not a mere symptom, complication, or terminal condition.
   - (*c*) The cause of which the contributory (secondary) cause is a frequent complication. See lists of "Frequent complications" under the various titles of the Tabular List.
   - (*d*) The physician may indicate the relation of the causes by words, although this is a departure from the way in which the blank was intended to be filled out. For example, "Bronchopneumonia *following* measles" (primary cause last) or "measles *followed by* brochopneumonia" (primary cause first).

[1] Manual, 1911, 1, p. 23.

2. If the relation of primary and secondary is not clear, prefer general diseases, and especially dangerous infective or epidemic diseases, to local diseases.

3. Prefer severe or usually fatal diseases to mild diseases.

4. Disregard ill-defined causes (Class XIV), and also indefinite and ill-defined terms (*e.g.*, "debility," "atrophy") in Classes XI and XII that are referred, for certain ages, to Class XIV, as compared with definite causes. Neglect mere modes of death (failure of heart or respiration) and terminal symptoms or conditions (*e.g.*, hypostatic congestion of lungs).

5. Select homicide and suicide in preference to any consequences, and severe accidental injuries, sufficient in themselves to cause death, to all ordinary consequences. Tetanus is preferred to any accidental injury and erysipelas, septicæmia, pyæmia, peritonitis, etc., are preferred to less serious accidental injuries. Prefer definite means of accidental injury (*e.g.*, railway accident, explosion in coal mine, etc.) to vague statements or statement of the nature of the injury only (*e.g.*, accident, fracture of skull).

6. Physical diseases (*e.g.*, tuberculosis of lungs, diabetes) are preferred to mental diseases as causes of death (*e.g.*, manic depressive psychosis), but general paralysis of the insane is a preferred term.

7. Prefer puerperal causes except when a serious disease (*e.g.*, cancer, chronic Bright's disease) was the independent cause.

8. Disregard indefinite terms and titles generally in favor of definite terms and titles. The precise line of demarcation is difficult to lay down, but may be indicated broadly by the kinds of type employed in the International List presented on page 35. The List in this form has been distributed by the Census to all physicians in the United

States,[1] so that the proportion of indefinite returns should become less.

Occupation. — During recent years the study of the relation between occupation and disease has received much attention, and this study has shown the very great importance of the industrial hazard. Fundamental to such a study is a proper classification of occupations. The list which follows was published by the Bureau of the Census in 1915. It is taken from a report entitled "Index to Occupations, alphabetical and classified," a book of 414 pages.

This classification contains 215 main groups, 84 of which are subdivided, making a total of 428 separate groups. The industrial field is divided into eight general divisions, and each occupation has been "classified in that part of the industrial field in which it is most commonly pursued." Clerical occupations are classified apart. The classification is along *occupational* rather than *industrial* lines. In the table each occupation is indicated by a symbol consisting of three figures, the first of which indicates one of the following main divisions:

0. Agriculture, forestry and animal husbandry.
1. Extraction of minerals.
2. }
3. } Manufacturing and mechanical industries.
4. }
5. Transportation.
6. Trade.
7. { Public Service.
   { Professional Service.
8. Domestic and Personal Service.
9. Clerical Occupations.

[1] See Physicians' Pocket Reference to the International List of Causes of Death.

The second and third figures of each symbol are used in combination and indicate the occupation under the given main division. Thus in the symbol 529, 5 stands for "Transportation" and 29 for "Brakeman-steam railroad."

The report emphasizes the need of great care in distinguishing between occupations and gives the following as examples of distinctions which must be made: —

An iron foundry and a brass foundry.
A felt hat factory and a straw hat factory.
A steam railroad and a street railway.
A paper box and a wooden box factory.
A locomotive engineer and a stationary engineer.
A wholesale and a retail merchant or dealer.
A clerk in a store and a salesman.
A machinist and a machine tender.
A paid housekeeper and a housewife in her own home.
A paid housekeeper and a servant girl.
A cook and a servant.
A proprietor and an employee, etc.

LIST OF OCCUPATIONS AND OCCUPATION GROUPS WITH THEIR SYMBOLS

Symbol.	Occupation and occupation group.
	Agriculture, Forestry, and Animal Husbandry
0 1 0....	Dairy farmers
0 1 2....	Dairy farm laborers
0 1 4....	Farmers[1]
	Farm laborers
0 2 1....	Farm laborers (home farm)
0 2 2....	Farm laborers (working out)
0 2 3....	Turpentine farm laborers
	Farm, dairy farm, garden, orchard, etc., foremen
0 2 5....	Dairy farm foremen
0 2 6....	Farm foremen[2]
0 2 7....	Garden and greenhouse foremen
0 2 8....	Orchard, nursery, etc., foremen

[1] Includes turpentine farmers.
[2] Includes turpentine farm foremen.

Symbol	Occupation and occupation group.
	Agriculture, Forestry, and Animal Husbandry — Continued
0 3 3....	Fishermen and oystermen
0 3 5....	Foresters
	Gardeners, florists, fruit growers, and nurserymen
0 4 2....	Florists
0 4 3....	Fruit growers and nurserymen
0 4 4....	Gardeners
0 4 5....	Landscape gardeners
	Garden, greenhouse, orchard, and nursery laborers
0 5	Cranberry bog laborers
0 5 5....	Garden laborers
0 5 6....	Greenhouse laborers
0 5 7....	Orchard and nursery laborers
	Lumbermen, raftsmen, and woodchoppers
0 6 5....	Foremen and overseers
0 6 6....	Lumbermen and raftsmen
0 6 7....	Teamsters and haulers
0 6 8....	Woodchoppers and tie cutters
0 7 5....	Owners and managers of log and timber camps
0 7 7....	Stock herders, drovers, and feeders
0 7 9....	Stock raisers
	Other agricultural and animal husbandry pursuits
0 8 5....	Apiarists
0 8 6....	Corn shellers, hay balers, grain threshers, etc.
0 8 7....	Ditchers (farm)
0 8 8....	Poultry raisers and poultry yard laborers
0 8 9....	Other and not specified pursuits
	Extraction of Minerals
	Foremen, overseers, and inspectors
1 0 0....	Foremen and overseers
1 0 1....	Inspectors
	Operators, officials, and managers
1 1 0....	Managers
1 1 1....	Officials
1 1 2....	Operators

Symbol.	Occupation and occupation group.
	Extraction of Minerals — Continued
1 2 2....	Coal mine operatives
1 3 3....	Copper mine operatives
1 4 4....	Gold and silver mine operatives
1 5 5....	Iron mine operatives
	Operatives in other and not specified mines
1 6 6......	Lead and zinc mine operatives
1 6 7......	All other mine operatives
1 7 7....	Quarry operatives
	Oil, gas, and salt well operatives
1 8 8......	Oil and gas well operatives
1 8 9......	Salt well and works operatives
	Manufacturing and Mechanical Industries
	Apprentices
2 0 0......	Apprentices to building and hand trades
2 0 1......	Dressmakers' and milliners' apprentices
2 0 2......	Other apprentices
2 1 0....	Bakers
	Blacksmiths, forgemen, and hammermen
2 1 1......	Blacksmiths
2 1 2......	Forgemen, hammermen, and welders
2 1 3....	Boiler makers
2 1 4....	Brick and stone masons
2 1 5....	Builders and building contractors
2 1 6....	Butchers and dressers (slaughterhouse)
2 1 7....	Cabinetmakers
2 1 8....	Carpenters
2 1 9....	Compositors, linotypers, and typesetters
2 2 0....	Coopers
2 2 1....	Dressmakers and seamstresses (not in factory)
2 2 2....	Dyers
2 2 3....	Electricians and electrical engineers
	Electrotypers, stereotypers, and lithographers
2 2 4....	Electrotypers and stereotypers
2 2 5....	Lithographers

Symbol.	Occupation and occupation group.
	Manufacturing and Mechanical Industries — Continued
2 2 6....	Engineers (mechanical)
2 2 7....	Engineers (stationary)
2 2 8....	Engravers
	Filers, grinders, buffers, and polishers (metal)
2 3 0......	Buffers and polishers
2 3 1......	Filers
2 3 2......	Grinders
2 3 3....	Firemen (except locomotive and fire department)
2 3 4....	Foremen and overseers (manufacturing)
	Furnace men, smelter men, heaters, pourers, etc.
2 3 5....	Furnace men and smelter men
2 3 6....	Heaters
2 3 7....	Ladlers and pourers
2 3 8....	Puddlers
2 3 9....	Glass blowers
	Jewelers, watchmakers, goldsmiths, and silversmiths
2 4 0....	Goldsmiths and silversmiths
2 4 1....	Jewelers and lapidaries (factory)
2 4 2....	Jewelers and watchmakers (not in factory)
	Laborers (n. o. s.[1])
	Building and hand trades
2 4 3....	General and not specified laborers
2 4 4....	Helpers in building and hand trades
	Chemical industries
2 4 5....	Fertilizer factories
2 4 6....	Paint factories
2 4 7....	Powder, cartridge, fireworks, etc., factories
2 4 8....	Other chemical factories
	Clay, glass, and stone industries
2 5 0....	Brick, tile, and terra-cotta factories
2 5 1....	Glass factories
2 5 2....	Lime, cement, and gypsum factories
2 5 3....	Marble and stone yards
2 5 4	Potteries

[1] Not otherwise specified.

Symbol.	Occupation and occupation group.
	Manufacturing and Mechanical Industries — Continued
	Iron and steel industries
2 5 5....	Automobile factories
2 5 6....	Blast furnaces and rolling mills[1]
2 5 7....	Car and railroad shops
2 5 8....	Wagon and carriage factories
2 5 9....	Other iron and steel works
	Other metal industries
2 6 3....	Brass mills
2 6 4....	Copper factories
2 6 5....	Lead and zinc factories
2 6 6....	Tinware and enamelware factories
2 6 7....	Other metal factories
	Lumber and furniture industries
2 7 0....	Furniture, piano, and organ factories
2 7 1....	Saw and planing mills[2]
2 7 2....	Other woodworking factories
	Textile industries
2 7 5....	Cotton mills
2 7 6....	Silk mills
2 7 7....	Woolen and worsted mills
2 7 8....	Other textile mills
	Other industries
2 8 0....	Charcoal and coke works
2 8 1....	Cigar and tobacco factories
2 8 2....	Clothing industries
2 8 3....	Electric light and power plants
2 8 4....	Electrical supply factories
	Food industries —
2 9 0....	Bakeries
2 9 1....	Butter and cheese factories
2 9 2....	Fish curing and packing
2 9 3....	Flour and grain mills
2 9 4....	Fruit and vegetable canning, etc.
2 9 5....	Slaughter and packing houses
2 9 6....	Sugar factories and refineries
2 9 7....	Other food factories

[1] Includes tinplate mills. [2] Includes wooden box factories.

Symbol.	Occupation and occupation group.
	Manufacturing and Mechanical Industries — Continued
3 0 0....	Gas works
3 0 1....	Liquor and beverage industries
3 0 2....	Oil refineries
3 0 3....	Paper and pulp mills
3 0 4....	Printing and publishing
3 0 5....	Rubber factories
3 0 6....	Shoe factories
3 0 7....	Tanneries
3 0 8....	Turpentine distilleries
3 0 9....	Other factories
3 1 0....	Loom fixers
	Machinists, millwrights, and toolmakers
3 1 1....	Machinists and millwrights
3 1 2....	Toolmakers and die setters and sinkers
3 1 3....	Managers and superintendents (manufacturing)
	Manufacturers and officials
3 1 4....	Manufacturers
3 1 5....	Officials
	Mechanics (n. o. s.[1])
3 1 6....	Gunsmiths, locksmiths, and bellhangers
3 1 7....	Wheelwrights
3 1 8....	Other mechanics
3 2 0....	Millers (grain, flour, feed, etc.)
3 2 1....	Milliners and millinery dealers
	Molders, founders, and casters (metal)
3 2 2....	Brass molders, founders, and casters
3 2 3....	Iron molders, founders, and casters
3 2 4....	Other molders, founders, and casters
3 2 6....	Oilers of machinery
	Painters, glaziers, varnishers, enamelers, etc.
3 2 7....	Enamelers, lacquerers, and japanners
3 2 8....	Painters, glaziers, and varnishers (building)
3 2 9....	Painters, glaziers, and varnishers (factory)

[1] Not otherwise specified.

Symbol.	Occupation and occupation group.
	Manufacturing and Mechanical Industries — (Continued)
3 3 0....	Paper hangers
3 3 1....	Pattern and model makers
3 3 2....	Plasterers
3 3 3....	Plumbers and gas and steam fitters
3 3 4....	Pressmen (printing)
3 3 5....	Rollers and roll hands (metal)
3 3 6....	Roofers and slaters
3 3 7....	Sawyers
	Semiskilled operatives (n. o. s.[1])
	Chemical industries
3 4 0....	Paint factories
3 4 1....	Powder, cartridge, fireworks, etc., factories
3 4 2....	Other chemical factories
3 4 4....	Cigar and tobacco factories
	Clay, glass, and stone industries
3 4 5....	Brick, tile, and terra-cotta factories
3 4 6....	Glass factories
3 4 7....	Lime, cement, and gypsum factories
3 4 8....	Marble and stone yards
3 4 9....	Potteries
	Clothing industries
3 5 5....	Hat factories (felt)
3 5 6....	Suit, coat, cloak, and overall factories
3 5 7....	Other clothing factories
	Food industries
3 6 0....	Bakeries
3 6 1....	Butter and cheese factories
3 6 2....	Candy factories
3 6 3....	Flour and grain mills
3 6 4....	Fruit and vegetable canning, etc.
3 6 5....	Slaughter and packing houses
3 6 6....	Other food factories
3 6 9....	Harness and saddle industries

[1] Not otherwise specified.

Symbol.	Occupation and occupation group.
	Manufacturing and Mechanical Industries — (Continued)
	Iron and steel industries
3 7 0....	Automobile factories
3 7 1....	Blast furnaces and rolling mills [1]
3 7 2....	Car and railroad shops [2]
3 7 3....	Wagon and carriage factories
3 7 4....	Other iron and steel works
	Other metal industries
3 8 0....	Brass mills
3 8 1....	Clock and watch factories
3 8 2....	Gold and silver and jewelry factories
3 8 3....	Lead and zinc factories
3 8 4....	Tinware and enamelware factories
3 8 5....	Other metal factories
	Liquor and beverage industries
3 9 0....	Breweries
3 9 1....	Distilleries
3 9 2....	Other liquor and beverage factories
	Lumber and furniture industries
3 9 4....	Furniture, piano, and organ factories
3 9 5....	Saw and planing mills [3]
3 9 6....	Other woodworking factories
4 0 0....	Paper and pulp mills
4 0 1....	Printing and publishing
4 0 2....	Shoe factories
4 0 3....	Tanneries
	Textile industries —
	Beamers, warpers, and slashers
4 0 5....	Cotton mills
4 0 6....	Silk mills
4 0 7....	Woolen and worsted mills
4 0 8....	Other textile mills
	Bobbin boys, doffers, and carriers
4 1 0....	Cotton mills
4 1 1....	Silk mills
4 1 2....	Woolen and worsted mills
4 1 3....	Other textile mills

[1] Includes tinplate mills. [2] Includes car repairers for street and steam railroads.
[3] Includes wooden box factories.

Symbol.	Occupation and occupation group.
	Manufacturing and Mechanical Industries — (Continued)
	Carders, combers, and lappers
4 1 5....	Cotton mills
4 1 6....	Silk mills
4 1 7....	Woolen and worsted mills
4 1 8....	Other textile mills
	Drawers, rovers, and twisters
4 2 0....	Cotton mills
4 2 1....	Silk mills
4 2 2....	Woolen and worsted mills
4 2 3....	Other textile mills
	Spinners
4 2 5....	Cotton mills
4 2 6....	Silk mills
4 2 7....	Woolen and worsted mills
4 2 8....	Other textile mills
	Weavers
4 3 0....	Cotton mills
4 3 1....	Silk mills
4 3 2....	Woolen and worsted mills
4 3 3....	Other textile mills
	Winders, reelers, and spoolers
4 3 5....	Cotton mills
4 3 6....	Silk mills
4 3 7....	Woolen and worsted mills
4 3 8....	Other textile mills
	Other occupations
4 4 0....	Cotton mills
4 4 1....	Silk mills
4 4 2....	Woolen and worsted mills
4 4 3....	Other textile mills
	Other industries
4 6 0....	Electrical supply factories
4 6 1....	Paper box factories
4 6 2....	Rubber factories
4 6 3....	Other factories

Symbol.	Occupation and occupation group.
	Manufacturing and Mechanical Industries — (Continued)
4 7 0....	Sewers and sewing machine operators (factory) [1]
4 7 1....	Shoemakers and cobblers (not in factory)
	Skilled occupations (n. o. s.[2])
4 7 2....	Annealers and temperers (metal)
4 7 3....	Piano and organ tuners
4 7 4....	Wood carvers
4 7 5....	Other skilled occupations
4 8 0....	Stonecutters
4 8 1....	Structural iron workers (building)
4 8 2....	Tailors and tailoresses
	Tinsmiths and coppersmiths
4 8 3....	Coppersmiths
4 8 4....	Tinsmiths
4 8 5....	Upholsterers
	Transportation
	Water transportation (selected occupations)
5 0 0....	Boatmen, canal men, and lock keepers
5 0 2 ...	Captains, masters, mates, and pilots
5 0 4....	Longshoremen and stevedores
5 0 6....	Sailors and deck hands
	Road and street transportation (selected occupations)
5 0 8....	Carriage and hack drivers
5 1 0....	Chauffeurs
5 1 2....	Draymen, teamsters, and expressmen [3]
5 1 4....	Foremen of livery and transfer companies
5 1 6....	Garage keepers and managers
5 1 8....	Hostlers and stable hands
5 2 0....	Livery stable keepers and managers
5 2 2....	Proprietors and managers of transfer companies
	Railroad transportation (selected occupations)
	Baggagemen and freight agents
5 2 4....	Baggagemen
5 2 5....	Freight agents

[1] Includes sewers and sewing machine operators in all factories except shoe and harness factories, and sack sewers in fertilizer, salt, and sugar factories, and cement, flour, and grain mills.

[2] Not otherwise specified.

[3] Teamsters in agriculture, forestry, and the extraction of minerals are classified with the other workers in those industries, respectively; and drivers for bakeries and laundries are classified with deliverymen in trade.

Symbol.	Occupation and occupation group.
	Transportation — (Continued)
5 2 7....	Boiler washers and engine hostlers
5 2 9....	Brakemen
5 3 0....	Conductors (steam railroad)
5 3 2....	Conductors (street railroad)
5 3 4....	Foremen and overseers
	Laborers
5 3 6....	Steam railroad
5 3 7....	Street railroad
5 3 9....	Locomotive engineers
5 4 0....	Locomotive firemen
5 4 2....	Motormen
	Officials and superintendents
5 4 4....	Steam railroad
5 4 5....	Street railroad
	Switchmen, flagmen, and yardmen
5 4 7....	Switchmen and flagmen (steam railroad)
5 4 8....	Switchmen and flagmen (street railroad)
5 4 9....	Yardmen (steam railroad)
5 5 0....	Ticket and station agents
	Express, post, telegraph, and telephone (selected occupations)
5 5 2....	Agents (express companies)
	Express messengers and railway mail clerks
5 5 4....	Express messengers
5 5 5....	Railway mail clerks
5 5 7....	Mail carriers
5 5 9....	Telegraph and telephone linemen
5 6 0....	Telegraph messengers
5 6 2....	Telegraph operators
5 6 4....	Telephone operators
	Other transportation pursuits
	Foremen and overseers (n. o. s.[1])
5 6 6....	Road and street building and repairing
5 6 7....	Telegraph and telephone companies
5 6 8....	Water transportation
5 6 9....	Other transportation

[1] Not otherwise specified.

Symbol.	Occupation and occupation group.
	Transportation — (Continued)
	Inspectors
5 7 0....	Steam railroad
5 7 1....	Street railroad
5 7 2....	Other transportation
	Laborers (n o. s.[1])
5 7 5....	Road and street building and repairing
5 7 6....	Street cleaning
5 7 7....	Other transportation
	Proprietors, officials, and managers (n. o. s.[1])
5 8 0....	Telegraph and telephone companies
5 8 1....	Other transportation
	Other occupations (semiskilled)
5 8 5....	Steam railroad
5 8 6....	Street railroad
5 8 7....	Other transportation
	Trade
	Bankers, brokers, and money lenders
6 0 0....	Bankers and bank officials
6 0 1....	Commercial brokers and commission men
6 0 2....	Loan brokers and loan company officials
6 0 3....	Pawnbrokers
6 0 4....	Stockbrokers
6 0 5....	Brokers not specified and promoters
6 1 1....	Clerks in stores
6 1 3....	Commercial travelers
6 1 5....	Decorators, drapers, and window dressers
	Deliverymen
6 2 0....	Bakeries and laundries
6 2 2....	Stores
	Floorwalkers, foremen, and overseers
6 2 4....	Floorwalkers and foremen in stores
6 2 5....	Foremen, warehouses, stockyards, etc.

[1] Not otherwise specified.

Symbol	Occupation and occupation group.
	Trade — (Continued)
6 2 7....	Inspectors, gaugers, and samplers
	Insurance agents and officials
6 3 0....	Insurance agents
6 3 1....	Officials of insurance companies
	Laborers in coal and lumberyards, warehouses, etc.
6 3 3....	Coal yards
6 3 4....	Elevators
6 3 5....	Lumberyards
6 3 6....	Stockyards
6 3 7....	Warehouses
6 4 0....	Laborers, porters, and helpers in stores
6 4 2....	Newsboys
	Proprietors, officials, and managers (n. o. s.[1])
6 4 4....	Employment office keepers
6 4 5....	Proprietors, etc., elevators
6 4 6....	Proprietors, etc., warehouses
6 4 7....	Other proprietors, officials, and managers
6 5 0....	Real estate agents and officials
6 5 5....	Retail dealers
	Salesmen and saleswomen
6 6 3....	Auctioneers
6 6 4....	Demonstrators
6 6 5....	Sales agents
6 6 6....	Salesmen and saleswomen (stores)
6 6 8....	Undertakers
6 7 7....	Wholesale dealers, importers, and exporters
	Other pursuits (semiskilled)
6 8 6....	Fruit graders and packers
6 8 7....	Meat cutters
6 8 8....	Other occupations

[1] Not otherwise specified.

Symbol.	Occupation and occupation group.
	Public Service (not Elsewhere Classified)
7 0 0....	Firemen (fire department)
7 0 2....	Guards, watchmen, and doorkeepers
	Laborers (public service)
7 0 6....	Garbage men and scavengers
7 0 7....	Other laborers
	Marshals, sheriffs, detectives, etc.
7 1 0....	Detectives
7 1 1....	Marshals and constables
7 1 2....	Probation and truant officers
7 1 3....	Sheriffs
	Officials and inspectors (city and county)
7 1 5....	Officials and inspectors (city)
7 1 6....	Officials and inspectors (county)
	Officials and inspectors (state and United States)
7 2 0....	Officials and inspectors (state)
7 2 1....	Officials and inspectors (United States)
7 2 5....	Policemen
7 2 7....	Soldiers, sailors, and marines
	Other pursuits
7 3 0....	Life-savers
7 3 1....	Lighthouse keepers
7 3 2....	Other occupations
	Professional Service
7 4 0....	Actors
7 4 2....	Architects
7 4 4....	Artists, sculptors, and teachers of art
	Authors, editors, and reporters
7 4 6....	Authors
7 4 7....	Editors and reporters
7 5 0....	Chemists, assayers, and metallurgists
	Civil and mining engineers and surveyors
7 5 2....	Civil engineers and surveyors
7 5 3 ...	Mining engineers
7 5 5....	Clergymen
7 5 7....	College presidents and professors
7 5 9....	Dentists

Symbol.	Occupation and occupation group.
	Professional Service — (Continued)
	Designers, draftsmen, and inventors
7 6 0....	Designers
7 6 1....	Draftsmen
7 6 2....	Inventors
7 6 4....	Lawyers, judges, and justices
7 6 6....	Musicians and teachers of music
7 6 8....	Photographers
7 7 0....	Physicians and surgeons
7 7 2....	Showmen
	Teachers
7 7 4....	Teachers (athletics, dancing, etc.)
7 7 5....	Teachers (school)
7 7 7....	Trained nurses
7 7 9....	Veterinary surgeons
7 8 0....	Other professional pursuits
	Semiprofessional pursuits
7 8 1....	Abstractors, notaries, and justices of peace
7 8 2....	Fortune tellers, hypnotists, spiritualists, etc.
7 8 3....	Healers (except physicians and surgeons)
7 8 4....	Keepers of charitable and penal institutions
7 8 5....	Officials of lodges, societies, etc.
7 8 6....	Religious and charity workers
7 8 7....	Theatrical owners, managers, and officials
7 8 8....	Other occupations
7 9 0....	Attendants and helpers (professional service)
	Domestic and Personal Service
8 0 0....	Barbers, hairdressers, and manicurists
8 0 2....	Bartenders
	Billiard room, dance hall, skating rink, etc., keepers
8 0 4....	Billiard and pool room keepers
8 0 5....	Dance hall, skating rink, etc., keepers
8 1 1....	Boarding and lodging house keepers
8 1 3....	Bootblacks
8 2 0....	Charwomen and cleaners
8 2 2....	Elevator tenders
8 3 0....	Hotel keepers and managers

Symbol.	Occupation and occupation group.
	Domestic and Personal Service — (Continued)
8 3 3....	Housekeepers and stewards
8 3 5....	Janitors and sextons
8 4 2....	Laborers (domestic and professional service)
8 4 4....	Launderers and laundresses (not in laundry)
8 4 6....	Laundry operatives
8 4 8....	Laundry owners, officials, and managers
	Midwives and nurses (not trained)
8 5 4....	Midwives
8 5 5....	Nurses (not trained)
8 6 6....	Porters (except in stores)
8 6 8....	Restaurant, cafe, and lunch room keepers
8 7 0....	Saloon keepers
	Servants
8 7 3....	Bell boys, chore boys, etc.
8 7 4....	Chambermaids
8 7 5....	Coachmen and footmen
8 7 6....	Cooks
8 7 7....	Other servants
8 8 8....	Waiters
	Other pursuits
8 9 5....	Bathouse keepers and attendants
8 9 6....	Cemetery keepers
8 9 7....	Cleaners and renovators (clothing, etc.)
8 9 8....	Umbrella menders and scissors grinders
8 9 9...	Other occupations
	Clerical Occupations
	Agents, canvassers, and collectors
9 5 5....	Agents
9 5 6....	Canvassers
9 5 7....	Collectors
9 6 6....	Bookkeepers, cashiers, and accountants
	Clerks (except clerks in stores)
9 7 6....	Shipping clerks
9 7 7....	Other clerks
	Messenger, bundle, and office boys [1]
9 8 7....	Bundle and cash boys and girls
9 8 8....	Messenger, errand, and office boys
9 9 9......	Stenographers and typewriters

[1] Except telegraph and telephone messengers.

Nosology not an exact science. — The following reported causes of death will enable the student to decide whether or not nosology is an exact science:

"Went to bed feeling well, but woke up dead."

"Died suddenly at the age of 103. To this time he bid fair to reach a ripe old age."

"Deceased had never been fatally sick."

"Last illness caused by chronic rheumatism, but was cured before death."

"Died suddenly, nothing serious."

"While cranking his automobile sustained what is technically known as a colles fracture of the right rib."

"Kick by horse showed on left kidney."

"Chronic disease."

"Deceased died from blood poison caused by a broken ankle, which is remarkable, as the automobile struck him between the lamp and the radiator."

"Death caused by five doctors."

"Delicate from birth."

"Artery lung busted."

"Collocinphantum."

"Typhoid fever, bronchitis, pneumonia and a miscarriage."
— "Vital Statistics."

EXERCISES AND QUESTIONS

1. What does Van Buren mean by the "Will-o'-the-wisp" of the statistics of causes of death? [See Am. J. P. H., Dec. 1917, p. 1016.]

2. What changes have taken place in the nomenclature of "Tuberculosis," during the last century?

3. Give ten examples of joint causes of death, indicating in each case which is primary and which secondary.

4. What preparations are being made to revise the present International List of Causes of Death?

5. Select the appropriate cause of death for statistical report from the following joint causes of death, and give reason for your selection.

a. Broncho-pneumonia and measles.
b. Infantile diarrhœa and convulsions.
c. Scarlet fever and diphtheria.

d. Nephritis and scarlet fever.
e. Pulmonary tuberculosis and puerperal septicemia.
f. Typhoid fever and pneumonia.
g. Pericarditis and appendicitis.
h. Cirrhosis and angina pectoris.
i. Saturnism and peritonitis.
j. Old age and bronchitis.

CHAPTER IX

ANALYSIS OF DEATH-RATES

Reasons for Analyzing a Death-rate. — We have now covered the principal methods used in the simpler forms of statistical study. We have seen the futility of using general death-rates for comparing the mortality of different places. We have learned how to compute specific rates for groups and classes, particular rates for different diseases and special rates of various kinds. Let us now put these ideas together and say that *the way to use a general death-rate is to analyze it.* Taken by itself it means very little, but if properly analyzed it will yield us useful information.

Two Methods of Analysis. — There are two methods of analyzing a general death-rate.

One is to sub-divide the numerator of the fraction into classes and groups, leaving the denominator of the fraction unchanged. The total population at mid-year is taken as the denominator of the fraction. This is sometimes done in separating all of the deaths in a year according to months and dividing each by the total population. It has the advantage that the sum of all the parts is equal to the whole. In the case mentioned the sum of all the monthly rates gives the yearly rate. It has the disadvantage that the figures cannot be compared or any standard easily carried in the mind.

Another and better method is to sub-divide both the numerator and denominator into classes and groups, that is, to find their specific rates. Here the sum of the rates

resulting from the separation does not equal the whole. The weighted average of the constituent rates will, however, equal the whole.

Let us take a simple example:

In 1910 in Massachusetts there were the following populations and deaths classified by sex.

TABLE 72

POPULATION AND DEATHS: MASSACHUSETTS

	Population.	Deaths.
(1)	(2)	(3)
Males	1,655,248	28,259
Females	1,711,168	26,148
Total	3,366,416	54,407

According to the first method of analysis the partial rates would be $28{,}259 \div 3366 = 8.4$ for males, and $26{,}148 \div 3366 = 7.7$ for females, the sum being 16.1, which is the same as dividing 54,407 by 3366, *i.e.*, 16.1 per 1000.

According to the second method the specific rate for males is $28{,}259 \div 1655 = 17.1$, and for females, $26{,}148 \div 1711 = 15.3$. In this case the weighted average would be $(17.1 \times 1655 + 15.3 \times 1711) \div 3366 = 16.1$ per 1000. The advantage of this second method is obvious, as one may readily compare the rate of 17.1 for males, and that of 15.3 for females, with 16.1, the death-rate for the entire population. In other words, this method of analysis gives us a chance to compare, and that is a prime object of statistical study.

Useful subdivisions. — For the purpose of analyzing a general death-rate we may subdivide the area geographically, finding the specific death-rate for each part. A state may

be subdivided into counties, boroughs, cities and towns; or into urban and rural districts. A large city may be divided into wards, precincts or blocks. The subdivisions must be so chosen that both the population and the deaths may be obtained for each one. This often limits the comparison to political subdivisions. Those who take the census and those who keep the death records should get together and see that the geographical subdivisions correspond. Having made these subdivisions and obtained the rates for each, the results should be arrayed and studied by the statistical method described in a later chapter.

We may subdivide the year into seasons, months, weeks, or even days and ascertain the specific death-rate for each subdivision. These results should be arranged for chronological study, and for comparing the results for similar seasons or months for different years.

We may subdivide the population by sex, by nationality, by occupation, and in all sorts of ways.

We may subdivide the deaths according to cause, using either individual causes or classes of causes.

And finally we may use these various separations in combination with each other.

Example of the analysis of a general death-rate for a state. — To give a complete example of an analysis of the general death-rate of a state would require a small volume. A few hints may be given by asking a number of questions in regard to Massachusetts for the year 1910.

According to the 73d Registration Report the general death-rate for the state was 16.1.

Q. Was the death-rate uniform throughout the state?

The answer is obtained by finding the rate for each county and placing them in array, that is, in order of magnitude. The result is as follows:

TABLE 73

DEATH-RATES BY COUNTIES: MASSACHUSETTS, 1910

County.	General death-rate.	County.	General death-rate.
(1)	(2)	(1)	(2)
Norfolk	13.3	Essex	15.9
Plymouth	14.2	Bristol	16.3
Middlesex	15.4	Hampden	16.8
Franklin	15.5	Suffolk	17.0
Worcester	15.6	Barnstable	18.1
Berkshire	15.6	Dukes	19.1
Hampshire	15.7	Nantucket	20.2

Q. What was the *median* death-rate for the different counties, that is, the rate for the county in the middle of the list?

It was 15.8, *i.e.*, between 15.7 and 15.9.

Q. Why is this median rate lower than 16.1, the rate for the entire state?

The more populous counties have death-rates relatively high and this brings up the average. An average of these county rates weighted according to their population would give 16.1.

Q. Why was the rate for Nantucket county so much higher than that for Norfolk?

In order to answer this question intelligently we need to find out *when* the deaths occurred (seasonal distribution), *where* the deaths occurred (geographical distribution), *who* died (distribution by sex, age, nationality), *what* was the cause of death. Knowing these facts we should then seek to correlate them with controllable conditions.

As a rule a county is not a good geographical unit for such a study as it is difficult to get the facts. A city is better.

Comparison of the death-rates of two cities. — In 1910 the general death-rates of the cities of Massachusetts which had populations exceeding 50,000 were as follows:

TABLE 74

DEATH-RATES OF CERTAIN CITIES IN MASSACHUSETTS 1910

City.	General death-rate.	City.	General death-rate.
(1)	(2)	(1)	(2)
Brockton	12.3	Boston	17.2
Lynn	13.1	Holyoke	17.7
Somerville	13.4	Lawrence	17.7
Cambridge	15.0	Fall River	18.4
Springfield	16.6	New Bedford	18.6
Worcester	16.9	Lowell	19.7

Q. Why was the death-rate so much higher in Lowell than in Brockton?

We naturally look first to differences in age and sex distribution. The U. S. Census gives us the following information:

TABLE 75

AGE AND SEX DISTRIBUTION OF POPULATION IN BROCKTON AND LOWELL, MASS.

	Brockton.	Lowell.
(1)	(2)	(3)
Per cent of population under 10 years		
Male	8.8	9.3
Female	8.6	9.3
Per cent of population over 45 years		
Male	10.1	9.2
Female	10.6	10.8

These differences are not striking, except that Lowell has a somewhat larger percentage of children under ten years of age. How about infants? There is not much difference. In Brockton the infant population was 2.15 per cent of the total, in Lowell, 2.19 per cent. The sex differences are not great except that in Lowell in the age-group 15–44 years there are more females than males, while in Brockton the numbers are about alike.

Let us next turn to nationality. Here we find a great difference. In Brockton, 72 per cent of the population were native white and 27 per cent foreign-born white, but in Lowell only 59 per cent were native white while 40 per cent were foreign-born white. Pursuing this further we find that in Lowell the foreign-born whites were made up of French Canadians, 28.3 per cent; Irish, 23.0 per cent; English, 10.5 per cent; Canadians other than French, 9.3 per cent; Greeks, 8.7 per cent. The corresponding figures for Brockton are not given in the census report.

With these fundamental differences in mind we must next turn to industrial conditions, living conditions, etc. Brockton is a shoe city, Lowell a textile city. The housing conditions of the working classes in Brockton are better than in Lowell. These matters should be studied in detail.

But what of the causes of death? The annual report of the State Board of Health shows that the death-rate for pneumonia was 118 per 100,000 in Brockton, but 210 in Lowell; tuberculosis 88 and 137 respectively, diarrhea and cholera morbus 23 and 184. This last is a very important difference.

Turning to the age distribution of deaths we find that in Brockton 18.5 per cent of the deaths were infants, in Lowell 25.2 per cent. Evidently the large number of infant deaths, the large numbers of deaths from dysentery and the large foreign population in Lowell point to certain environmental conditions which influence mortality.

In order to get these facts it was necessary to consult the State Registration Report, the Annual Report of the State Board of Health and the Census Report. The annual reports of the local boards of health should have contained these essential data; in fact they should have contained the following specific death-rates for 1910:

TABLE 76

SPECIFIC DEATH-RATES BY AGE-GROUPS FOR BROCKTON AND LOWELL: 1910

Age-group.	Specific death-rates per 1000.			
	Brockton.		Lowell.	
	Male.	Female.	Male.	Female.
(1)	(2)	(3)	(4)	(5)
0–1	123.0	101.0	286.0	237.0
1–4	8.0	13.0	31.0	35.0
5–9	3.5	6.5	5.2	4.6
10–14	2.1	3.0	1.6	2.7
15–19	4.0	3.2	4.7	3.1
20–24	3.1	2.6	5.2	5.0
25–34	3.9	6.0	7.5	6.8
35–44	4.7	4.3	9.8	10.7
45–64	18.4	11.8	24.0	23.0
65–	106.0	90.0	99.0	95.0

These figures show directly that the infant death-rate was much higher in Lowell than in Brockton, that the death-rate for young children was also higher. This would point at once to home environment. But the rates were also higher in Lowell for the middle-age groups, which would point to greater industrial hazards there.

"Rates" not the only method of comparison. — So much has been said about rates and specific rates that there is danger that the student may come to think of them as

the only method of statistical comparison. That is far from being the case.

The seasonal changes in mortality may be shown in three ways, each of which has its use. In Massachusetts the general death-rate for 1910 was 16.1 per 1000. It varied seasonally as follows:

TABLE 77

SEASONAL DISTRIBUTION OF MORTALITY

Massachusetts, 1910

Month.	Death-rate.	Percentage of total deaths.	Ratio of monthly deaths to average number for each month.
(1)	(2)	(3)	(4)
January	17.1	8.9	106
February	17.1	8.1	106
March	17.8	9.5	110
April	17.2	8.8	107
May	15.2	8.0	94
June	14.4	7.3	89
July	17.2	9.0	107
August	16.6	8.7	103
September	15.8	8.0	98
October	14.7	7.7	91
November	14.8	7.5	92
December	16.2	8.5	101
Year	16.1	100.0	100

Column (2) gives the death-rate for each month as compared with the yearly rate. Columns (3) and (4) are most useful in comparing one year with another. They do not involve population, an uncertain factor in all but the census years, but on the other hand a change in one month affects the figures in all the other months.

EXERCISES AND QUESTIONS

1. Make a statistical analysis of the general death-rates of Boston and Baltimore for the year 1910.

2. Make a statistical analysis of the general death-rates of Chicago and New Orleans for the year 1910.

3. Make a statistical analysis of the general death-rates of other cities to be assigned by instructor.

4. Find the median death-rate for the counties of New York state for 1910.

5. Compare the seasonal mortalities of San Francisco and Boston for 1910, using several different methods of statement.

CHAPTER X

STATISTICS OF PARTICULAR DISEASES

In studying particular diseases we commonly use four ratios which, though described in different ways, may be distinguished by the terms, (*a*) mortality rate; (*b*) proportionate mortality; (*c*) morbidity rate and (*d*) fatality or case fatality. In addition to these ratios the number of cases of a particular disease may be arranged in groups and classes, by age, sex, nationality, occupation, date of onset and in other ways without using ratios; and the same is true of deaths from a particular disease.

Mortality rate. — The mortality rate for a particular disease is obtained by dividing the number of deaths from that disease by the mid-year population expressed in hundred thousands.

Proportionate Mortality. — The proportionate mortality of a particular disease is the per cent which the number of deaths from that disease is of the total number of deaths from all causes. The interval of time is usually taken as one year, but shorter periods may be used. This method is sometimes spoken of as the percentage of mortality, or per cent distribution.

Percentages of mortality are not as commonly published as they were some years ago. They do not involve the population, hence they are especially useful where the population is not known or cannot be correctly estimated. Since the custom of estimating population by a uniform system has become general there has been less need for considering

the percentage of mortality. A theoretical disadvantage of the method is the fact that the number of deaths from the particular disease appears in both the numerator and the denominator of the fraction; that is, the number of deaths from the particular disease helps to make up the total number of deaths.

Morbidity rate. — The morbidity rate is the ratio between the number of cases of a particular disease in a year and the mid-year population expressed, in thousands, or better in hundred thousands. It is sometimes called the "case rate." The morbidity rate is very useful in epidemiological investigations. It is usually based on the entire population, but just as in the case of death-rates, or mortality rates, from particular diseases it may be computed for specific age-groups or classes.

Fatality. — The fatality of a disease is the ratio between the number of deaths and the number of cases. It is best expressed as a percentage. The fatality of any disease is far from being the same at all ages.

Example. — In 1915 the population of Cambridge, Mass., was 108,822; the total number of deaths from all causes 1460; the number of cases and deaths from scarlet fever were 379 and 5, respectively. From these facts we have the following rates and ratios:

General death-rate, $1460 \div 108.822 = 13.45$ per 1000.
Scarlet-fever, mortality rate, $5 \div 1.08822 = 4.6$ per 100,000.
Scarlet-fever, proportionate mortality, $5 \div 14.60 = 0.34$ per cent.
Scarlet-fever, morbidity rate, $379 \div 1.08822 = 347$ per 100,000.
Scarlet-fever, fatality, $5 \div 379 = 1.32$ per cent.

Inaccuracy of morbidity and fatality rates. — It must not be forgotten that rates for morbidity are based on reported cases and that not all cases are reported. Nearly all morbidity rates are too low. It follows therefore, that nearly all fatality percentages are too high. In the case of typhoid-fever, for example, a comparison of deaths and

reported cases, has led to the popular idea that the fatality is about 10 per cent, that is, one death for every ten cases. But in a number of epidemics, where the cases were accurately obtained by a house to house canvas, it has been found that there were from twelve to fifteen cases for each death, that is, the fatality was only about 7 per cent.

It is interesting to see how an epidemic of typhoid fever will result in an increased proportion of cases being reported. In Cleveland, Ohio, in the year 1902 there were but 3.7 times as many reported cases as deaths, but the following year, when a severe epidemic occurred, there were 7.3 times as many reported cases as deaths. If the figures for 1902 had been correct it would have meant a fatality of 27 per cent, which is most unlikely.

Causes of death in Massachusetts. — In 1915 the principal causes of death in Massachusetts were as follows. They are arranged according to the Abridged International List.

TABLE 78

PRINCIPAL CAUSES OF DEATH IN MASSACHUSETTS

Rank.	Cause of death.	Per cent of mortality.
(1)	(2)	(3)
1	Pneumonia (92)	8.8
2	Tuberculosis of the lungs (28, 29)	8.3
3	Organic diseases of the heart (79)	7.4
4	Diarrhea and enteritis (104)	6.9
5	Congenital debility and malformations (150, 151)	6.8
6	Cerebral hemorrhage and softening (64, 65)	6.1
7	Cancer and other malignant tumors (39–45)	5.6
8	Acute nephritis and Bright's disease (119, 120)	5.6
9	Other diseases of respiratory system (86–88, 91, 93–98)	4.8
10	Violent deaths, suicide excepted (164–186)	4.4

It will be seen that these ten causes account for nearly two-thirds of all the deaths.

The ten most important causes of death for the U. S. registration area in 1914 were not placed in the same order, but were as follows:

TABLE 79

PRINCIPAL CAUSES OF DEATH: UNITED STATES, 1915

Rank.	Cause of death.
(1)	(2)
1	Organic diseases of the heart (79)
2	Tuberculosis of the lungs (28, 29)
3	Bright's disease (119, 120)
4	Pneumonia (92)
5	Violent deaths (164–186)
6	Cancer (39–45)
7	Cerebral hemorrhage (64, 65)
8	Congenital debility and malformations (150, 151)
9	Diarrhea and enteritis (104)
10	Bronchitis (89, 90)

The proportionate mortality differs more or less in different places. It is not the same for the two sexes. It differs greatly at different ages. It is not the same at all seasons. It is different to-day from what it was a generation ago. The control of communicable diseases has considerably altered the relative importance of the different causes of death.

Study of tuberculosis by age and sex. — In attempting to study any particular disease in order to determine its relation to age and sex one will be surprised to find how difficult it is to get a complete statement of the necessary facts for any given place. Obviously we need to have both the cases and deaths classified by age and sex, and we also need the population and the deaths from all causes arranged by sex and according to the same age grouping. If we attempt to use the U. S. Census reports we find that no data for cases are given; if we attempt to use the state board of health

reports we may find that the deaths are classified by age and sex, but that only the total numbers are given for cases; in some city board of health reports we may find cases and deaths duly classified but no populations given for the corresponding groups and classes. As an illustration of unsatisfactory current practice let us study the statistics of tuberculosis for the city of Cambridge, Mass., for the year 1915. The data in the following table were taken from the annual report of the local board of health, except the population statistics, which were taken from the state census of that year. These data are more than ordinarily complete, yet they are not satisfactory, due chiefly to incomplete reports of cases. It may be assumed that the numbers of deaths are reasonably precise, yet they do not strictly represent local conditions as they include deaths in hospitals. The numbers of cases and deaths are small and this also makes the derived rates subject to erratic fluctuations.

The fundamental data are given in columns (2) to (9), the derived figures in the subsequent columns. Column (10) was obtained from columns (2) and (8); column (12) from column (8); column (14) from columns (6) and (2); column (16) from column (6); column (18) from columns (6) and (4); column (20) from columns (6) and (8).

If we take the figures at their face value we notice first that both the morbidity and mortality rates are high in infancy and low in childhood. The male morbidity rate reaches its highest point in age group 30–39 years, but the male mortality rate is highest between 40 and 50. In females the morbidity rate rises earlier and is highest in age-group 20–29. The highest female mortality rate is also found in the same group. Forty per cent of all the cases and 37.9 per cent of all the deaths from tuberculosis among females occurred between the ages of twenty and thirty.

If we study the figures for proportionate mortality we see

TABLE 80

CAMBRIDGE, MASS., 1915

Statistics of Tuberculosis (28-35) Cases and Death, Arranged by Age and Sex

Age-group.	Population.		Deaths, all causes.		Tuberculosis Deaths.		Tuberculosis Cases.	
	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0–1	1,114	1,080	138	105	0	1	1	1
1–4	4,161	4,120	38	40	2	1	6	1
5–9	4,996	5,000	13	13	0	1	4	5
10–14	4,488	4,533	7	6	1	1	4	0
15–19	4,569	4,901	22	12	9	7	9	17
20–29	10,424	11,326	44	51	23	31	40	50
30–39	8,334	9,190	64	43	29	19	49	31
40–49	6,552	7,177	80	76	32	10	31	9
50–59	4,133	4,823	88	85	13	6	14	8
60–	3,224	4,678	229	306	10	5	9	3
Total	51,995	56,808	723	737	119	82	167	125

Age-group.	Morbidity (case), rate per 100,000.		Percentage distribution of cases.		Mortality (death) rate.		Percentage distribution of deaths.		Proportionate mortality, per cent.		Fatality, per cent.	
	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.
(1)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
0–1	90	93	0.6	0.8	0	93	0	1.2	0	1	0	100
1–4	144	41	3.6	0.8	48	41	1.7	1.2	5	3	33	100
5–9	80	100	2.4	4.0	0	20	0	1.2	0	8	0	20
10–14	89	0	2.4	0.0	22	26	0.8	1.2	14	17	25	. .
15–19	197	347	5.4	13.6	197	143	7.6	8.5	39	58	100	41
20–29	383	441	23.8	40.0	220	274	19.3	37.9	52	61	57	62
30–39	588	337	29.4	24.8	348	207	24.4	23.2	45	44	59	61
40–49	473	125	18.6	7.2	488	139	26.9	12.2	40	13	103	111
50–59	339	166	8.4	6.4	315	124	10.9	7.3	15	7	92	75
60–	279	64	5.4	2.4	310	107	8.4	6.1	4	2	111	167
Total	321	220	100.0	100.0	229	144	100.0	100.0	16.5	11.1	71	66

that tuberculosis caused 16.5 per cent of all deaths among males and 11.1 per cent of all deaths among females. In age-group 20–29 this disease caused nearly two-thirds of all deaths of females and more than half of all deaths of males. In comparing the figures for proportionate mortality it should be observed that the age-groups are not of equal value throughout the table; some cover ten years, some five, one covers four years, and one only one year.

The fatality rates are practically worthless. Sometimes the number of reported cases was less than the number of deaths, thus making the fatality rate higher than 100 per cent. This would be an absurdity, if we did not know that the tuberculosis deaths of one year may represent cases of the year before or the year before that. Tuberculosis is a disease of long duration, sometimes several years. The fatality of such a disease as this cannot be computed in this way. Yet imperfect as these figures are we can gather from them the main facts. We can see that tuberculosis is essentially a disease of early manhood and womanhood, and at those ages we naturally look to working conditions as contributory factors. The disease continues as an important cause of death up to old age, especially among males.

If we take the figures for the U. S. Registration Area as given in the Mortality Report for 1914 we obtain a more uniform set of figures, as they are based on 898,059 deaths instead of 1460 deaths. (Table 81.) Here the highest proportionate mortality for tuberculosis was for age-group 20–24 years; for males it was 34.3 per cent, for females 39.2. These figures are considerably lower than for Cambridge. The percentage distribution of tuberculosis deaths showed a maximum in age-group 20–24 for females, and in age-group 25–29 for males. The morbidity, mortality and fatality could not be computed as no records of cases and no population by age-groups were given in the Mortality Report.

TABLE 81

DEATHS FROM TUBERCULOSIS OF THE LUNGS (28)

U. S. Registration Area, 1914, by Age and Sex

Age-group.	Deaths, all causes.		Deaths (28)		Percentage distribution.		Proportionate mortality.	
	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0–4	118,375	95,735	3,416	2,832	6.1	6.9	2.9	3.0
5–9	10,162	9,140	832	878	1.5	2.1	8.2	9.6
10–14	6,819	6,054	702	1,235	1.3	3.0	10.3	20.3
15–19	10,934	10,322	2,719	3,801	4.9	9.2	24.8	36.8
20–24	17,516	15,408	6,002	6,061	10.8	**14.7**	**34.3**	**39.2**
25–29	19,407	16,300	6,634	5,795	**11.9**	14.1	34.0	35.5
30–34	20,212	15,878	6,466	4,701	11.6	11.4	32.2	29.7
35–39	23,154	17,155	6,428	3,922	11.5	9.5	27.8	22.9
40–44	24,116	16,803	5,761	2,950	10.3	7.2	23.8	17.6
45–49	25,283	17,779	4,640	2,172	8.3	5.3	18.3	12.2
50–54	28,809	20,294	3,853	1,679	6.9	4.1	13.4	8.3
55–59	28,896	21,006	2,905	1,388	5.2	3.4	10.1	6.6
60–64	31,255	24,275	2,139	1,217	3.8	3.0	6.8	5.0
65–69	32,728	27,075	1,460	966	2.6	2.3	4.5	3.6
70–74	32,760	29,109	929	777	1.6	1.9	2.8	2.7
75–79	27,365	26,410	515	465	0.9	1.1	1.9	1.8
80–84	19,132	20,537	188	205	0.3	0.5	1.0	1.0
85–89	9,600	11,447	48	57	0.1	0.1	0.5	0.5
90–94	3,129	4,165	9	16	....	....	0.3	0.4
95–99	704	1,007	6	3	....	....	....	....
100–	179	288	1	2	....	....	....	....
Total	491,416	406,643	55,724	41,179	100.0	100.0	11.4	10.2

Seasonal Distribution of deaths from tuberculosis. — A natural way of studying the seasonal distribution of deaths from tuberculosis is to subdivide the annual number of deaths into the numbers which occurred each month and then find what per cent each is of the whole. It is common to arrange the results in a horizontal line thus:

TABLE 82

SEASONAL DISTRIBUTION OF DEATHS FROM TUBERCULOSIS (28-35)

U. S. Registration Area, 1914

	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Total.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Number of deaths....	7522	7524	8537	8238	7782	6901	6528	6209	6031	6009	6212	6873	84,366
Per cent of total for the year...............	8.9	8.9	10.5	9.8	9.2	8.1	7.6	7.4	7.1	7.1	7.4	8.0	100.0%

These figures show that the largest numbers of deaths occur during the spring months, but the difference between winter and summer is not great. It must not be forgotten in such a comparison as this that the months are of unequal length. While the above figures show that 8.9 per cent of the deaths occurred in February and 10.5 per cent in March the average number of deaths per day was 269 per day in February and 275 per day in March. The U. S. Mortality Report, from which these figures were taken, do not distribute the deaths in each month according to age.

Another way of studying the seasonal distribution is to find the proportionate mortality for tuberculosis for each month.

TABLE 83

PROPORTIONATE MORTALITY FROM TUBERCULOSIS BY MONTHS (28-35)

U. S. Registration Area, 1914

Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
9.1	9.7	9.5	10.1	10.2	13.3	9.3	8.7	8.8	8.8	9.0	9.1	9.4%

Here the highest per cent was found in June. These figures are influenced, of course, by the numbers of deaths from causes other than tuberculosis.

Chronological study of tuberculosis. — The death-rate from tuberculosis has decreased steadily during the last generation in Massachusetts as shown by Fig. 50. This curve does not tell us many of the things which we desire to

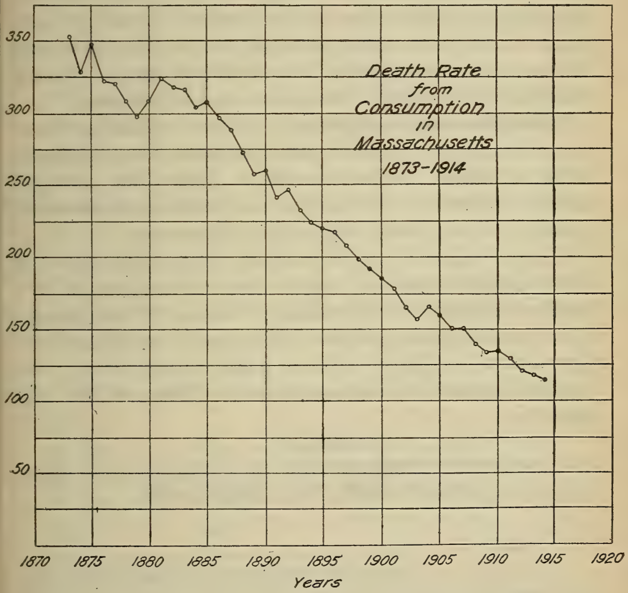


FIG. 50. — Death-rates from Tuberculosis, Massachusetts, 1873–1914.

know. It shows that prior to 1885 the death-rate exceeded 300 per 100,000 but that now it is in the vicinity of 100. It is not decreasing arithmetically, however. Tuberculosis will not disappear by 1940, or thereabouts as one might think by a hasty forward projection of the plotted line. The curve is

losing slope. Even if the rate of decrease remained the same from year to year, it would take many, many years for the curve to reach the zero line.

The curve does not tell us whether it is the lives of the young or the old which are being saved. It is not easy to obtain specific death-rates for tuberculosis by sex and age-groups which cover a long period of years. Even if we had the figures they would not be very reliable because of changes which are being made in the diagnosis of the disease.

Tuberculosis and occupation. — Many misleading statistics relating to tuberculosis and occupation are continually being published. As statements of facts they may be correct, but they are often subject to the fallacy of concealed classification and therefore give false impressions.

A recent report of the New Jersey State Department of Health gives statistics of deaths from tuberculosis in 1916 classified by age and occupation. This is a better arrangement than is sometimes used, but even in studying these figures, it is necessary to be on guard against wrong conclusions because of inadequate data. Thus we find the following:

TABLE 84

DEATHS FROM TUBERCULOSIS CLASSIFIED BY AGE AND OCCUPATION: NEW JERSEY, 1916

Class.	Age.									
	Total	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90+
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Farmers	47	2	6	1	9	11	9	6	3	0
Farm laborers	11	2	2	3	1	2	1	0	0	0
Clerks	110	11	45	31	20	2	1	0	0	0
Housekeepers and stewards	858	34	276	269	158	65	36	18	1	1
General laborers	364	12	61	96	115	49	27	4	0	0
Stone cutters	19	0	0	2	4	7	5	1	0	0

Why is the number of deaths from tuberculosis so high among housekeepers? Not because housekeeping imposes a special hazard, but because there are so many housekeepers in the state. Obviously what is needed here are the specific rates for this particular disease by age-groups. But to compute them it is necessary to know how many housekeepers there are in the state in each age-group, and who knows these facts? Also are the "stewards" referred to male or female?

Why is the number of deaths among stone cutters so small? This occupation is certainly hazardous from the standpoint of tuberculosis, as the fine, sharp, stone dust tends to lacerate the lungs. We cannot draw any reliable conclusion from the figures because we do not know how many stone cutters there are in each group.

We notice that the largest number of deaths from tuberculosis among farmers occurred in age-group 50–59, but that among farm laborers in age-group 30–39. What is a farmer and what is a farm laborer? We must know that. Also do farm laborers ultimately become farmers? Is there a shifting of individuals from one class to the other as they grow older?

So also in the case of clerks. The largest number of deaths is in age-group 20–29. Do the clerks die off at this early age or do they cease to be clerks? Are the clerks male or female?

The student of statistics must persistently cultivate this critical faculty until it becomes a habit. It may result in a cynical and pessimistic frame of mind in regard to published vital statistics, but even this is better than an easy lapse into an unthinking acceptance of all statistics at their face value. Statistics should be used with truth or they had better not be used at all.

Racial composition of population and tuberculosis death-rate. — The following interesting and at first puzzling

situation will serve to emphasize the importance of the careful analysis of death-rates and the necessity of taking into account not only specific death-rates but the composition of the population.

In 1910 the death-rate from tuberculosis of the lungs was 226 per 100,000 in Richmond, Va., and 187 in New York City, and yet the specific death-rates from this disease for both white and colored persons were greater in New York than in Richmond. The following figures were taken from the U. S. Census reports.

TABLE 85

TUBERCULOSIS DEATH-RATES IN NEW YORK AND RICHMOND

Class.	Population.		Number of deaths.		Death-rate per 100,000.	
	New York.	Richmond.	New York.	Richmond.	New York.	Richmond.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
White	4,675,174	80,895	8368	131	179	162
Colored	91,709	46,733	513	155	560	332
Total	4,766,883	127,628	8881	286	187	226

The explanation of this anomaly lies, of course, in the fact that in Richmond more than one-third of the population is colored, while in New York the colored population is less than two per cent.

Many similar comparisons can be found between northern and southern cities. This is merely a striking case.

Diphtheria in Cambridge, Mass. — Applying the same methods to the study of diphtheria we have the following figures:

TABLE 86

CAMBRIDGE, MASS., 1915

Statistics of Diphtheria, Cases and Deaths Arranged by Age and Sex

Age-group.	Population.		Deaths, all causes.		Deaths from diphtheria.		Cases of diphtheria.	
	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0–1	1,114	1,080	138	105	1	3	7	4
1–4	4,161	4,120	38	40	4	6	56	67
5–9	4,996	5,000	13	13	5	5	63	80
10–14	4,488	4,533	7	6	1	1	15	24
15–19	4,569	4,901	22	12	1	0	7	4
20–29	10,424	11,326	44	51	1	0	8	12
30–39	8,334	9,190	64	43	0	0	4	5
40–49	6,552	7,177	80	76	0	0	0	1
50–59	4,133	4,823	88	85	0	0	0	1
60–	3,224	4,678	229	306	0	0	0	0
Total	51,995	56,808	723	737	13	15	160	198

Age-group.	Morbidity (case) rate per 10,000.		Percentage distribution of cases.		Mortality (death) rate.		Percentage distribution of deaths.		Proportionate mortality, per cent.		Fatality, per cent.	
	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.
(1)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
0–1	629	371	4.4	2.0	90	278	7.7	20.0	0.7	2.8	14	75
1–4	1340	1630	3 .0	33.9	96	146	30.7	40.0	10.5	15.0	7	9
5–9	1262	1600	39.3	40.4	100	100	38.5	33.3	38.4	38.4	8	6
10–14	334	528	9.4	12.1	22	22	7.7	6.7	14.3	16.7	7	4
15–19	153	82	4.4	2.0	22	0	7.7	0.0	4.5	0.0	14	0
20–29	76	106	5.0	6.1	9	0	7.7	0.0	2.2	0.0	13	0
30–39	48	55	2.5	2.5	0	0	0.0	0.0	0.0	0.0	0	0
40–49	0	14	0.0	0.5	0	0	0.0	0.0	0.0	0.0	0	0
50–59	0	21	0.0	0.5	0	0	0.0	0.0	0.0	0.0	0	0
60–	0	0	0.0	0.0	0	0	0.0	0.0	0.0	0.0	0	0
Total	308	349	100.0	100.0	25	26	100.0	100.0	1.8	2.0	8	7

Here we see that the maximum age incidence occurs between the ages of one and ten for both males and females. The morbidity rate in Cambridge for this year was higher for females than for males, but the percentage distribution of the cases was about the same for the two sexes. The mortality rates followed the morbidity rates rather closely, and the fatality was fairly constant except for infant females. The proportionate mortality was highest in age-group 5–9. It must be remembered that these rates are computed from small numbers of cases and deaths, hence no very uniform or significant conclusions can be drawn from them. It is only by using large numbers of events that significant tendencies can be shown. The differences between the occurrence of diphtheria and tuberculosis are, however, very striking.

The seasonal distribution of reported cases of diphtheria was as follows:

TABLE 87

SEASONAL DISTRIBUTION OF DIPHTHERIA CASES: CAMBRIDGE, MASS., 1915

	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Total.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Number of cases....	24	35	37	44	31	32	21	15	20	25	38	36	358

It will be noticed that the lowest numbers were reported during the summer vacation months.

Knowing that the incidence of the disease was greatest during the school ages does this indicate that schools played an important part in spreading the infection? Do these statistics *prove* it? If not, what other statistics would be necessary to prove it?

Age susceptibility to diphtheria. — Dr. Charles V. Chapin the Superintendent of Health, of Providence, R. I., has been in the habit of computing what he calls the *attack rate*. This is a ratio between the number of cases and the number of persons exposed, that is, all the members of the family where the disease occurred, including the cases and those who were removed from home after the disease developed. The following figures, given in Dr. Chapin's report for the year 1915, are based on a study of 53,280 exposed persons during 1889–1915.

TABLE 88

DIPHTHERIA ATTACK RATE: PROVIDENCE, R. I., 1915

Age-group.	Attack rate (per cent).	Age-group.	Attack rate (per cent).
(1)	(2)	(1)	(2)
0–1 yr.	16.70	12+ yr.	31.12
1+	43.65	13+	26.08
2+	54.55	14+	22.41
3+	55.61	15+	18.92
4+	55.91	16+	18.58
5+	53.99	17+	17.85
6+	53.82	18+	16.86
7+	49.33	19+	17.33
8+	44.31	20+	23.56
9+	40.91	Adults	6.83
10+	36.42	Total	25.45
11+	35.35		

These figures indicate that the chance of exposed persons acquiring the disease in recognizable form is highest at age four and decreases steadily as the age increases. At the most susceptible period more than half of those exposed came down with diphtheria.

It was found that between the years 1889 and 1915 out of 6822 families who lived in houses where the disease existed in other families, only 474 of these exposed families were

attacked. This is only 6.9 per cent. In most of these cases of attacked families there had been some sort of intercourse with the infected families, that is enough to transmit the disease by contact.

Fatality of diphtheria. — Dr. Chapin has also kept a careful record of the fatality of diphtheria in Providence. In 1884 it was 30 per cent, and a few years later it rose to 42 per cent. Between 1895 and 1896 it dropped from 20 per cent to 14 per cent, since which date it has fallen until now it is only about 8 per cent, that is, there is only one death for each 12 cases. The fatality is not the same at all ages as the following table [1] shows:

TABLE 89

DIPHTHERIA CASE FATALITY AT DIFFERENT AGES

Age.	1889–1914.			1915.		
	Cases.	Deaths.	Fatality.	Cases.	Deaths.	Fatality.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0–1	280	96	34.28	21	2	9.52
1+	706	247	34.99	43	6	13.95
2–4	3,322	697	20.98	181	24	13.26
5–9	4,541	460	10.13	219	15	6.85
10–14	1,801	83	4.61	9	3	3.79
15–19	616	26	4.22	20	1	5.00
20+	1.670	40	2.39	62	2	3.23
Total	12,936	1649	12.74	625	53	8.48

The greatest decrease in the fatality of the disease has occurred among young children.

To a large extent the decreased fatality has been due to the use of antitoxin, which decreased the number of deaths. To some extent it may have been due to better diagnosis by

[1] Ann. Report Providence Supt. of Health, 1915, p. 64.

culture. If this practice increased the number of recognized cases it would decrease the fatality rates.

Diphtheria is a short disease. Hence the fatality can be computed far more accurately than in the case of tuberculosis.

Chronological study of diphtheria. — Fig. 51[1] shows the decrease in the death-rate from diphtheria in Massachusetts since 1873. In 1876 the rate was very high, about 195 per 100,000. It has decreased very greatly until now it is usually less than 20 per 100,000. Recurrences of diphtheria have occurred at intervals of five or six years. After the great epidemic of 1876 there was no important recurrence until 1889, but after that recurrences were noted in 1894 and 1900. Since then, thanks no doubt to preventive medicine, the recurrences have been so slight as to be almost unnoticed.

What is the reason for these recurrences, for this periodic development of diphtheria? In a general way, whooping cough, scarlet fever, measles, all children's diseases, have similar recurrences. It is commonly explained on the theory of susceptibility. It has already been seen that the rate of attack of exposed persons is very high among young children. It is known, too, that one attack usually makes the victim relatively immune against a second attack. After a period of relative quiescence during which the class of susceptible children has been annually recruited it is natural to expect that an epidemic may spread. This is apparently what happened until the methods of preventive medicine came to be generally used. It probably still happens, but to a less extent than formerly.

Urban and rural distribution of diphtheria. — It is not easy to obtain complete statements of the facts to show the differences between the occurrence of diphtheria in cities and

[1] State Sanitation, Vol. I, p. 167.

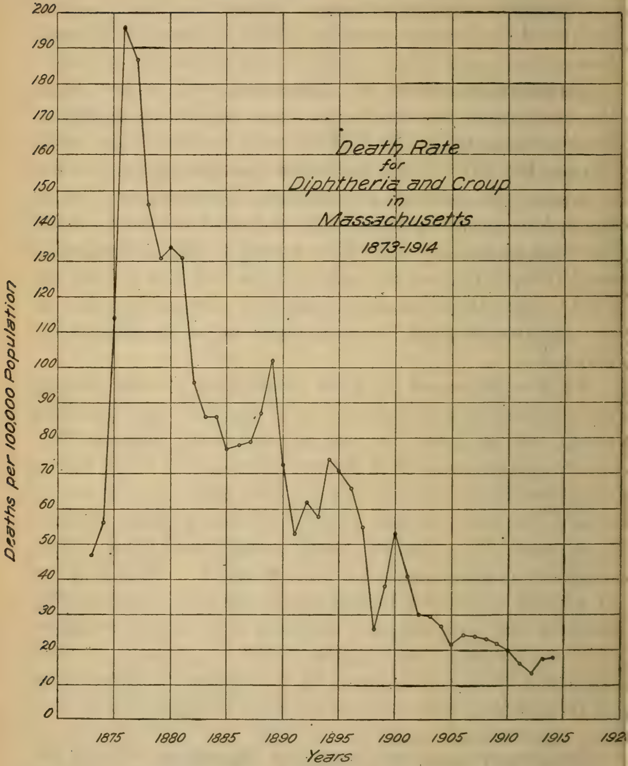


FIG. 51. — Death-rates from Diphtheria, Massachusetts, 1873–1914.

rural districts. Occasionally partial statements are published. In the annual report of the Michigan State Board of Health for 1916–17 it is stated that for the period 1904–15 the morbidity rate was 213 per 100,000 in urban districts and 82 in rural districts; the mortality rates for 1908–1915 were 16.2 and 12.2 per 100,000 respectively. The fatality was 10.9 per cent for cities and 15.7 for urban districts. No separations were made according to age and sex and it is difficult to find these facts. There is, however, quite a difference in the age distribution of diphtheria between the city and the country. In general the average age of diphtheria cases, as well as of persons dying from this disease, is lower in the city. The following facts were taken almost at random from the Mortality Statistics of 1914:

TABLE 90.

PERCENTAGE AGE DISTRIBUTION OF DEATHS FROM DIPHTHERIA

Age.	Cities.		Rural states.		
	New York.	Boston.	Vermont.	New Hampshire.	Maine.
(1)	(2)	(3)	(4)	(5)	(6)
0+	10.7	7.7	0.0	6.7	9.3
1+	25.9	21.4	5.2	17.8	10.5
2+	16.5	13.7	18.4	11.1	16.3
3+	13.7	14.9	18.4	8.9	9.3
4+	9.7	6.0	13.1	17.8	7.0
5–9 (per year)	3.7	5.3	6.3	4.0	7.0
10–19 "	0.26	0.6	0.52	1.55	0.7
20–29 "	0.14	0.12	0.0	0.0	0.23
30–39 "	0.05	0.06	0.0	0.0	0.23

Statistical study of typhoid fever. — Typhoid fever has been given a great deal of attention from the statistical point of view. Hundreds of scientific papers describing

local outbreaks of the disease, variations in the typhoid fever death-rate and so on have been published. For the most part these have been extensive and not intensive studies. It is rather surprising, when we view this enormous mass of statistics, how little we know about certain important points, such as the morbidity and fatality rates at different ages. Our interest has been engrossed by the more important matter of causation. There are many ways in which the disease may be communicated from one person to another and this question must be answered for each particular outbreak or epidemic. The interest in statistical studies of typhoid fever has, therefore, centered around the subject of correlation, a phase of statistics which we shall consider in Chapter XIII. It will be useful to consider at this point some of the fundamental relations of this disease to human beings. Those who are interested in the epidemiology of the subject are referred to the author's book on Typhoid Fever. This book, it should be said, is to-day somewhat out of date, although its historical value remains.

Age distribution of typhoid fever. — The largest number of deaths from typhoid fever is generally found in age-group 20–29 years. Table 91 shows the percentage distribution of deaths by ages according to the U. S. Census [1] for 1900.

In the case of epidemics caused by a widely scattered infection, as through the public water-supply, the age distribution of the deaths usually approximates these figures. If, however, the outbreak occurs in a school-house or is caused by infected milk, which is used more freely by children than by adults, the larger numbers of deaths may occur in the lower ages; in fact, this is a test often applied in the study of typhoid fever outbreaks.

[1] Vital Statistics, Vol. III, Part I, page cxlvi.

TABLE 91

PERCENTAGE DISTRIBUTION OF DEATHS FROM TYPHOID FEVER; UNITED STATES: 1900

Age-group.	Per cent of deaths.	Age-group.	Per cent of deaths.
(1)	(2)	(1)	(2)
0–4	4.09	50–54	3.52
5–9	5.05	55–59	2.55
10–14	5.20	60–64	1.95
15–19	11.23	65–69	1.12
20–24	17.78	70–74	0.91
25–29	15.09	75–79	0.34
30–34	11.46	80–84	0.11
35–39	9.12	85–89	0.09
40–44	5.77	Total	100.00
45–49	4.62		

If we take the specific death-rates by sex and ages we obtain the following figures:

TABLE 92

SPECIFIC DEATH-RATES FOR TYPHOID FEVER

United States: 1900

Age-group.	Rate per 100,000.		Age-group.	Rate per 100,000.	
	Males.	Females.		Males.	Females.
(1)	(2)	(3)	(1)	(2)	(3)
0–4	12	16	45–49	34	29
5–9	15	21	50–54	30	30
10–14	17	31	55–59	30	33
15–19	45	53	60–64	29	33
20–24	66	57	65–69	22	40
25–29	61	48	70–74	27	43
30–34	53	43	75–79	20	23
35–39	48	39	80–84	10	26
40–44	34	37	85–89	16	35

It will be noticed that these differences are not as great as in the previous table. This is because there are fewer persons living at the ages above 50 and even if the specific rate remained high there would not be as many deaths. It is for this reason that both the age distribution of deaths and the specific death-rate are important tabulations. The specific rates just given represent practical conditions and take into account the important element of exposure. The difference between the death-rates of males and females at ages 25–29 must not be regarded as having a physiological basis, for at those ages males are more exposed to the disease than females.

Except at times of epidemics typhoid fever is not a well-reported disease. It is difficult therefore to obtain reliable specific morbidity rates by sex and ages. Such as have been computed, however, show an age distribution very similar to that of deaths, but with a tendency towards larger percentages of cases in the earlier years.

The fatality of the disease at different ages is not as well-established as it ought to be. Computations by the author, by Newsholme, by A. W. Freeman,[1] seem to warrant the following approximate figures:

TABLE 93

APPROXIMATE CASE FATALITY IN TYPHOID FEVER

Age.	Per cent.	Age.	Per cent.
(1)	(2)	(1)	(2)
0	15	40	21
10	8	50	25
20	15	60	42
30	18	All ages	14

[1] Case Fatality in Typhoid Fever, Public Health Reports, Dec. 8, 1916.

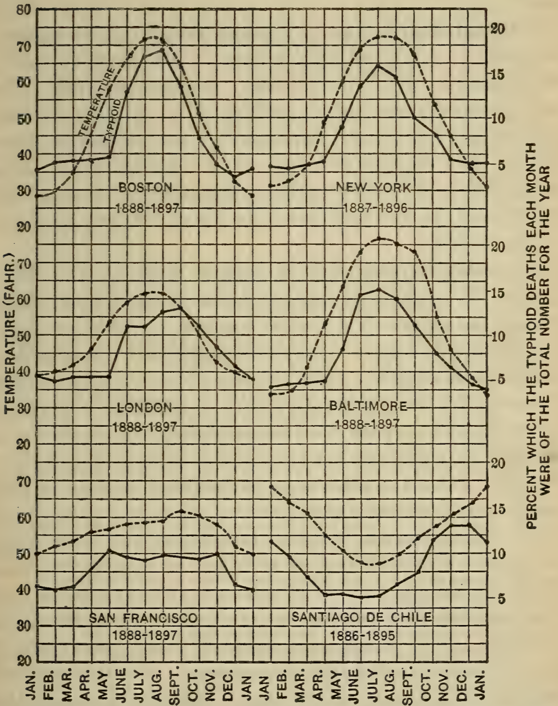


FIG. 52. — Diagram Showing the Relation between Atmospheric Temperature and Seasonal Distribution of Typhoid Fever. (After Sedgwick and Winslow.)

It is probable, therefore, that there is much unreported typhoid fever among children, some of it doubtless unrecognized.

Seasonal distribution of typhoid fever. — The seasonal distribution of typhoid fever appears to be closely related to the manner in which the infection is communicated. Normally there appears to be a fairly close relation between typhoid death-rates and atmospheric temperature, as shown in Fig. 52. Water-borne typhoid is most common during the colder months of the year. Examples of seasonal distribution of typhoid fever are to be found in many epidemiological studies.

Chronological reduction in typhoid fever. — The reduction in the amount of typhoid fever in the United States during the last twenty-five years has been general and steady. From being one of our most dreaded diseases it seems likely to almost disappear. This has been due to many things. George A. Johnson,[1] in an exhaustive compilation of statistics, gives the chief credit to the purification of public water supplies by filtration, his conclusions being summed up in Fig. 53. His main contention is doubtless correct, but the purification of water is only one of many factors in the problem. The safeguards thrown around milk and other foods, the better understanding of the idea of contact, the constantly decreasing number of typhoid carriers since the Civil War and since the purification of water-supplies, the recent extensive use of vaccination methods, have contributed to the present low and falling death-rates. The typhoid-fever death-rates in many cities using unfiltered water-supplies have fallen along with the others. This, however, is no argument against the need of water purification. It does show the need of very careful analysis of the data. Extensive studies, like those of Johnson, have their value, but they are

[1] The Typhoid Toll, Jour. Am. Water Works Assoc., June, 1916.

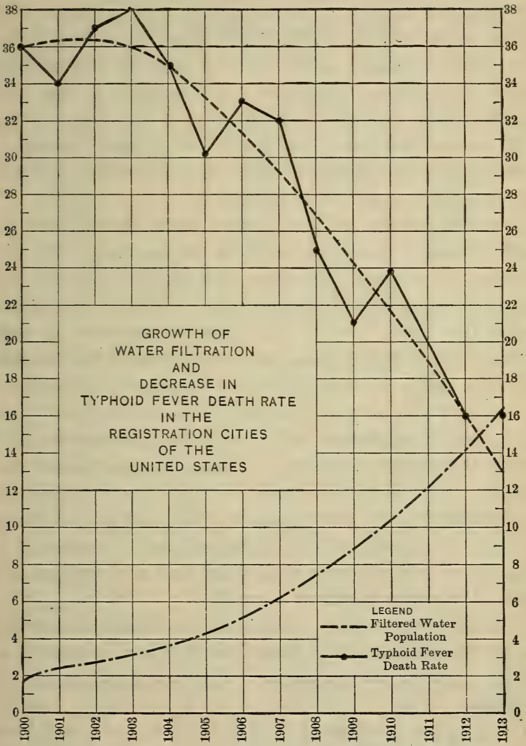


FIG. 53. — Relation between Typhoid Fever and Water Filtration. After Johnson.

not to be compared in importance with more fully analyzed and more critical statistical studies.

Statistics of cancer. — Let us now take up a disease which is quite different from tuberculosis, diphtheria and typhoid fever, namely, cancer. This involves some interesting applications of statistical principles. The subject is one which demands most careful investigation and the reader should by all means read a paper on "The Alleged Increase of Cancer," by Prof. Walter F. Willcox, of Cornell University, as a splendid example of critical work.[1]

Extensive studies of death-rates have shown that as a *reported* cause of death cancer is on the increase. Dr. F. L. Hoffman in a most elaborate monograph entitled "The Mortality of Cancer throughout the World,"[2] has demonstrated this fact. But is this reported increase an actual increase? Is it due to changing conceptions of the statistical unit, to a better recognition of the disease, to differences in the composition of populations? Messrs. King and Newsholme,[3] take the ground that the alleged increase is due to statistical fallacies. Their conclusion is based on intensive studies. Willcox, in the article referred to, has made a critical comparison of these two points of view, and his conclusions are that "improvements in diagnosis and changes in age composition explain away more than half and perhaps all of apparent increase in cancer mortality."

It is admitted at the start that no reliable statistics of cancer morbidity exist. Therefore, neither morbidity nor fatality rates can be computed. The entire discussion rests on deaths.

The increase in reported deaths from cancer is well shown by the following figures:

[1] Quar. Pub. Am. Sta. Asso., Sept. 1917, Vol. XV, p. 701.

[2] Published in 1915, by the Prudential Press, Newark, N. J.

[3] Proc. Royal Society, 1893, liv, pp. 209–242.

TABLE 94

DEATH-RATES FROM CANCER

U. S. Registration Area of 1900

Year.	Rate per 100,000.	
	Male.	Female.
(1)	(2)	(3)
1900	47.0	80.7
1905	53.0	92.1
1910	62.6	103.7
1915	72.3	111.9

The death-rate for females is considerably higher than for males.

The specific death-rate by ages and sex runs as follows:

TABLE 95

SPECIFIC DEATH-RATES FOR CANCER

U. S. Registration Area of 1900 for the Year 1910

Age-group.	Rate per 100,000.		Age-group.	Rate per 100,000.	
	Male.	Female.		Male.	Female.
(1)	(2)	(3)	(1)	(2)	(3)
0–5	4.1	2.8	35–44	33.0	88.9
5–9	1.5	1.2	45–54	106.7	230.7
10–14	1.8	1.4	55–64	272.0	411.3
15–19	2.9	3.5	65–74	493.6	616.2
20–24	4.9	4.1	75–	693.7	867.8
25–34	9.5	21.9			

By applying the method of adjustment to the Standard Million of Population, Willcox finds that for England and Wales in 1911 the death-rate from cancer should have been

91.5 instead of 99.3 per 100,000. In 1901 it was 84.3; hence the increase would have been 8.7 per cent, instead of 17.8 per cent, if computed on the basis of similar populations. Similar comparisons are made for other populations, from all of which the conclusion is drawn that about one-third of the increase in all the populations considered is due to changes in sex and age composition.

In regard to diagnosis many interesting facts are presented. There are differences between the statistics for accessible and inaccessible cancer, the increase being chiefly in the latter. "Laymen seldom report cancer as a cause of death," and there appears to be a correlation between cancer increase and an increase in the number of physicians per 100,000 of population, and the number of medical certificates signed by competent persons. The increase in hospitals is also a factor. Deaths ascribed to tumor, and to "old age," have been decreasing, as cancer has increased, and the implication is that there has been a shifting of these statistical units. For all of these deaths the reader should consult Willcox's paper. He gives also, by way of analogy, a comparison between cancer and appendicitis, which shows that the rate of increase in reported causes of death are substantially the same for the two diseases, namely, 44 per cent and 40 per cent respectively between 1900 and 1915. The upshot of this investigation is that there is no more reason for people to fear dying from cancer now than there was a generation ago. The disease has not changed, people have not changed; it is the reports of the physicians which have changed because of their increased knowledge. Is this the last word on the subject? Probably not.

Further studies of particular diseases. — It is not possible to take up the hundred or more particular diseases and discuss them by means of statistics. This, however, is one of the chief uses of vital statistics. Enough has been given

perhaps to illustrate the method of procedure, and to emphasize the importance of critical statistical analysis. The necessity of considering specific rates, and varying compositions of populations in all these studies cannot be too strongly insisted on.

The student will find it a fascinating and highly valuable study to take up some disease in which he may be interested and study it intensively. There is room in statistical literature for many monographs treating of the statistics of particular diseases.

EXERCISES AND QUESTIONS

1. Describe the cycles of whooping cough in New York State since 1885. [N. Y. State Dept. of Health, Monthly Bulletin, March 1917, p. 70.]

2. How would you find out what proportion of all children born have whooping cough at some time in their lives? Try to make up a table from morbidity statistics of whooping cough classified by age in years, and see if you cannot determine this. Select, for example, the year 1910. How many babies were born that year and how many had had whooping cough while infants? In 1911 how many cases of whooping cough were in age-group 1–2; in 1912 how many in age-groups 2–3, etc.? Add these together and compare the result with the births in 1910. Then do the same starting with 1909, and then 1908, etc. Compare all the results. Are they more uniform than the ordinary annual statistics for whooping cough?

3. Make the same sort of study for measles.

4. Make the same sort of study for diphtheria.

5. Make the same sort of study for scarlet fever.

6. Compare death-rates for whooping cough, measles, etc., in urban and rural districts. What foundation is there for Farr's law that contagious diseases increase as the density of population?

7. Explain the recent finding that the death-rates from measles in the U. S. army cantonments has varied inversely as the density of population (percentage of urban population) in the states from which the soldiers came.

8. Describe the periodicity of whooping cough in Sweden. (See Stephenson and Murray's Textbook of Hygiene.)

9. Describe the age distribution of Pellagra. [See Amer. J. P. H., July, 1918, p. 488.]

10. What reduction in diphtheria has occurred as a result of the use of anti-toxin. [See Am. J. P. H., May, 1917, p. 445.]

11. Is appendicitis increasing? [See Am. J. P. H., July, 1916.]

12. Make a statistical summary of the influenza epidemic of 1889–90. Consult reports of state and city departments of health.

13. Make a statistical study of the influenza epidemic of 1918 for some state, city or town.

14. Prepare a short statistical summary of cancer, — its geographical distribution, its occurrence among different age-groups, its chronology, etc. [Hoffman, Frederick L. The Mortality from Cancer throughout the World. Newark. The Prudential Press, 1915.]

CHAPTER XI

STUDIES OF DEATHS BY AGE PERIODS

Infant mortality. — No part of vital statistics is attracting more attention nowadays than the subject of infant mortality. It is, indeed, a serious problem and worthy of most careful study. It is a complex problem and one difficult to understand. It is a problem which goes beyond itself. Newsholme says that "infant mortality is the most sensitive index of social welfare and of sanitary improvements which we possess." Another says that "infant mortality is to the health officer what the clinical thermometer is to the physician." People who will not take care of their offspring will not take care of themselves.

Some definitions. — The term infant is applied to any child from the day of its birth up to one year of age. A child born dead is not regarded as having been born. It is not included among either births or deaths; it is a still-birth. But if the child is born alive and dies almost immediately it is to be regarded as an infant and both its birth and its death is to be recognized statistically. In the past health officials were not careful to make this distinction and many of the old statistics are for that reason not comparable with present-day records. This should be kept in mind in comparing statistics which extend over long periods of time.

The specific death-rate for infants, that is, for age-group 0–1, is computed in the same way as the specific death-rate for any other age-group, namely, by dividing the annual number of deaths in the group by the mid-year population of

the group, expressed in thousands. By *infant mortality*, as the term is generally understood, is meant something slightly different, namely, the number of infant deaths in a calendar year divided by the number of births during the same year.

Prenatal deaths. — Fœtal deaths which occur before the sixth or seventh month of gestation are known as miscarriages and are not reportable or recognized in ordinary statistical work; those which occur later than this are called still-births and must be reported. The time limit is sometimes stated as twenty-eight weeks, sometimes it is made dependent upon the apparent condition of the fœtus. Still-births, although reportable, should always be kept apart from true births.

The following figures show how in Boston the ratio of still-births to total population and the ratio between still-births and living births have changed during the last twenty years.

TABLE 96

STILL-BIRTHS, BOSTON

Year.	Number per 100 living births.	Number per 1000 inhabitants.	Year.	Number per 100 living births.	Number per 1000 inhabitants.
(1)	(2)	(3)	(1)	(2)	(3)
1891	4.2	1.3	1904	4.0	1.1
1892	4.2	1.2	1905	4.2	1.1
1893	4.1	1.3	1906	3.8	1.1
1894	4.5	1.4	1907	4.0	1.2
1895	3.8	1.2	1908	3.4	1.0
1896	3.9	1.3	1909	4.0	1.1
1897	3.6	1.2	1910	3.0	1.0
1898	3.7	1.1	1911		
1899	3.3	1.0	1912		
1900	3.5	1.0	1913		
1901	3.6	1.0	1914		
1902	3.9	1.1	1915		
1903	4.0	1.1			

The monthly records show no appreciable variation in the rate of still-births during the year. The ratio of still-births to living births is much greater for illegitimate than for legitimate children, especially among mothers less than twenty years of age. There are marked differences in the still-birth rates in different countries.

At Johnstown, Pa., 4.5 per cent of all births were still-births, and 8.7 per cent of all mothers reporting had suffered miscarriages.

The percentages of still-births arranged according to the age of the mothers gave the very high percentage of 11.1 per cent for mothers under 20 years of age, 4 per cent for age group 20–24, 5.1 for 25–29 years, 4.4 for 30–39 years, and 3.3 for ages over 40. The percentage for native mothers was 5.2 per cent, for foreign mothers, 3.8 per cent.

Infant mortality and the specific death-rate for infants. — There are two reasons why the specific death-rate for age group 0–1 year is not used more. The first is the difficulty of finding the actual number of infants alive at the middle of any calendar year. Of course, a census might be taken on July 1, but even that figure would not be very satisfactory for births are not uniformly distributed through the year and the ages of infants are often given incorrectly. It is possible to compute the number alive July 1 from the monthly records of births and deaths, but rarely, if ever, in this country are the data for doing this published. The reports of vital statistics of Hamburg contain each year a table like that shown in Table 97 from which this computation can be made.

Starting in 1911 we see that in January 1853 children were born, of which 260 died the same year; 5, however, died in January, 1912, before reaching their first birthday. Of the children born in February, 1911, 8 died in January, 1912, and 3 in February, 1912. By keeping up this tabulation we

TABLE 97

BIRTHS AND DEATHS OF CHILDREN UNDER ONE YEAR

Hamburg, 1911 and 1912

Year and mo.	Births.	Died in 1911.	Died in 1912 before reaching age of one year.												Total	Died in first year.	Reached the first year alive.		Living Jan., 1913.
			J.	F.	M.	A.	M.	J.	J.	A.	S.	O.	N.	D.			No.	%	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	( )	(20)
1911.																			
Jan.	1,853	260	5	...	...	...	...	...	...	...	...	...	...	...	5	265	1,588	85.70	.....
Feb.	1,700	254	8	3	...	...	...	...	...	...	...	...	...	...	11	265	1,435	84.41	.....
Mar.	1,752	253	4	6	6	...	...	...	...	...	...	...	...	...	16	269	1,483	84.65	.....
April	1,713	250	6	4	12	3	...	...	...	...	...	...	...	...	25	275	1,438	83.95	.....
May	1,777	251	11	3	10	43	...	...	...	...	...	...	...	...	31	282	1,495	84.13	.....
June	1,670	242	11	8	11	7	8	0	...	...	...	...	...	...	45	287	1,383	82.81	.....
July	1,791	255	14	8	9	10	4	13	4	...	...	...	...	...	62	317	1,474	82.30	.....
Aug.	1,760	188	15	10	12	6	9	8	6	4	...	...	...	...	70	258	1,502	85.34	.....
Sept	1,670	124	18	11	8	8	11	6	2	7	2	...	...	...	73	197	1,473	88.20	.....
Oct.	1,668	131	19	18	17	6	14	5	8	4	6	2	...	...	99	230	1,438	86.21	.....
Nov.	1,603	90	27	27	8	16	8	15	9	8	7	5	3	...	133	223	1,380	86.09	.....
Dec.	1,705	58	53	27	25	14	10	7	13	9	3	10	6	2	179	237	1,468	86.10	.....
Sum	20,662.	2356	191	125	118	74	67	54	42	32	18	17	9	2	749	3105	17,557	84.97	.....
1912.																			
Jan.	1,829	.....	78	45	30	19	17	13	12	9	8	4	9	3	247	.....	.....	.....	1,582
Feb.	1,722	.....	...	56	44	20	12	15	15	15	9	4	6	3	199	.....	.....	.....	1,523
Mar.	1,810	.....	...	...	63	29	21	20	15	19	7	6	15	6	201	.....	.....	.....	1,609
April	1,721	.....	...	...	...	57	37	17	20	19	6	7	6	8	177	.....	.....	.....	1,544
May	1,723	.....	...	...	...	...	76	26	20	28	12	12	13	8	195	.....	.....	.....	1,528
June	1,777	.....	...	...	...	...	...	60	41	26	21	19	7	12	186	.....	.....	.....	1,591
July	1,833	.....	...	...	...	...	...	...	68	43	21	19	16	11	178	.....	.....	.....	1,655
Aug.	1,817	.....	...	...	...	...	...	...	...	70	25	30	20	21	166	.....	.....	.....	1,651
Sept.	1,748	.....	...	...	...	...	...	...	...	...	67	27	25	22	141	.....	.....	.....	1,607
Oct.	1,781	.....	...	...	...	...	...	...	...	...	...	71	38	33	142	.....	.....	.....	1,639
Nov.	1,688	.....	...	...	...	...	...	...	...	...	...	...	69	35	104	.....	.....	.....	1,584
Dec.	1,799	.....	...	...	...	...	...	...	...	...	...	...	...	70	70	.....	.....	.....	1,729
Sum	21,248	.....	78	101	137	125	163	151	191	229	176	199	224	232	2006	.....	.....	.....	19,242
Died in year 1912..			269	226	255	199	230	205	233	261	194	216	233	234	2755				

obtain all the deaths in 1912 of babies born in 1911. In the same way we obtain the deaths in 1912 of infants born in 1912. These added to the preceding give us all the infant deaths for 1912, namely, 2755. The number of children living Jan. 1, 1913, was 21,248 – 2006, or 19,242. By starting with July 1, 1910, we could obtain the number of children living July 1, 1912. It requires monthly records extending over two years to get this result, and even after we get it it may not be exact as there may have been errors in the records.

The *infant mortality* is much simpler to compute, but it is not without its errors. Birth reporting is notoriously bad, and there is often doubt as to the stated age of the dying child. Children within a few months of their first birthday are sometimes said to be a year old.

In the long run, the average specific death-rates for age group 0–1 agree fairly well with the infant mortalities, but in any particular place and year they may vary from each other as much as twenty-five or fifty per cent. The infant mortality should be stated in whole numbers as the accuracy of the data does not warrant the use of decimals.

The deaths of infants in one year will include some who were born in the preceding year. In our infant mortality ratios, therefore, we are not dealing wholly with the same infants in the denominator and numerator. Fluctuations in the birth-rate in successive years may influence this ratio.

First-year death-rate. — In the case of Hamburg we see from the table that in the year 1912 there were 21,248 births and 2755 infant deaths. From these figures we may compute the infant mortality ratio in the usual way and obtain 130 per 1000. Yet if we consider the 20,662 births in the twelve months of 1911 and follow them through their first year, we find that 3105 died, that is the ratio was 150 per 1000. In

other words, 850 per 1000, or 85 per cent reached, their first year of life.

In the printed table we find in the last column for 1911 figures which show these percentages by months. It is interesting to notice how this percentage of born children who reach their first year varies with the season. According to the 1911 figures for Hamburg, September is the most favorable month for a birth because 88.2 per cent of the children born that month reached the age of one year; 118 per thousand died. July is the most unfavorable month as only 82.3 per cent reached the first year; 177 per thousand died.

Very few American cities keep their records in such shape that facts like these can be easily secured. In Hamburg they publish both the infant mortality rates and the percentage of infants who die in their first year of life. They also publish the proportionate infant mortality, that is, the per cents which the infant deaths are of the total deaths. It is interesting to compare these figures.

TABLE 98

INFANT DEATHS, HAMBURG

Year.	Proportionate mortality.	Infant mortality (per 1000 births.)	Number of infants per 1000 who died in their first year.
(1)	(2)	(3)	(4)
1908	26.2	156	184
1909	23.7	142	159
1910	24.4	149	160
1911	23.4	158	159
1912	20.8	130	141

This method of studying infant mortality by determining the percentage of first-year deaths was used in the Johnstown, Pa., investigation in 1914. It was here referred to as

the "absolute infant mortality," the conventional method of comparing births and infant deaths for a calendar year being regarded, as indeed it is, as an approximate method, chosen for convenience only. In Johnstown the results were obtained by an intensive study of individual infants. Contrary to the results in Hamburg the "percentage of first-year deaths" was less than the "infant mortality," the figures being 13.4 per cent (134 per 1000) and 165 respectively.

Various methods of stating infant mortality. — It will be seen that infant mortality may be expressed in various ways:

1. Rate of deaths in first year — the true, or "absolute," method.
2. Infant mortality, the calendar ratio between infant deaths and births — the common method.
3. Specific death-rate for age-group 0–1 year — difficult to compute, but useful as hereafter shown.
4. Proportionate mortality — the ratio between infant deaths and total deaths.
5. Infant death-rate per 1000 inhabitants — a ratio rarely used and of little value.

It should be noticed that all of these ratios except the first are calendar ratios, that is, they are based on one year or some other interval of time. The true rate of infant deaths considers the calendar only as to births, the period covered by the deaths being one year from the date of each birth. In the following paragraphs all of these ratios are used in order that the student may learn to discriminate between them.

Chronological reduction in infant mortality. — The infant mortality rates in nearly all civilized countries are falling. In recent years the fall has been more rapid than it was a generation ago. In Sweden we have a very long record, a part of which is given in Table 99. Prior to 1800 the infant

TABLE 99

INFANT AND CHILD MORTALITY IN SWEDEN

1751 to 1900 by 5-Year Periods

Period.	Total death-rate per 1000.	Age-group in years.			
		0–1.[1]	1–3.[2]	3–5.[2]	0–5.[2]
(1)	(2)	(3)	(4)	(5)	(6)
1751–55	26.52	205.75	52.17	27.31	86.07
1756–60	28.25	203.41	49.50	26.26	81.64
1761–65	29.08	221.73	53.94	28.49	90.46
1766–70	26.38	210.41	50.12	27.06	85.14
1771–75	33.07	212.89	66.55	36.15	92.88
1776–80	24.86	192.02	56.21	29.13	83.74
1781–85	27.80	193.98	62.64	36.16	86.44
1786–90	27.61	205.70	48.78	23.04	81.67
1791–95	25.21	192.59	44.63	20.76	77.09
1796–00	25.65	199.53	48.02	23.47	79.55
1801–05	24.35	186.08	41.48	18.70	70.65
1806–10	31.45	211.46	59.09	29.09	87.42
1811–15	27.11	191.76	56.46	20.57	81.54
1816–20	24.63	175.51	45.93	17.96	71.00
1821–25	22.07	158.85	36.24	14.33	61.63
1826–30	25.10	175.76	37.72	17.07	64.53
1831–35	23.05	167.31	33.44	14.44	60.32
1836–40	22.53	166.35	35.47	15.42	60.31
1841–45	20.20	153.77	30.38	14.39	56.18
1846–50	20.95	152.56	33.39	16.48	57.34
1851–55	21.65	148.89	35.32	18.79	58.83
1856–60	21.73	143.47	39.12	23.87	61.96
1861–65	19.76	136.17	40.95	21.78	58.48
1866–70	20.54	141.93	38.78	19.59	56.14
1871–75	18.28	133.57	29.78	14.64	51.47
1876–80	18.26	126.28	36.26	19.80	53.01
1881–85	17.53	116.08	31.82	17.09	47.18
1886–90	16.37	105.00	25.88	13.41	40.06
1891–95	16.61	102.76	23.97	12.65	38.21
1896–00	16.12	100.50	21.78	9.72	35.65

[1] Per 1000 births during the given period, *i.e.*, " infant mortality."

[2] Per 1000 of population at middle of period, *i.e.*, specific death-rates.

mortality was upwards of 200, but by 1900 it was only about half as much. In Stockholm in 1912 it was only 82.

In Massachusetts [1] the rate of infant mortality has varied as follows:

TABLE 100

RATE OF DEATHS DURING FIRST YEAR

Massachusetts

Year.	Per 1000.	Year.	Per 1000.	Year.	Per 1000.	Year.	Per 1000.	Year.	Per 1000.
(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
1851	133	1864	154	1877	152	1890	167	1903	138
1852	126	1865	147	1878	150	1891	162	1904	133
1853	135	1866	138	1879	145	1892	162	1905	140
1854	131	1867	136	1880	163	1893	164	1906	138
1855	135	1868	140	1881	163	1894	163	1907	133
1856	123	1869	149	1882	163	1895	156	1908	134
1857	118	1870	162	1883	159	1896	158	1909	127
1858	122	1871	151	1884	159	1897	147	1910	133
1859	130	1872	194	1885	157	1898	151	1911	119
1860	134	1873	178	1886	155	1899	150	1912	117
1861	146	1874	164	1887	160	1900	157	1913	110
1862	131	1875	175	1888	162	1901	138	1914	106
1863	150	1876	158	1889	160	1902	140	1915	102

The report speaks of these as "first-year deaths" but does not state how they were computed. Apparently the figures refer to infant mortality computed in the conventional way. The figures show a substantial decrease only during the last ten years.

The following figures show the decrease in infant mortality computed in the conventional way between 1908 and 1915 for Massachusetts, Boston and the remainder of the state outside of Boston.

[1] Mass. Registration Report, 1915, p. 153.

TABLE 101

INFANT MORTALITY: MASSACHUSETTS

Year.	Massachusetts.	Boston.	Remainder of state.
(1)	(2)	(3)	(4)
1908	134	149	129
1909	127	121	129
1910	133	127	134
1911	119	126	118
1912	117	117	116
1913	110	110	110
1914	106	105	107
1915	102	104	101

It would be possible to print page after page of such figures as these taken from the records of our American cities.

In Boston the rate of infant deaths per 1000 of total population has decreased as follows:

TABLE 102

INFANT MORTALITY: BOSTON

Year.	Rate per 1000 of population.	Year.	Rate per 1000 of population.
(1)	(2)	(1)	(2)
1875	6.6	1895	5.1
1880	5.6	1900	4.3
1885	5.5	1905	3.7
1890	5.1	1910	3.3

This ratio is one which depends upon the number of children born as well as upon their rate of death, and involves the varying composition of the population as to age, marriage, nationality, and so on.

Reasons for the decreasing infant mortality. — As most of the current discussions of infant deaths are based on the

calendar ratio between *reported* deaths of infants and *reported* births, it is well to remember that a falling ratio may result from an increase in the denominator as well as from a decrease in the numerator of the fraction. We have already learned that the birth registration is increasing in accuracy, that a larger percentage of births are reported now than formerly. This fact alone will account for a part of the drop in infant mortality; in some places it may account for nearly all of it. In comparing the infant mortality rates in different places this difference in the relative accuracy of reports of births and deaths must not be overlooked. To understand just what is being accomplished by present-day activities in infant welfare it is necessary to dig deeper into the subject and to analyze the statistics of infant births and deaths.

Infant mortality in different places. — If we examine the statistics for different countries we shall discover great differences in infant mortality. We have space here for only a few figures.

TABLE 103

INFANT MORTALITY IN A FEW FOREIGN CITIES

Cities.	1881–1885.	1886–1890.	1891–1895.	1896–1900.	1901–1905.	1906–1910.	1911.	1912.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
London	150	153	156	162	139	114	129	91
Edinburgh	127	136	140	144	131	119	118	113
Dublin (registration area)	176	175	169	175	158	146	156	140
Sydney	173	155	138	130	107	85	71	76
Melbourne	171	173	136	129	113	94	78	90
Montreal	...	246	237	258	271	261	242	...
Toronto	...	...	200	231	205	166	114	...
Paris	162	152	135	119	110	106	118	103
Amsterdam	203	199	168	146	122	90	91	64
Rotterdam	209	207	191	167	144	105	103	79
Stockholm	208	182	170	169	136	103	77	82
Christiania	156	168	158	152	119	96	116	107
St. Petersburg	301	243	242	251	246	256	231	249
Moscow	340	320	316	286	262	313	321	333
Berlin	279	264	242	218	202	164	173	142
Hamburg	222	260	226	182	174	150	158	130
Vienna	196	196	219	195	178	172	166	149
Prague	218	200	194	170	163	156	186	139
Budapest	244	237	199	174	149	151	161	141
Trieste	212	238	233	218	201	213	215	184
Milan	156	161	158	147	146	129	?	102
Buenos Aires	185	209	151	120	93	100	105	96

If we take the different cities of the United States we shall find ranges of infant mortality almost as great as in the cities of different countries. In 1915 the New York Milk Committee made an extensive study of the infant mortality rates of 144 United States cities. The minimum, median and maximum rates for the year 1915 were as follows:

TABLE 104

INFANT MORTALITIES IN UNITED STATES CITIES

Population group.	Number of cities.	Infant mortality.		
		Minimum.	Median.	Maximum.
(1)	(2)	(3)	(4)	(5)
500,000–	10	82	104	120
200,000–500,000	20	53	84	133
100,000–200,000	16	47	100	182
50,000–100,000	31	62	98	193
30,000– 50,000	31	31	86	185
20,000– 30,000	21	37	98	167

These figures indicate that there was little difference in the median infant mortality between the large and the small cities, but that in the larger cities there was a greater uniformity in the figures. The very low rates as well as the very high rates were found in relatively small cities. The inaccuracies of the birth registration may account for many of these differences.

We may go even further and take the different wards of a single city, and find these same differences. In the twenty-five wards of Boston in 1910 the infant mortalities ranged from 75 to 210, the median being 117 and the average 122. In eleven districts of Johnstown in 1911 the "absolute infant mortality" varied from 50 to 271, the figure for the entire

city being 134. And in the same way we could find differences block by block. In any intensive study it is of fundamental importance to find out the geographical location of infant deaths.

Deaths of infants at different ages. — A year is a long time in the life of an infant. One can learn no more from a study of the infant mortality, when all ages up to one year are considered together, than from a study of the general death-rate of a community where all deaths from zero to a hundred years of age are considered together. It is necessary to study the infant death-rate by months, weeks and even days.

The need of such study is obvious. During early life many of the deaths are from troubles incident to birth; later the question of feeding, and especially of the effect of artificial food becomes important. Some of the welfare activities are directed towards one end, some towards another. The establishment of milk stations, for example, might affect the death of weaned babies, but have little influence on babies less than a month old. There are many things which will occur to the reader which will show the importance of this specific information.

Let us first consider some of these subdivisions. In doing so, we may use several of the methods with which we have already become familiar.

In Hamburg, in 1912, the percentage age distribution of infant deaths was as follows:

TABLE 105

INFANT DEATHS AT DIFFERENT AGES: HAMBURG

Age-group, months.	Per cent of infant deaths in group.	Age, months.	Per cent of infants who died at less than stated age.
(1)	(2)	(3)	(4)
0+	38.5	1	38.5
1+	12.3	2	50.8
2+	9.8	3	60.6
3+	8.4	4	69.0
4+	6.1	5	75.1
5+	4.7	6	79.8
6+	4.3	7	84.1
7+	4.4	8	88.5
8+	3.0	9	91.5
9+	2.9	10	94.4
10+	2.6	11	97.0
11+	3.0	12	100.0

An irregular grouping is more common, because of the greater importance of the subdivisions at the very early ages. Thus for Boston, in 1912, we find the following figures:

TABLE 106

AGE DISTRIBUTION OF INFANT DEATHS: BOSTON, 1912

Age-group.	Per cent of infant deaths.		Age.	Per cent of infants who died at less than stated age.	
	Male.	Female.		Male.	Female.
(1)	(2)	(3)	(4)	(5)	(6)
0– days	15.4	11.2	1 day	15.4	11.2
1+ days	4.0	4.5	2 days	19.4	15.7
2+ days	3.9	2.4	3 days	23.3	18.1
3+ days	6.8	7.7	1 week	30.1	25.8
1+ weeks	3.7	4.9	2 weeks	33.8	30.7
2+ weeks	5.4	4.4	3 weeks	39.2	35.1
3+ weeks	2.8	2.5	1 month	42.0	37.6
1+ months	10.9	9.0	2 months	52.9	46.6
2+ months	7.0	9.5	3 months	59.9	56.1
3+ months	18.1	19.2	6 months	78.0	75.3
6+ months	13.4	13.6	9 months	91.4	88.9
9 to 1 year	8.6	11.1	1 year	100.0	100.0
0 to 1 year	100.0	100.0			

It will be noticed that on the basis of the summation results of the last column the figures may be readily compared with those for Hamburg where the figures are given by regular monthly groups.

The important fact is that the death-rate of infants is much higher during the first weeks and days of life than it is after the first six months. The apparent increase from the eleventh to the twelfth month, often found, is very likely due to inaccuracies in stating the age at one year — the error of round numbers.

Specific death-rates of infants at different ages. — A better appreciation of the early infant mortality can be obtained by studying the specific death-rates of infants at different ages. In Glover's United States Life Tables we find the following figures which give the *monthly* specific death-rates.

TABLE 107

SPECIFIC DEATH–RATES OF INFANTS BY MONTHS, 1910

Original Registration States as Constituted in 1900

Age interval, months.	Number dying in age interval among 1000 alive at beginning of age interval.	
	Males.	Females.
(1)	(2)	(3)
0 – 1	48.94	38.33
1 +	13.17	10.44
2 +	10.91	9.01
3 +	9.29	7.82
4 +	8.21	6.96
5 +	7.41	6.36
6 +	6.76	5.90
7 +	6.25	5.47
8 +	5.81	5.09
9 +	5.40	4.74
10 +	5.03	4.39
11 +	4.70	4.04
0 – 1 yr.	124.95	103.77

Expectation of life at different ages. — The expectation of life is given for infants as follows:

TABLE 108

EXPECTATION OF LIFE[1]

Original Registration States as Constituted in 1900

Age interval, months.	Average length of life remaining to each one alive at beginning of age interval. Years.	
	Males.	Females.
(1)	(2)	(3)
0 – 1	49.86	53.24
1 +	52.35	55.28
2 +	52.96	55.78
3 +	53.46	56.20
4 +	53.88	56.56
5 +	54.24	56.87
6 +	54.56	57.15
7 +	54.85	57.41
8 +	55.11	57.64
9 +	55.35	57.85
10 +	55.57	58.05
11 +	55.76	58.22

[1] Based upon deaths in 1909, 1910 and 1911.

The expectation of life of a male child at birth is about the same as that of a 11-year old boy. The expectation of life increases from birth to about the third year when it reaches its maximum.

Infant mortality by age periods. — Another way of showing the infant mortality by age periods is to find the ratios between the numbers of deaths in each period and the total number of births. These results may also be expressed cumulatively. Thus for Boston, in 1910, we have:

TABLE 109

INFANT MORTALITY, BOSTON, 1910

Age period.	Deaths per 1000 births.	Age period.	Deaths per 1000 births.
(1)	(2)	(1)	(2)
0–2 days	20	0–2 days	20
2 days–1 week	12	0–1 week	32
1 week–1 month	16	0–1 month	48
1–3 months	21	0–3 months	69
3–6 months	21	0–6 months	90
6–9 months	17	0–9 months	107
9–12 months	15	0–1 year	122

Causes of infant deaths. — In Boston, in 1910, the principal causes of infant deaths were as follows:

TABLE 110

	Number of deaths.	
	Male.	Female.
(1)	(2)	(3)
I. General diseases, total	**113**	**100**
Measles	16	12
Diptheria and croup	18	8
Whooping cough	8	16
Erysipelas	10	8
Tubercular meningitis	15	14
Syphilis	11	12
II. Diseases of the nervous system, total	**47**	**47**
Meningitis	21	20
Convulsions	18	16
III. Diseases of the circulatory system, total	**4**	**4**
IV. Diseases of the respiratory system	**209**	**161**
Acute bronchitis	37	30
Broncho-pneumonia	88	71
Pneumonia	79	56
V. Diseases of the digestive system, totals	**355**	**270**
Diseases of stomach, except cancer	25	12
Diarrhea and enteritis	320	245
VI. Diseases of genito-urinary system	**2**	**6**
VIII. Diseases of skin and cellular tissue	**8**	**2**
IX. Diseases of bones, etc	**2**	**5**
X. Malformations	**69**	**51**
XI. Early infancy	**392**	**319**
Congenital debility	302	238
XIII. External causes	**9**	**7**
XIV. Ill-defined diseases	**35**	**32**
Total from all causes	**1245**	**1004**

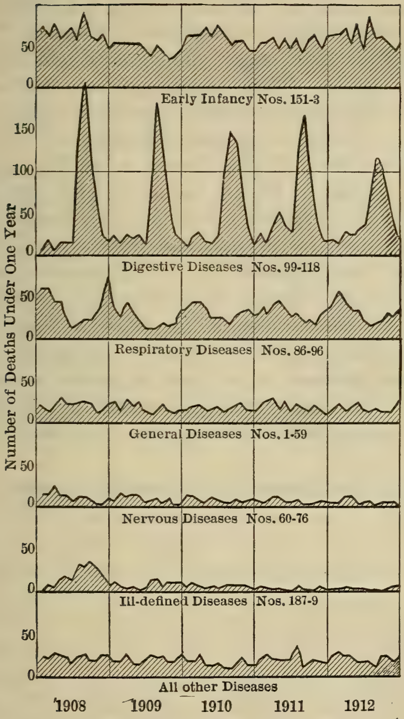


FIG. 54. — Infant Mortality by Months, Classified According to Cause: Boston, Mass.

Among both male and female infants 37 per cent of the deaths were from malformations and diseases of early infancy; about 27 per cent were from digestive diseases; about 17 per cent were from respiratory diseases. Together the deaths from these causes amounted to four-fifths of all the infant deaths. These percentages are not constant. There is an important seasonal variation; there are also differences according to age and nationality.

In 1912, Dr. Wm. H. Davis made an excellent analysis of the infant deaths in Boston for a five-year period. Fig. 54, drawn from figures in his report, shows how the deaths from digestive diseases have fallen during the summer season, but remained almost unchanged during the winter; how the deaths from respiratory diseases are higher in the winter than in the summer; and how the deaths from diseases of early infancy, the general diseases and nervous diseases do not have a marked seasonal distribution. The diagram also shows how the diseases from ill-defined causes have decreased, due, it is said, to better diagnosis.

In Boston the diseases were classified by cause and age as follows:

TABLE 111

CAUSES OF INFANT DEATHS: BOSTON, 1910

Cause	Number of deaths.							
	0–1 Days.	2–6 Days.	1–30 Days.	1–2 Months.	3–5 Months.	6–8 Months.	9–11 Months.	Total Infant.
(1)	(2)	(3)	(4	(5)	(6)	(7)	(8)	(9)
I. General disease	0	2	21	37	43	58	52	213
II. Nervous system	1	8	6	17	17	24	21	94
III. Circulatory system	0	1	2	4	1	0	0	8
IV. Respiratory system	1	9	54	68	80	89	69	370
V. Digestive system	0	10	41	153	201	112	108	625
VI. Genito-urinary system	0	0	0	1	2	2	3	8
VIII. Skin and tissue	0	0	3	0	3	2	2	10
IX. Bones	0	0	0	0	2	3	2	7
X. Malformations	41	32	24	11	7	3	2	120
XI. Early infancy	305	147	135	99	12	8	5	711
XIII. External causes	5	0	2	3	4	1	1	16
XIV. Ill-defined causes	0	4	3	7	33	15	5	67
All causes	353	213	292	400	405	317	270	2249

As would be expected from the definition the largest numbers of deaths from causes incident to early infancy occur among early infants. This is true also of malformations. The intestinal diseases reach their maximum effect between the third and fifth month, the respiratory diseases and the general diseases, which are chiefly communicable, a little later — say between the sixth and eighth months.

In Johnstown important differences were noted between the causes of death among infants of native and foreign mothers. Thus, during the first year of life the following absolute infant mortalities, with subdivisions by cause were found.

TABLE 112

CAUSES OF INFANT DEATHS: JOHNSTOWN

(1)	Native mothers. (2)	Foreign mothers. (3)
All causes	104	171
Diarrhœa and enteritis	21	54
Respiratory diseases	23	48
Premature births	14	20
Congenital debility or malformations	6	21
Injuries at birth	7	2
Other cause, or not reported	33	26

In Boston the following figures were given for 1910 for deaths of infants born to native and foreign mothers:

TABLE 113

CAUSES OF INFANT DEATHS: BOSTON

	Rates per 1000 births				
	Native mothers.	Canadian mothers.	Irish mothers.	Italian mothers.	Russian mothers.
(1)	(2)	(3)	(4)	(5)	(6)
Congenital debility and malformations	50	31	49	24	20
Diarrhœa and enteritis	34	37	43	22	19
Pneumonia and broncho-pneumonia	15	18	12	29	16
Diseases of early infancy	10	13	16	4	7
Tuberculosis	2	4	1	2	2
Measles, scarlet fever, whooping cough and diphtheria	3	3	3	8	4

The Johnstown studies. — In 1915 the Children's Bureau of the U. S. Department of Labor,[1] published an important intensive study of the Infant Mortality of Johnstown, an industrial city of Pennsylvania. Miss Julia C. Lathrop is the Chief of this bureau. The field work was in charge of Miss Emma Duke. This was essentially a sociological study. Only a few of the simple correlations can here be presented. The report is one which the student may profitably read in full.

TABLE 114

INFANT MORTALITY AND TYPE OF HOME

Housing condition.	Infant mortality.
(1)	(2)
Clean, dry	105
" damp	127
Moderately clean, dry	171
" " damp	158
Dirty, dry	162
" damp	204
Water supply in house	118
Water supply outside	198
Water closet	108
Yard privy	159

[1] Infant Mortality Series No. 3.

TABLE 115

INFANT MORTALITY AND SLEEPING ROOMS

	Infant mortality.
(1)	(2)
Number of others sleeping in same room with baby:	
2 or less	67
3 to 5	98
Over 5	123
Baby sleeping in separate bed:	
Yes	56
No	109

TABLE 116

INFANT MORTALITY AND VENTILATION

Ventilation of baby's room.	Infant mortality.
(1)	(2)
Good	28
Fair	92
Poor	169

TABLE 117

INFANT MORTALITY AND ATTENDANT AT BIRTH

	Infant mortality.
(1)	(2)
Physician	100
Midwife	180

TABLE 118

INFANT MORTALITY AND EDUCATION OF FOREIGN MOTHERS

	Infant mortality.
(1)	(2)
Literate	148
Illiterate	214
Speak English	146
Do not speak English	187

TABLE 119

INFANT MORTALITY AND AGE OF MOTHER

Age of mothers.	Infant mortality.
(1)	(2)
Under 20	137
20–24	121
25–29	143
30–39	136
40 and over	149

The study of feeding was made by months. The following figures show the rate of mortality per 1000 babies alive at the specified time.

TABLE 120

INFANT MORTALITY AND FEEDING

Age.	Specific infant mortality (absolute)		
	Breast feeding only.	Mixed feeding.	Artificial feeding only.
(1)	(2)	(3)	(4)
Second month	72	78	237
Third "	54	92	217
Fourth "	47	57	166
Fifth "	38	40	127
Sixth "	26	32	92
Seventh "	29	22	72
Eighth "	26	20	53
Ninth "	18	16	25
Tenth "	14	11	11

In the early ages the difference between deaths of breast fed infants and those artificially fed is very great, but the difference becomes less as the baby grows older.

TABLE 121

INFANT MORTALITY AND HOUSEHOLD DUTIES

Household duty.	Infant mortality.
(1)	(2)
Cessation of duties before confinement:	
None or less than one month	137
One or more months	113
Time of resuming all household duties after confinement:	
8 days or less	169
9 to 13 days	165
14 days or more	117

TABLE 122

INFANT MORTALITY AND EARNINGS OF FATHER

Annual earnings of husband.	Infant mortality.	
	Native wives.	Foreign wives.
(1)	(2)	(3)
Under $521	.....	251
$521 to $624	146	162
$625 to $779	70	130
$780 to $899	131	167
$900 to $1199	76	152
$1200 or more	.....	.....
Ample	78	108

The report also contains statistics relating to reproductive histories of the mothers studied during the investigation.

Other studies of the Children's Bureau. — Besides the Johnstown studies, here emphasized because they were first made, the Children's Bureau has made at this writing (1918), intensive studies in Manchester, N. H., Saginaw, Mich., Waterbury, Conn., Brockton and New Bedford, Mass., Akron, Ohio, and Baltimore. A brief account of these most important intensive investigations, based on a first-hand collection of the facts may be found in the Quarterly Publication of the American Statistical Association.[1]

Two tables from this report are of interest:

[1] Robert M. Woodbury, Infant Mortality Studies of the Children's Bureau, June 1918, pp. 30–53.

TABLE 123

INFANT MORTALITY AND FATHER'S EARNINGS, BALTIMORE

Earnings of father per year.	True infant mortality rate.	Earnings of father per year.	True infant mortality rate.
(1)	(2)	(1)	(2)
No earnings	207.7	$1050–1249	66.6
Under $450	156.7	1250–1449	74.0
$450–$549	118.0	1450–1849	86.3
550– 649	108.8	1850 and more	37.2
650– 849	96.06	Not reported	140.2
850–1049	71.5	All classes	103.5

TABLE 124

INFANT MORTALITY AND ORDER OF BIRTH—MOTHERS OF ALL AGES

Number of birth in order.	True infant mortality.	Number of birth in order.	True infant mortality.
(1)	(2)	(1)	(2)
1	115.8	7	128.2
2	102.7	8	162.6
3	111.5	9	142.1
4	127.0	10	181.1
5	129.3	11	146.8
6	132.2		

Although in general the average infant mortality is less for the second child than for the first or subsequent children, this is a matter which varies somewhat with the age of the mother. For mothers under twenty the mortality is lowest for first children; for mothers aged 30–34 years it is lowest for the third children; and for mothers aged 35–39 it is lowest for fourth children. Perhaps, if nationality were considered, other differences would be noticed.

Infant mortality problems. — There are many practical problems relating to infant mortality which must be studied with the aid of statistics. The object of the tables here given is to show the complexity of the problem and the futility of depending alone upon the current approximate method of stating infant mortality. Extensive compilations of data for various places and for different years make easy reading and give one a superficial knowledge of the subject, but they do not help us very much in solving real problems. It is the intensive studies which count. What kind of welfare work deserves the largest appropriations? The answer depends upon where the babies are dying, at what age they are dying, under what social conditions, under what remediable conditions, and so on. Are the milk stations of our large cities a paying life-saving agency? The answer cannot be told by comparing the conventional infant mortality rates; perhaps the reduction of infant mortality may be among the earliest weeks of life, an age at which artificial feeding is less common. What relation is there between density of population and infant mortality? The answer cannot be found without splitting up the infant mortality into its constituent parts.

The lesson is one which the author wishes to teach in every chapter of this book, namely, that the vital statistician must train himself to analyze his statistics; to be specific; to think first what kind of facts he needs in order to answer a specific question and then go after them, remembering that a small number of well-directed statistics are worth more than vast numbers of general statistics, piled together without regard to internal differences which may make them worthless.

Maternal mortality. — Closely associated with infant mortality we have the problem of maternal mortality. Since the long-ago studies of Dr. Oliver Wendell Holmes, but especially since the rise of bacteriology, there has been

a very great decrease in death-rates from child-bed fever, but even within very recent years we can see an added improvement, which can be attributed to the general attention being given to pre-natal care, to laws in regard to mid-wives and similar causes. The following condensed figures for New York city[1] illustrate this decrease.

TABLE 125

MATERNAL MORTALITY-RATE, CITY OF NEW YORK

Quinquennial period.	Rate per 100,000 females (age 15–45).	
	Puerperal sepsis.	Other deaths.
(1)	(2)	(3)
1898–1902	25.9	40.5
1903–1907	26.1	41.3
1908–1912	18.3	35.7
1913–1917	15.3	29.8

These figures might more properly have been based on married women within the given ages, or upon births and still-births taken together instead of on all females of child-bearing age, but the chronological differences are so great as to leave no room for doubt as to the main facts.

Childhood mortality. — The period of life between the ages of one and five years represents a peculiar environment which may be described by the words home and play. In this period the physiological influence of the mother on the child becomes less, but her intelligence, her social and economic condition, the general environment of the house and the neighborhood become greater. During these four years the specific death-rate of children decreases greatly and the diseases to which they are subject change in character.

[1] Weekly Bulletin, Dept. of Health, March, 1918.

TABLE 126

SPECIFIC DEATH-RATES OF CHILDREN[1]

U. S. Registration Area, 1910–1915

Age.	Rate per 1000	
	Male.	Female.
(1)	(2)	(3)
0 – 1 year	125.8	101.1
1 +	27.3	25.0
2 +	11.0	10.1
3 +	6.9	6.3
4 +	5.1	4.7
0–5 years	36.0	30.0
5–9	3.3	3.0
10–14	2.3	2.1

[1] From Dr. Dublin's paper.

The diseases which occur during childhood are especially amenable to preventive measures, a fact which makes their study one of especial importance from the standpoint of life saving.

Diseases of early childhood. — Dr. Louis I. Dublin, Statistician of the Metropolitan Life Insurance Company has discussed these diseases in an article on the Mortality of Childhood,[1] from which the figures for proportionate mortality given in Table 127 are taken.

This table gives only those diseases for which the proportionate mortality was more than 3 per cent of all deaths. One rather unexpected cause of death looms large in this table — namely, burns. In the second year of life the proportionate mortality was 1.7 per cent, the next year 4.3 per cent, the next 5.9 per cent, the next 5.7 per cent. Dr.

[1] Quarterly Publications, Am. Statistical Assoc., March, 1918, p. 921.

TABLE 127

PROPORTIONATE MORTALITY, AGES 1 TO 5

U. S. Registration Area 1910–15

Age 0–1.	Per cent.	Age 1–2.	Per cent.	Age 2–3.	Per cent.	Age 3–4	Per cent.	Age 4–5	Per cent.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Congenital debility	25.9	Diarrhœa and enteritis	27.3	Diarrhœa and enteritis	13.2	Diphtheria	17.0	Diphtheria	18.8
Diarrhœa and enteritis	24.4	Broncho-pneumonia	14.4	Diphtheria	12.2	Broncho-pneumonia	7.8	Scarlet fever	7.6
Broncho-pneumonia	8.1	Pneumonia	9.5	Broncho-pneumonia	11.1	Pneumonia	7.6	Pneumonia	6.8
Diseases of early infancy	6.1	Diphtheria	5.8	Pneumonia	8.9	Diarrhœa and enteritis	7.3	Broncho-pneumonia	6.0
Malformations	5.6	Measles	5.9	Measles	5.5	Scarlet fever	7.0	Burns	5.7
		Whooping cough	4.6	Scarlet fever	4.7	Burns	5.9	Diarrhœa and enteritis	5.0
		Tuberculous meningitis	3.1	Burns	4.3	Measles	4.4	Tuberculous meningitis	3.8
				Whooping cough	4.0	Tuberculous meningitis	4.0	Measles	3.5
				Tuberculous meningitis	3.9	Whooping cough	3.2		

Dublin in the paper referred to gives the specific death-rates as well as the proportionate mortalities. The figures for burns are for the second year 44.1 per 100,000, for the third, 44.8, for the fourth, 39.4, for the fifth, 28.1. The increasing importance of such communicable diseases as diphtheria, whooping cough, measles and the like during this period shows the increasing influence of environment and association of children with each other.

Proportionate mortalities during school age. — During the ages from 5 to 15 when children are at school we have what is perhaps the maximum opportunity for contact infection. During these ages, therefore, we may expect to see communicable diseases coming to the front in our proportionate mortalities. But we also find weaknesses in the human mechanism making themselves felt. Tuberculosis and typhoid fever also begin to loom up as great menaces.

TABLE 128

PROPORTIONATE MORTALITY

U. S. Registration Area, 1910–15

Ages 5–9.	Per cent.	Ages 10–14.	Per cent.
(1)	(2)	(3)	(4)
Diphtheria and croup	15.8	Tuberculosis of lungs	10.2
Scarlet fever	7.1	Organic diseases of heart	8.6
Pneumonia	5.9	Typhoid fever	6.4
Organic diseases of heart	4.4	Appendicitis	6.3
Vehicular accidents	4.4	Diphtheria	5.6
Typhoid fever	3.7	Pneumonia	5.3
Broncho-pneumonia	3.5	Drowning	4.4
Tuberculosis of lungs	3.5	Vehicular accidents	4.4
Appendicitis	3.4	Scarlet fever	3.0
Tuberculous meningitis	3.4	Acute-articular rheumatism	3.0
Burns	2.7		
Drowning	2.6		
Measles	2.5		

Proportionate mortalities at higher ages. — The following statistics show the proportionate mortalities for age-groups 30–34, 50–54 and 70–74 years.

TABLE 129

PROPORTIONATE MORTALITY

U. S. Registration Area, 1914, Males

Age 30–34 years.	Per cent.	Age 50–54 years.	Per cent.	Age 70–74 years.	Per cent.
(1)	(2)	(3)	(4)	(5)	(6)
Tuberculosis	32.0	Tuberculosis	13.4	Organic diseases of heart	21.6
Accidents	16.1	Organic diseases of heart	11.7	Bright's disease	13.4
Pneumonia	6.9	Bright's disease	11.0	Apoplexy	12.8
Organic diseases of heart	5.4	Cancer	8.1	Cancer	8.4
Angina pectoris	5.2	Accidents	8.0	Pneumonia	4.9
Suicide	4.2	Pneumonia	7.7	Diseases of arteries	4.5
Bright's disease	4.1	Apoplexy	6.8	Accidents	3.2
Typhoid fever	2.8	Suicide	3.2	Tuberculosis	2.8
Homicide	2.8	Cirrhosis of liver	2.9	Old age	2.0
Appendicitis	1.9	Diabetes	1.7	Broncho-pneumonia	1.9
Cancer	1.6	Paralysis	1.5	Diseases of prostate	1.7
		Acute endocarditis	1.4	Paralysis	1.6
		Pleurisy	1.4	Cirrhosis of liver	1.5
		Alcoholism	1.2	Diabetes	1.5
				Angina pectoris	1.4
				Influenza	0.9

Tuberculosis stands at the head of the list until age 70. Organic diseases of the heart increase with age. Accidents diminish. Bright's disease increases. Suicide decreases. Cancer increases, and so on.

Of course in a complete study all of these diseases at different ages should be studied by the use of specific rates as well as proportionate mortality. Studies by sex, by season, by nationality, and so on, should also be made.

Average age of persons living. — The average age of a community is, of course, the weighted average of the differ-

ent age-groups. It is the sum of the ages of all the people divided by the total population.

In 1880 the average age of the aggregate population of the U. S. registration area was 24.6 years, in 1890 it was 25.6, in 1900, 26.3 years. There has apparently been an increase although the figures do not stand for exactly the same areas. But this result might be due to a lessening of the birth-rate, to an increase in infant mortality, to an influx of immigrants of middle age or to a reduced death-rate among the aged. That the native birth-rate has been declining is true, that immigrants of middle age have been entering the country is also true. These would tend to increase the average age. But the infant mortality has been decreasing, not increasing, and the mortality in the higher age-groups has rather increased than diminished. These factors would tend to decrease the average age. Evidently the problem is so complicated that the average age of the living cannot be fairly taken as an index of hygienic conditions.

Median age of persons living. — Instead of finding the average age of the living the median might be used, but the objections to the average age of the living would apply also to the median, although the magnitude of their influence would be somewhat different. The median age of the population of the United States has greatly increased during the last century as the following figures show:

TABLE 130

MEDIAN AGE OF POPULATION: UNITED STATES

Year.	Median age.
1800	16.0
1810	16.0
1820	16.5
1830	17.2
1840	17.9
1850	19.1
1860	19.7
1870	20.4
1880	21.3
1890	21.9
1900	23.4
1910	24.4

Average age at death. — Nor does the average age at death afford a fair index of the healthfulness and physical welfare of a community. The reasons are similar to those just mentioned. A high average age at death may mean simply that the birth-rate is low.

There has been, in recent years, a general rise in the average age at death. In Rhode Island, for example, the increase has been as follows:

TABLE 131

AVERAGE AGE AT DEATH: RHODE ISLAND

Period.	Average age at death.	Period.	Average age at death.
(1)	(2)	(3)	(4)
1861–65	29.32	1881–85	33.99
1866–70	32.42	1886–90	33.42
1871–75	30.16	1891–95	33.96
1876–80	31.21	1896–00	34.53

In 1900 the average age at death in the registration states of the U. S. was 36.8 years. For the cities it was 32.4; for the rural districts 44.7 years. In Mass., in 1910, the average age at death was 39.51. In 1913 the average age at death for the U. S. Registration Area was 39.2 years for males, 40.6 years for females, and 39.8 years for the entire population.

In a general way, however, the prolongation of life may be regarded as an index of human progress, as Professor W. F. Willcox has pointed out.

EXERCISES AND QUESTIONS

1. Compare the infant mortalities for certain assigned large cities and rural districts.

2. Compare the infant mortalities for California cities with those of eastern cities.

3. Compute the seasonal variations of infant mortality for California cities.

4. What is the average infant mortality in New South Wales? Why is it so low?

5. Do the statistics of infant mortality justify the continuance of the milk stations in New York City?

6. In what direction will efforts to reduce infant mortality yield the most profitable results?

7. Is poverty, ignorance, race or climate the greatest factor in causing high infant mortalities?

8. Make a statistical study of some cause of death, to be assigned by the instructor, according to age periods.

CHAPTER XII

PROBABILITY

In the second chapter it was shown that the average, or mean, of a number of figures gave a very inadequate idea of the figures themselves; that two sets of figures may have the same average yet differ among themselves in a striking manner. It is often important to find out what these differences, or *variations*, are. We have seen that one way to do this is to arrange the items in array, that is, in order of magnitude and find the median, the mode, the quartiles and so on, but even this is not enough; it is necessary, if possible, to find some mathematical relation between the variations.

Natural frequency. — It is a curious and important fact that if we measure natural objects, such as the lengths of the leaves on a tree, or the heights of a regiment of men, or the lengths and breadths of nuts, to use illustrations studied by the Eldertons in their Primer of Statistics, we shall find that most of the observations will be very close to the mean of all, that a few will differ from it considerably and that a very small number will differ from it very greatly. In a thousand observations a certain number are almost sure to differ from the mean by a definite amount, and a certain other number are almost sure to differ from the mean by twice that amount. In fact these relations are so regular as to amount to what may be called a law of nature, a sort of natural frequency. In these variations we shall find some observations larger than the mean and some smaller. Natural frequency can best be understood by an example.

In a certain army the results of measurement of the heights of 18,780 soldiers were as follows:

TABLE 131

HEIGHTS OF SOLDIERS

Height in inches.	Number of soldiers.	Per cent of soldiers.
(1)	(2)	(3)
60 +	197	1.05
61 +	317	1.69
62 +	692	3.69
63 +	1,289	6.86
64 +	1,961	10.44
65 +	2,613	13.91
66 +	2,974	15.84
67 +	3,017	16.07
68 +	2,287	12.18
69 +	1,599	8.52
70 +	878	4.67
71 +	520	2.77
72 +	262	1.39
73 +	174	0.92
Total............	18,780	100.00

It will be seen that the *mode*, the most commonly observed height, was in height-group 67 +, *i.e.*, 5 feet 7 inches and 5 feet 8 inches. The mean was 67.24 inches. If we should attempt to stand these 18,780 in array we should have an impossible task. We might try it, however, and obtain something like this:

There are 18,780 soldiers in all. The middle one would be number 9390, or between this and 9391. By counting up from the left we find that the median is just a little below 67 inches. There are, of course, differences in height in each group and with care we could get the median exactly. By taking a weighted average, as described in the second chapter, we could get the mean. But just now we are interested in

the variations. We can plot the number of soldiers by height groups, as in Fig. 55. This will give us a characteristic curve highest in the middle and sloping downwards gently

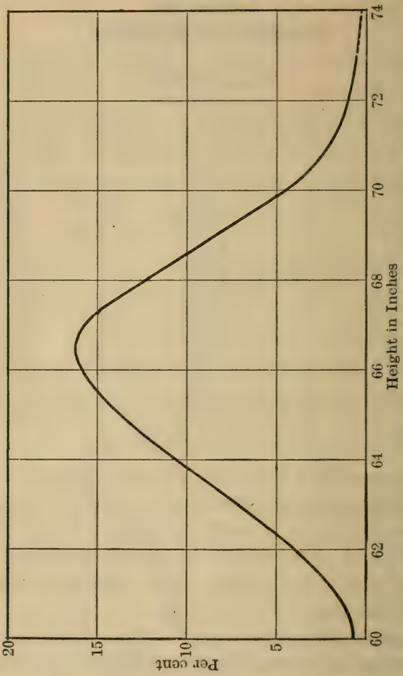


FIG. 55. — Percentage Distribution of Soldiers According to Height.

towards either end. This is called a *frequency curve.* It should be noticed that whereas in the array the height was indicated by the vertical scale it is in this diagram indicated by the horizontal scale.

Coin tossing. — Ten coins were tossed into the air by the students in one of my classes an aggregate of 1250 times, records being kept of the number of heads which came up. The results were as follows:

TABLE 132

RESULTS OF COIN TOSSING

Number of heads up at once.	Number of throws.	Number of heads up at once.	Number of throws.
(1)	(2)	(1)	(2)
0	1	6	266
1	15	7	128
2	62	8	55
3	156	9	13
4	265	10	1
5	288		

These results when plotted gave a frequency curve which was much like that obtained for the soldiers. This curve was evidently the result of chance. One cannot tell for any given throw how many heads will come up, yet in the long run we always get some such result as that obtained by the coin tossing students.

What is meant by "chance."— What determines whether a coin thrown into the air will fall with the head up? Many things, of course, — the way it is held when thrown, the twist with which it starts, the height to which it rises, the manner in which it strikes the floor, the way it rolls, and many other factors. The sum total of these many causes gives what we call "chance." Chance is not the absence of cause, it is the result of a multiplicity of causes. In chance we must judge the result by the combination of these many causes. Often it is the only way we can judge the result. In chance we can never tell exactly any particular result,

but we can form an idea as to the frequency with which any possible result will occur.

In the case of the coins we could not tell in advance the result of any particular throw of ten coins but we could safely predict that five heads would be thrown more often than any other number, and that no heads or ten heads would happen least frequently. Is there any way by which the frequency that other numbers of heads would be thrown can be ascertained? There is, and it is quite simple.

We will start with a single coin. We toss it up. It is an even chance as to whether it comes up a head or a tail. If we should toss the coin a hundred times we would probably have a head in fifty of the throws. In practice it might not come out exactly 50, it might be 48 or 55, but if we tossed the coin an enormously large number of times a head would come up half the time. Let us now take two coins which we will call *a* and *b*. If we indicate a head by heavy type then we have the following possible combinations:

ab; **a**b; a**b**; ab.

We thus have the following results:

Heads	0	1	2	
Number of throws	1	2	1	Total 4

If we have three coins we have the following possible chances:

abc; **ab**c, **a**b**c**, a**bc**; **a**bc, a**b**c, ab**c**; abc.

Heads	0	1	2	3	
Number of throws	1	3	3	1	Total 8

If we have four coins we have:

abcd; **abc**d, **ab**c**d**, **a**b**cd**, a**bcd**; **ab**cd, **a**b**c**d, **a**bc**d**, a**bc**d, a**b**c**d**, ab**cd**; **a**bcd, a**b**cd, ab**c**d, abc**d**; abcd.

Heads	0	1	2	3	4	
Number of throws	1	4	6	4	1	Total 16

And so it goes on until for 10 coins we have:

Heads	0	1	2	3	4	5	6	7	8	9	10	
Number of throws	1	10	45	120	210	252	210	120	45	10	1	Total 1024

Theoretically, therefore, the coins in 1250 throws should have given us the following numbers: These compare reasonably well with those obtained by the students.

TABLE 133

THEORETICAL RESULT OF TOSSING 10 COINS 1250 TIMES

Number of heads.	Number of throws.	Number of heads.	Number of throws.
(1)	(2)	(1)	(2)
0	1	6	257
1	12	7	147
2	55	8	55
3	147	9	12
4	257	10	1
5	354		

Binomial theorem. — Another interesting fact is that these numbers which we have just obtained as representing what would result from applying the laws of chance to the tossing of two, three and more coins, are the same as are obtained by expanding the sum of two quantities by the binomial theorem, $(a + b)^n$ in which each quantity, a and b is taken as 1, *i.e.*, $(1 + 1)^n$. In the problem a head was just as likely to come up as a tail. In this expression n is the number of coins. If

$$
\begin{aligned}
n &= 1, \quad (1 + 1)^1 = 1 + 1,\\
n &= 2, \quad (1 + 1)^2 = 1 + 2 + 1,\\
n &= 3, \quad (1 + 1)^3 = 1 + 3 + 3 + 1,\\
n &= 4, \quad (1 + 1)^4 = 1 + 4 + 6 + 4 + 1,\\
n &= 5, \quad (1 + 1)^5 = 1 + 5 + 10 + 10 + 5 + 1.
\end{aligned}
$$

The binomial theorem, therefore, gives us a method of finding the shape of any natural frequency curve if we know the number of terms. It should be observed that only the even values of n give an odd number of terms with a middle highest term.

Some interesting conclusions may be predicted from this application of the binomial theorem. One of them is that the larger the number of terms the more closely are the items clustered around the median figure. It follows that the average of a large number of observations is much more precise than the average of only a few observations. In fact, it can be shown that the error of a set of observations varies inversely as the square of the number of observations. If we multiply the number of observations by four, we halve the probable error.

Chance and natural phenomena. — Does it follow therefore that the measurements of natural phenomena result from chance? Certainly, if they follow the binomial law as pointed out. How is it in the case of the heights of soldiers? Here we had 18,780 soldiers. Theoretically, these should have been distributed as shown in Column 3. Actually they were distributed as in Column 2. The differences are very slight.

What are the many causes which determine a person's height? It is difficult to say. Possibly inheritance, age, nationality, food supply during the period of growth, early illnesses, habits of sleeping, sitting, standing and many other factors. It would be an interesting subject for discussion. Whatever the causes are they are combined in so many ways that we have no better method of predicting the heights of the soldiers in a regiment than by the application of this law of chance.

TABLE 134

HEIGHTS OF SOLDIERS

Height in inches.	Per cent of soldiers.	
	Actual.	Theoretical.
(1)	(2)	(3)
60 +	1.05	1.00
61 +	1.69	1.71
62 +	3.69	3.68
63 +	6.86	6.75
64 +	10.44	10.51
65 +	13.91	13.99
66 +	15.84	15.84
67 +	16.07	15.31
68 +	12.18	12.60
69 +	8.52	8.84
70 +	4.67	5.31
71 +	2.77	2.67
72 +	1.39	1.18
73 +	0.92	0.61
	100.00%	100.00%

Skew curves. — In plotting natural phenomena it will be found that not all frequency curves are symmetrical. The median is not always the mean; there may be more items on one side of the mean than on the other. The asymmetrical curves are known as skew curves. They are not susceptible of mathematical analysis except by the use of complicated and rather uncertain methods.

There are four common types of asymmetrical curves commonly met with in demographic studies. These are shown in Fig. 56. In this diagram *A* represents the symmetrical frequency curves, the two sides of which are symmetrical about the mode. This type of curve has already been discussed. Type *B* is represented by the age distribution of deaths from measles. In early childhood the curve

rises sharply. Type C is a variant of B. Type D starts off with the mode and steadily diminishes. Age distribution of infant deaths by months gives us an example of this curve. Type E, the U-shaped curve, is already familiar to us. It is substantially the curve of specific death-rates by ages. All of these skew curves take many forms.

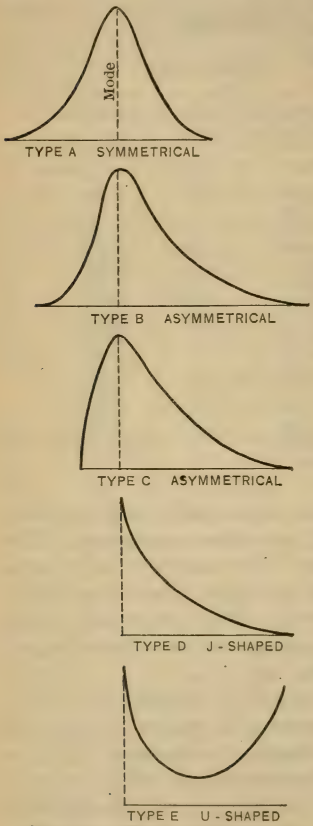


FIG. 56. — Types of Frequency Curves.

It will be remembered that in the case of the law of chance it was assumed that the chance of an event happening and of its not happening were equal. The chance of the coin falling as a head was the same as that of its falling as a tail, and one or the other was bound to happen. But we can imagine a result depending upon many factors, one of which was much more likely to occur than not to occur. This would result in producing a skew curve. There might be many such factors, and these might exist in all sorts of combinations.

When statistics naturally plot out as a skew curve it

is a sign that they should be investigated to determine, if possible, what the influence is which is producing the skewness. Sometimes it can be found. For example, in a case recently studied the quantity of butter fats in a series of analyses of milk samples was slightly skewed at one end. This was found to be due to the adulteration of about five per cent of the samples with water.

Beyond recognizing the skewness of a curve and making some attempt to account for it, the student of vital statistics will do well to let the mathematics of skew curves alone. Karl Pearson and others of his school have suggested certain methods of mathematical analysis.

Frequency shown by summation diagrams. — Another way of expressing "frequency" is by the use of the summation, or cumulative, diagram. In some respects this is more useful than the method of plotting by separate groups. Let us return to our 18,780 soldiers whose heights were measured. If 197 soldiers were between 60″ and 61″ then 197 were less than 61″; if 317 were between 61″ and 62″ then 197 + 317, or 514, were less than 62″; and so on. If these results are plotted we shall obtain a characteristic ogee curve. If the distribution is exactly in accordance with the law of natural frequency then the upper and lower parts of the curve will be symmetrical.

Instead of using the actual numbers of soldiers beginning with 197 and running up to 18,780, we might have plotted the percentage distribution from 1.05 per cent to 100 per cent. The result would have been the same.

Deviation from the mean. — Still another way of studying these figures is to find the extent to which the heights of the soldiers differed from the average, or mean, height. The mean height was 67.24″. For the sake of simplicity let us call it $67\frac{1}{4}$″. We may fairly assume that the height measurements were measured accurately and that the average height

of the 197 soldiers in height group $60'' - 61''$ was $60\frac{1}{2}''$. Then the average deviation of the height of these 197 soldiers from the mean was $67\frac{1}{4} - 60\frac{1}{2}$, or $6\frac{3}{4}''$. In the same way the 317 soldiers had an average deviation of $5\frac{3}{4}$; and so on. The average deviation of group $73 - 74''$ was $73\frac{1}{2} - 67\frac{1}{4}$ or $6\frac{1}{4}$. Some of these deviations are positive and some are negative, because some of the soldiers are shorter than the mean and some are taller.

If we plot these results we obtain the curve shown in Fig. 57. This is the curve of error, so-called. The deviations from the mean are regarded as errors. It is similar to the summation curve of variation. In fact it is the same curve, the only difference being the scale. Mathematicians, physicists and engineers look at their data from the standpoint of errors of observation, and therefore their text books which treat of this subject are called "Precision of Measurements," "Theory of Least Squares," and the like. Natural scientists however speak of "Variation." It is all one. The figures show us that small errors occur very often, large errors occur less frequently, and very large errors rarely occur.

In any set of measurements we may assume that errors will exist, and that in natural phenomena there will be variations caused by many factors. We are naturally interested to find out the extent of these variations. We want to know the average deviation and the variation most likely to occur. It will not be possible to go into these matters in great detail in this book. Readers who want to know the theory of these matters must study the theory of probability, or "Least Squares." A few methods of dealing with the subject practically will be given because they have an important use even in elementary statistics. In doing so we will consider first a very simple set of figures, and then come back to some more measurements of men, but lest we tire of our 18,780 soldiers we will consider some more recent measurements

made by Drs. Frankel and Dublin of the Metropolitan Life Insurance Company.

Standard deviation. — Let us suppose that we have five figures, or statistics, which represent something, no matter

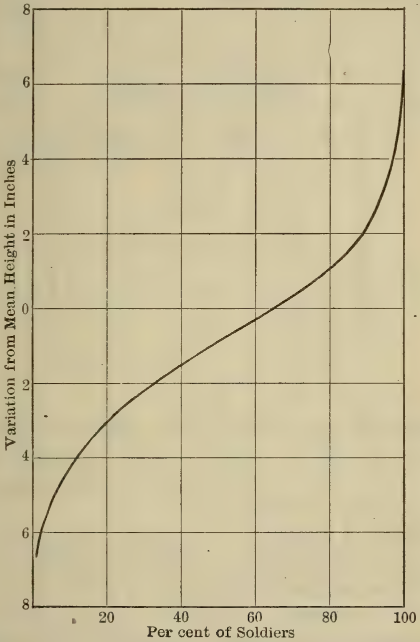


FIG. 57. — Percentage Deviation of the Heights of Soldiers from the Mean.

what. They are 6, 8, 2, 4, 5. The mean of these figures is 5. The deviations from the mean are respectively 1, 3, −3, −1, and 0. The average deviation, disregarding signs, is

their sum divided by 5, or 1.6. A more useful quantity is that called the *standard deviation.* It is obtained by squaring these deviations, finding the average square and taking its square root. If we average the data in tabular form we shall better understand the process.

TABLE 135

STATISTICAL DATA

Item.	Deviation from Mean.	Square of Deviation.
(1)	(2)	(3)
6	1	1
8	3	9
2	−3	9
4	−1	1
5	0	0
Sum 25	8[1]	20
Ave. 5	1.6	4

[1] Neglecting signs.

The average square is 4 and $\sqrt{4}$ is 2. Hence 2 is the standard deviation. It will be noticed that the standard deviation gives greater weight to the large deviations than a mere averaging of the deviations does.

Coefficient of variation. — The ratio between the standard variation and the mean is called the coefficient of variation. In the case just mentioned it is $2 \div 5$, or 0.40. The coefficient of variation is usually expressed decimally. If the variations are very small the coefficient of variation is small. If the variations are large the coefficient of variation is large. In some parts of the country the annual rainfalls do not vary much from year to year. In Massachusetts the coefficient of variation is about 0.17. In other parts of the country there are great fluctuations from year to year. In

Arizona the coefficient of variation is 0.50. A low coefficient means, in general, that the figures are more dependable; a high coefficient means that they are likely to be untrustworthy because of their fluctuations. This coefficient is very useful in the study of vital statistics.

Computing the coefficient of variation when data are grouped. — This is likely to cause trouble to the beginner. It is necessary to use care or mistakes will be made. Suppose we have the following items divided into magnitude groups between 0 and 5, the measurements being made to the nearest tenth as shown in columns (1) and (2).

TABLE 136

STATISTICAL DATA

Magnitude group.	Number in group.	Average magnitude.	Product.	Deviation of number in group.	Square of deviation.	Product.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0–0.9	6	0.45	2.70	1.76	3.10	18.60
1–1.9	8	1.45	11.60	0.76	0.58	4.64
2–2.9	2	2.45	4.90	−0.24	0.06	0.12
3–3.9	4	3.45	13.80	−1.24	1.54	6.16
4–4.9	5	4.45	22.25	−2.24	5.02	25.10
Total	25	....	55.25	....	....	54.62
Mean	....	....	2.21	....	....	2.18

Here we first find the average magnitudes of the numbers in each group, column (3). By multiplying these by the number of items in each group and dividing by the number of items we have (4) the weighted average, or the mean of all the items. This is 2.21. Subtracting the figures in column (3) from 2.21 we have the group deviations in column (5). These are squared (6) and then multiplied by the number of items in each group (7). The sum of the squares divided by 25 gives the average square, *i.e.*, 2.18 and $\sqrt{2.18}$ is 1.48, the

standard variation. $1.48 \div 2.21$ gives 0.67 the coefficient of variation. Unthinking students sometimes multiply the figures in column (5) by those in column (2) before squaring. This is wrong. It is the deviations which are squared. The subsequent process is merely to get the weighted average of the squares.

Probable error. — Neither the average deviation from the mean nor the standard deviation is the one most likely to occur. It is the median deviation, or the median error, which is most likely to occur. It can be shown by calculus that when observations follow the normal law of error, or the normal frequency distribution, *i.e.*, the binomial distribution, the median deviation is about two-thirds of the standard deviation. To be exact, the figure is 0.6745. If we let r stand for this median deviation, this probable error, and if we let x be any individual error, and if n = the number of observations, then, remembering that the sign Σ means "the sum of," we shall see that

$$r = 0.6745\sqrt{\frac{\Sigma x^2}{n}}.$$

This is merely the mathematical way of stating what we have just done. Σx^2 means the sum of all the squares of the deviations, $\frac{\Sigma x^2}{n}$ means the average square, and $\sqrt{\frac{\Sigma x^2}{n}}$ means the square root of the average square, *i.e.*, the standard deviation. Where does 0.6745 come from? If we take the curve of error (Fig. 57), and consider the side to the left of the middle ordinate, it will be possible to draw a vertical line somewhere to the left (or the right) of the middle which will divide the area included between the curve and the base line into two equal parts. The height of the ordinate which will do this is 0.6745 that of the middle ordinate.

This probable error is quite useful in statistics. One use is that of throwing out of consideration doubtful observations.

Doubtful observations. — Scientists make a distinction between errors and mistakes. Errors are supposed to fall within the limits of probability; mistakes are supposed to be glaring, erratic observations which really ought to be left out of account, or at least not included when the average is computed. We have all had experiences of this kind. In a daily record of the number of bacteria in a filtered water we may find that where most of the figures are less than 25 per cubic centimeter there is one which exceeds 1000. Shall we include this in the average for the month? If we do we unduly raise the average for the month and bring discredit on the filter. And yet there may be no reason for excluding it. It may have been a fact. And a fact is not to be discarded.

The theory of probability gives us a means of telling whether it should be included in the average or not. If we know the probable error r, as above described, then we shall find that there is an allowable ratio of $\frac{x}{r}$ which depends upon the number of observations. If we had only three observations then any value of the ratio $\frac{x}{r}$ which is greater than about 2 should be regarded as outside the probable variations resulting from the law of chance. If n is 10 the limit of $\frac{x}{r}$ is 3; if $n = 30$, then the limit of $\frac{x}{r}$ is 3.5; if n is 100, the limit is 4; if n is 500, the limit is 5, and so on. These values are merely approximate.

The probability scale. — This ratio of $\frac{x}{r}$, the ratio of any error to the mean, or most probable error, is useful in another way because on the basis of the binomial distribution we can

compute the frequency with which any value of $\frac{x}{r}$ is likely to occur. We call this the probability of its occurrence.

If x is any error and r is the most probable error then when $\frac{x}{r} = 1$ the chances are even that the error will be x. There are as many chances that the error will be larger than x as that it will be smaller. We may call this a "fifty-fifty" chance, and we may write the probability of its occurrence as $\frac{1}{2}$ or 0.5. If $\frac{x}{r}$ is less than 1 the probability that any error will be less than $\frac{x}{r}$ is less, and if $\frac{x}{r}$ is greater than 1 the probability that any error will be less than $\frac{x}{r}$ is greater. In fact we shall find that the following relations hold:

TABLE 137

PROBABILITY

$\frac{x}{r}$.	Probability that any error will be less than $\frac{x}{r}$.	$\frac{x}{r}$.	Probability that any error will be less than $\frac{x}{r}$.
(1)	(2)	(1)	(2)
0.0	0.0000	1.7	0.7485
0.1	0.0538	1.8	0.7753
0.2	0.1073	1.9	0.8000
0.3	0.1603	2.0	0.8227
0.4	0.2127	2.1	0.8433
0.5	0.2641	2.2	0.8622
0.6	0.3143	2.3	0.8792
0.7	0.3632	2.4	0.8945
0.8	0.4105	2.5	0.9082
0.9	0.4562	2.6	0.9205
1.0	0.5000	2.7	0.9314
1.1	0.5419	2.8	0.9410
1.2	0.5872	2.9	0.9495
1.3	0.6194	3.0	0.9570
1.4	0.6550	4.0	0.9930
1.5	0.6883	5.0	0.9993
1.6	0.7195	∞	1.000

If we compute the values of $\frac{x}{r}$ which correspond to certain probabilities we have the following approximate figures:

TABLE 138

PROBABILITY

Probability.	$\frac{x}{r}$.	Probability.	$\frac{x}{r}$.
(1)	(2)	(1)	(2)
0.01	0.02	0.80	1.90
0.02	0.04	0.90	2.44
0.03	0.06	0.95	2.91
0.05	0.09	0.98	3.45
0.10	0.19	0.99	3.82
0.20	0.38	0.999	4.887
0.30	0.58	0.9999	5.783
0.40	0.77	0.99999	6.592
0.50	1.00	0.999999	7.258
0.60	1.25	0.9999999	7.967
0.70	1.54		

Probability paper. — Until recently it has been difficult to use the theory of probability in statistical work, but it is now easy. In 1913, my partner, Dr. Allen Hazen, devised a new kind of plotting paper. The percentage scale was so spaced that any set of figures which follow the natural law of probability would plot out not as an ogee curve, but as a straight line. The spacing was based fundamentally on the preceding figures, but it was necessary to take account of the sign of the error, whether positive or negative, and make allowance for this in designing the plotting paper. The 50 per cent, or median line, was placed in the middle of the percentage scale. The other relative distances were as follows. The figures given cover only one side of the 50 per cent line.

TABLE 139

DATA FOR PREPARING PROBABILITY PAPER

Line.	Relative distance.	Line.	Relative distance.	Line.	Relative distance.
(1)	(2)	(1)	(2)	(1)	(2)
Per cent.		Per cent.		Per cent.	
50	0.000	17	1.415	0.8	3.573
48	0.074	16	1.474	0.7	3.646
46	0.149	15	1.537	0.6	3.727
44	0.224	14	1.602	0.5	3.821
42	0.300	13	1.670	0.4	3.933
40	0.376	12	1.742	0.3	4.077
38	0.453	11	1.818	0.2	4.267
36	0.531	10	1.906	0.1	4.585
34	0.611	9	1.988	0.09	4.630
32	0.693	8	2.083	0.08	4.685
30	0.777	7	2.188	0.07	4.748
28	0.864	6	2.305	0.06	4.817
26	0.954	5	2.439	0.05	4.900
24	1.047	4	2.596	0.04	5.000
22	1.145	3	2.789	0.03	5.120
20	1.248	2	3.045	0.02	5.290
19	1.302	1	3.450	0.01	5.550
18	1.357	0.9	3.507		

As first used the percentage scale was used horizontally, as in Fig. 63. There are some advantages plotting the percentages as ordinates as in Figs. 58, 59 and 60.

In the latter the horizontal scale is the ordinary arithmetical scale. The vertical scale may be labeled from 0 to 100 per cent, in either direction, or it may read from 0 to 50 on either side of the median line. It depends upon whether we want to keep the positive and negative errors separate or add them together and consider their magnitude alone.

A few examples of the use of this probability paper will now be given.

For a more complete description of this paper the reader is referred to the author's monograph on the "Element of Chance in Sanitation." [1]

[1] Jour. Franklin Institute, July, 1916.

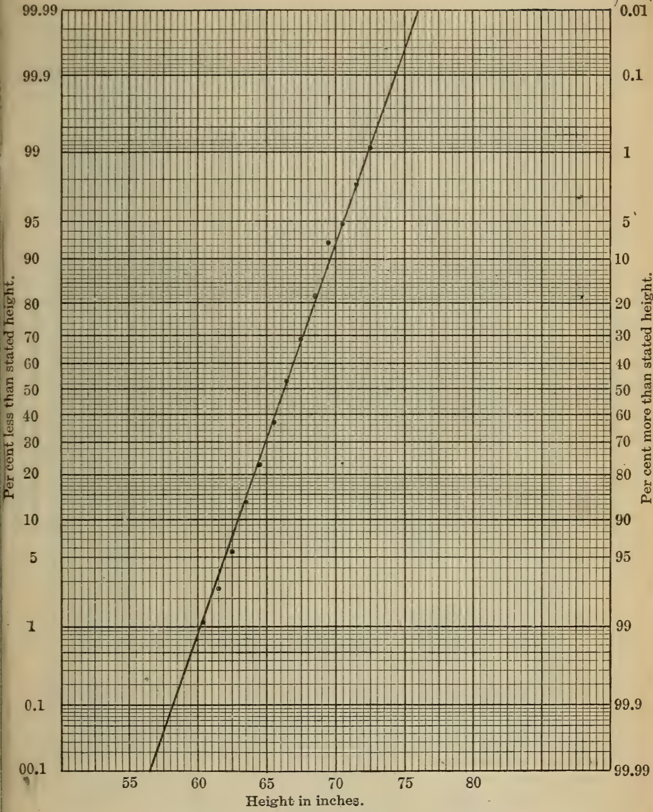


Fig. 58. — Distribution of Soldiers According to Height. Plotted on Arithmetic-Probability Paper.

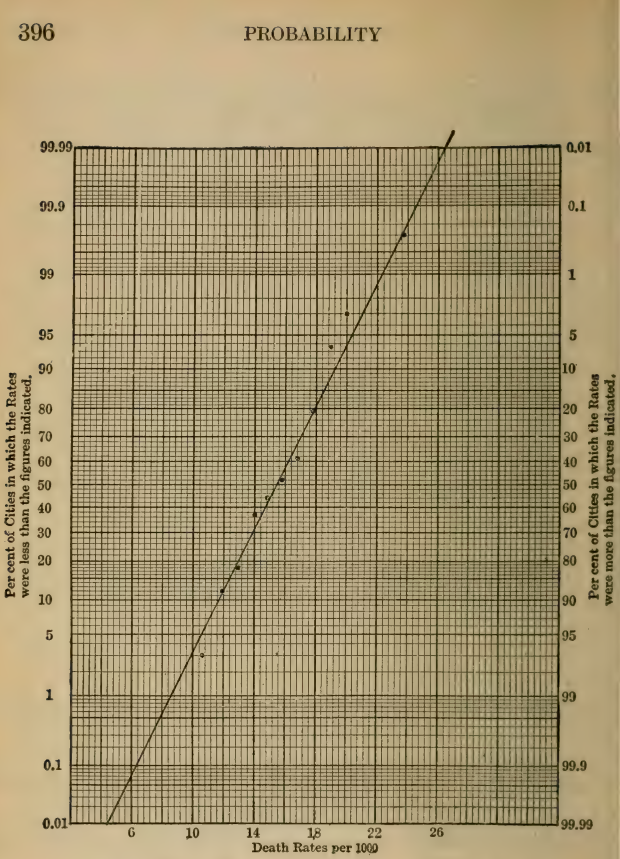


FIG. 59. — Death-rates of Massachusetts Cities and Towns. Plotted on Arithmetic-Probability Paper.

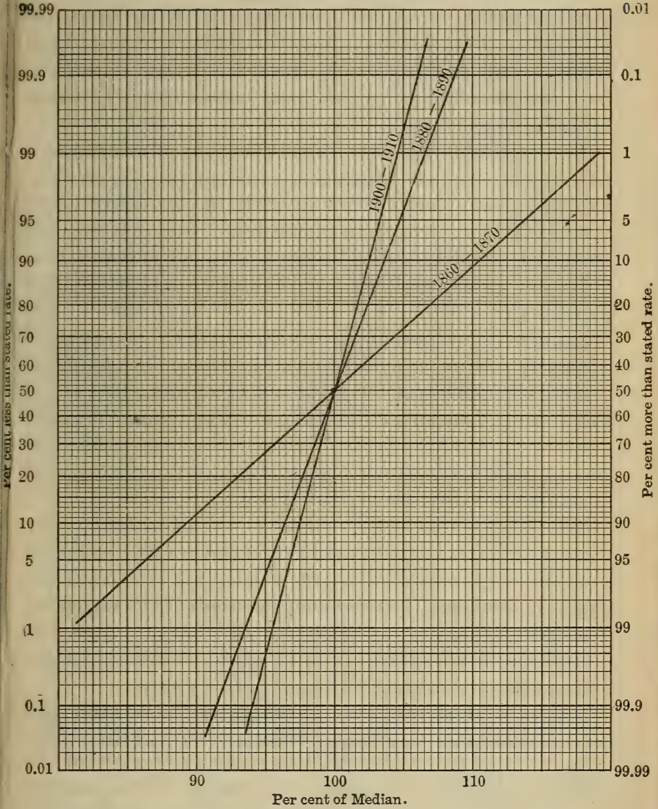


Fig. 60. — Percentage Variation of Death-rates of Massachusetts Cities and Towns for Three Different Decades. Plotted on Arithmetic-Probability Paper.

Examples of use of probability paper. — Fig. 58 shows the distribution of soldiers according to height plotted on probability paper. This is based on the observations with which we have already become familiar. It will be noticed that instead of forming the usual ogee curve the points fall on a straight line.

Fig. 59 shows that the death-rates for Massachusetts cities and towns also plot out on this paper as a straight line. Fig. 60 shows that in 1900–10 the death-rates throughout the state have been more uniform than in 1860–70. This is indicated by the different slope of the lines.

For an example of the use of logarithmic probability paper, see Fig. 63.

Another use of probability. — Bernouilli's theorem gives us another interesting application of the theory of probability.

If we let p represent the frequency of an event happening and q the frequency of its not happening, then obviously $p + q = 1$. Unless this fundamental condition holds, the laws of probability do not hold. It is always well to see if there are any other factors than p and q.

If we let n represent the number of cases considered, and ϵ the mean error, then Bernouilli says, $\epsilon = \sqrt{npq}$. We need not stop here to prove this, but we may see how it can be used. If n is large then ϵ is a fair measure of the deviation from the standard for it is said that in 2 out of 3 cases the deviation will be less than ϵ; in 19 out of 20 cases less than $2\,\epsilon$; and deviations greater than $4\,\epsilon$ are very rare.

Let us suppose that in a population of 10,000 the general death-rate was 15 per 1000, *i.e.*, 150 deaths in all. Then $n = 10{,}000$; $p = \frac{15}{1000}$; $q = \frac{985}{1000}$; then

$$\epsilon = \sqrt{10{,}000 \times \frac{15}{1000} \times \frac{985}{1000}} = \sqrt{147.75} = 12.15 \text{ deaths},$$

or 1.2 per 1000. A fluctuation of this amount from year to year would not be outside of the bounds of chance phenomena. If the population were 1,000,000 then ϵ would be $\sqrt{1.4775}$ or 1.2 deaths in 1,000,000 or 0.012 per thousand.

Other criteria than Bernouilli's have been suggested for this computation. The results differ considerably, and none of the methods must be taken as mathematically exact.

Let us suppose we are studying an epidemic of typhoid fever in which all the cases, 120, were actually caused by the public water supply. The population was 50,000. There were two milk dealers: A served 40,000 persons; B served 10,000 persons. What would be a chance distribution of cans among these two dealers? If they were distributed uniformly we should expect to find among A's customers $\frac{40,000}{50,000}$ of 120, or 96; and among B's customers $\frac{10,000}{50,000}$, or 24. Now what is a reasonable variation from these figures? In the case of A,

$$\epsilon = \sqrt{40,000 \times \frac{120}{50,000} \times \frac{49,880}{50,000}} = 10 \text{ approximately.}$$

Therefore, if A had any number between 86 and 106 it would be within the bounds of chance. If he had 116 cases it might be a suspicious circumstance. In the case of B,

$$\epsilon = \sqrt{10,000 \times \frac{120}{50,000} \times \frac{49,880}{50,000}} = 2.4 \text{ approximately.}$$

If, therefore, B had more than 27 cases it would be suspicious.

The frequency curve as a conception. — The frequency curve is something far beyond a statistical tool. Properly conceived it stands for a universal principle. Not all the leaves of a tree are alike, not all shells are alike, soldiers are not all of the same height or weight. We cannot well compare the tallness of pine trees and elm trees with-

out resorting to the frequency curve. One man may say, "The elm tree is the taller; I have seen elms taller than pines." Another says, "That is nothing; pine trees have a greater average height." But the first man is not convinced. He goes back to his own observation and insists that he has seen elms taller than pines. To give the true picture both men need to know the frequency of different heights of both elms and pines.

Are young women as good scholars as young men? Assuming that we have an adequate definition of what is meant by a good scholar, can we settle the question by saying that we have seen young women who were better scholars than young men, or that the average of scholarship is higher among men; must we not know the frequency with which we find good scholars and poor scholars among both men and women? It is quite conceivable that among women we have greater extremes of scholarship than among men, or *vice versa.*

Are women as well fitted for voting as are men? Suffragists point to drunken sots and say, "We are better fitted to vote than they are." When they say this they are comparing the end of one frequency curve with the middle or the upper end of the other. Such comparisons are utterly meaningless.

Sometimes we need to make comparisons on the basis of lower limits, sometimes on the basis of upper limits, sometimes we ought to compare modes, sometimes medians, sometimes averages, sometimes we do not know the facts well enough to make comparisons at all: but throughout all realms of thought an appreciation of the fundamental importance of the frequency curve will help us to reason soundly and will prevent us from making false comparisons.

The frequency curve contains in itself the element of

beauty. Moons wax and wane; the tide rises and falls; the flowers of spring come, first a few, then many; and they disappear in the same way, a few lingering into summer. It is said that we live in a world of chance. Nothing is more true. We live in a world where many causes are acting with and against each other. We live in a world of frequency curves. Artists and architects recognize this. The ogee curve is the line of beauty.

EXERCISES AND QUESTIONS

1. Find data for and plot an example of a typical symmetrical frequency curve. (Anthropometrical measurements.)

2. Find data for and plot an asymmetrical frequency curve (specific death-rates for scarlet fever, diphtheria, etc.).

3. Describe the application of Bernouilli's Theorem to the chance distribution of cases among milk customers? [See Am. J. P. H., Apr., 1912, p. 296.]

4. Construct a model to illustrate the general law of probability. [See Rosenau's Preventive Medicine, Chapter on Heredity and Eugenics.]

5. Repeat the coin tossing experiment described in this chapter.

6. Find the height of 50 males (or females) above eighteen years of age, and compute:

a. The average deviation from the mean.
b. The standard deviation.
c. The coefficient of deviation.
d. The probable error.

7. Plot the height records of these persons on "probability paper."

8. Discuss the use of the law of chance in public health studies. (Whipple, Geo. C. The Law of Chance in Sanitation, Jour. Franklin Institute, July, 1916.)

9. Prepare a short statistical abstract of the stature of recruits, U. S. A., 1906–15. [Hoffman, Frederick L. Army Anthropometry and Medical Rejection. Newark. Prudential Press, 1918.]

CHAPTER XIII

CORRELATION

Correlation is the word by which the statistician describes the correspondences or relations between series, classes or groups of data; in fact, it is largely for the study of these relationships that statistics are collected.

Deaths from typhoid fever are arranged by months in order to ascertain if there is a fixed relation between the frequency of such deaths and the season of the year; or they are arranged by the age, occupation or place of residence of the decedants in order to learn of any other correspondences which may exist. The heights and weights of men, or women, are compared to see if the variations in height are related to variations in weight; the length of the arm is compared with some other measurement of the body; the heights of sons are compared with the heights of their fathers. These are all simple correlations. Two sets of measurements only are compared.

Often the problem is more complicated. The infant mortality in cities varies with the season, being highest in the summer; the temperature of the air also varies with the season, being highest in the summer; and the statistician desires to ascertain if there is any definite relation, any correlation, between atmospheric temperature and infant mortality. Here there are three elements to be considered — season, temperature and infant mortality. Also the number of flies ordinarily increases with an increased atmospheric temperature, and the question arises "Is there a fixed re-

lation between the increase in the number of flies and the infant mortality?" One naturally asks: "Why not eliminate the temperature of the air and study the direct and simple correlation between flies and mortality? That would, indeed, be the best and safest method, but unfortunately the data may not exist, or cannot be obtained in comparable form. It is, therefore, necessary to devise some way of studying this problem by indirect correlation, or secondary correlation.

Causal relations. — Sometimes statistics are studied merely to determine whether correlation exists between two variables, this result being practically useful. The knowledge that infant mortality increases with the atmospheric temperature is in itself of value to the physician and the health officer. More often perhaps the underlying motive in correlation studies is that of determining cause and effect. In the illustration given the question is, Is the increase in atmospheric temperature the cause of the increased mortality among infants? Is the increase in the number of flies in the summer the cause of the increased infant mortality? Or, to go back to the examples of simple correlation, Is the increased height of men the cause of their increased weight? Is the tallness of a son the effect of the tallness of his father? Does the establishment of correlation also mean that a causal relation has been established? To answer this we must consider what is meant by *cause.*

Jevons[1] says: "By the cause of an event we mean *the circumstances which must have preceded* in order that the event should happen. It is not generally possible to say that an event has one single cause and no more. The cause of the loud explosion in a gun is not simply the pulling of the trigger, which is only the last apparent cause or the occasion of the explosion; the qualities of the powder, the proper form of

[1] Lessons in Logic, p. 239.

the barrel; the existence of some resisting charge; the proper arranging of the percussion cap and powder; the existence of a surrounding atmosphere, are among the circumstances necessary to the loud report of the gun; any of them being absent it would not have occurred." In the above phrase, "the circumstances which must have preceded in order that the event should happen," emphasis must be placed on the word *must,* otherwise our reasoning is *post hoc non propter hoc.*

It is obvious that statistics do not in themselves establish these causal relations. The laws of logic are the primary laws, and the rules of statistics must be subsidiary to them. Westergaard, the celebrated Danish statistician, has recently said (Jour. Am. Stat. Asso., Sept. 1916, p. 259), "that the task of the statistician is not so much to find the causality himself as to help others to find it. The statistician must be content if he can show that certain groups of numbers have marked differences, leaving it to physiology, meteorology and other sciences to explain these differences."

The statistician can prove nothing by his statistics unless he uses them logically.

On the other hand, the statistical arrangements of facts are of the greatest aid in helping to establish causal relations, because by expressing facts by numbers it is possible to concentrate extended experiences into quantities which may be easily and quickly compared.

Correlation and causality. — In studying correlation as a process for determining causality it is necessary to distinguish between the simple correlation which may exist between two variables and the more indirect correlation, or secondary correlation, which occurs when two series of events, both correlated to a third factor, are compared to each other. The former may be safely used to establish a causal relation; in fact, King says (Elements of Statistics, p. 197), that

"correlation means that between two series or groups of data there exists some causal relation." In stating this he evidently had in mind the simple correlation between two variables. And, of course, "causal relation" does not mean "sole cause." Besides correlation we must also establish *connection* between the two variables. It is not the task of the statistician to do this. It would be more exact to say that a *causal relation may be shown by establishing a definite correlation between two series, classes or groups of connected data.*

It is chiefly in secondary correlations that we err in our logical processes. In mathematics we learned that "two things which are equal to a third are equal to each other," but it is not necessarily true that two series of events which vary as a third are equal to each other, or even are related to each other at all. Infant mortality increases with the atmospheric temperature in summer; the softness of the asphalt pavements increases with the atmospheric temperature in summer; but we cannot infer that there is any relation between infant mortality and the softness of asphalt pavements.

The actual connection between events is not shown by statistics or by the statistical methods except as the data are interpreted according to the laws of logic.

Let the reader try to answer questions like these. Why is it not true that there is a causal relation between the softness of pavements and infant mortality? Is it, or is it not, true that there is a causal relation between the presence of flies and infant mortality? Which shows the higher degree of correlation with infant mortality — the presence of flies or the softness of asphalt?

Laws of causation. — While we are thinking about correlation and its relation to causation it will not be out of place to refer to the three methods of induction as stated

by John Stuart Mill. The cause of an event may be said to be "the circumstances which must have preceded in order that the event should happen."

Mill's first canon is, "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon." This is the method of agreement. The epidemiologist follows this principle when he studies case after case of disease looking for some common antecedent circumstance. Here one instance does not establish proof of a cause, and the larger the number of instances the stronger the proof.

The second canon is "if an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstances in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon. This is the method of difference, the method of experiment. This principle also is used in epidemiology.

The third canon is called the joint method. "If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances (always or invariably) differ, is the effect, or the cause or an indispensable part of the cause, of the phenomenon."

These are sometimes expressed as follows, the large letters, A, B, C, etc., representing antecedents, and the small letters, a, b, c, etc., the consequents.

Method of Agreement

$A\ B\ C$	$a\ b\ c$
$A\ D\ E$	$a\ d\ e$
$A\ F\ G$	$a\ f\ g$
$A\ H\ K$	$a\ h\ k$

Method of Difference

$A\ B\ C$	$a\ b\ c$
$B\ C$	$b\ c$

Joint Method

$A\ B\ C$	$a\ b\ c$
$A\ D\ E$	$a\ d\ e$
$A\ F\ G$	$a\ f\ g$
$A\ H\ K$	$a\ h\ k$
....	....
$P\ Q$	$p\ q$
$R\ S$	$r\ s$
$T\ V$	$t\ v$
$X\ Y$	$x\ y$
	....

Methods of correlation. — Correlations may be divided into two classes: — (1) simple, or primary, and (2) secondary.

Simple correlations are studied as between two variables, these two variables being compared on the basis of magnitude, that is they are compared by grouping.

Secondary correlations are studied when two variables are compared with each other after first being compared to a third variable — such as time or place.

When two variables are so correlated that the numerical values increase and decrease together the correlation is said to be *direct*.

When the correlation is such that the numerical value of one variable increases as that of the other decreases the correlation is said to be *inverse.*

The closeness of correlation is termed the *degree of correlation.* There are mathematical methods of determining the degree of correlation, according to which perfect correlation is represented by *unity* and complete absence of correlation by *zero.*

The following are some of the methods used in the study of correlation:

Simple correlations (two variables compared directly):

1. Plotting of original data.
2. Correlation table (grouping by lines and columns).
3. Correlation model (correlation surface).
4. Plotting of group means (Galton).
5. Computation of coefficient of correlation:
    - (*a*) Galton's method (see Elderton's Primer of Statistics).
    - (*b*) Karl Pearson's method.
6. Use of mathematical formulæ.

Secondary correlations (two variables compared on the basis of a third variable):

1. Comparisons between two plotted lines representing original data, as to:
    - (*a*) Parallelism.
    - (*b*) Correspondence of fluctuation in time of occurrence and in magnitude.
    - (*c*) Correspondence of cycles.
    - (*d*) Lag.
    - (*e*) Inverse relations.

2. Comparison between two plotted lines, each representing variations from the mean.

3. Comparison between two plotted lines, each representing variations from the moving average (or some smoothed line showing trend).

Galton's coefficient of correlation. — Let us suppose that we have the following pairs of observations. Each a has a corresponding b. What is the correlation between a and b? Offhand one can see that in a general way a and b rise and fall together. But how can we express this relation?

TABLE 140

EXAMPLE OF CORRELATION

a	b	x	x^2	y	y^2	xy
(1)	(2)	(3)	(4)	(5)	(6)	(7)
7	4	1	1	0	0	0
5	2	−1	1	−2	4	2
6	5	0	0	1	1	0
3	1	−3	9	−3	9	9
9	8	3	9	4	16	12
Sum 30	20	......	20	....	30	23
Average 6	4	......	4	....	6	4.6
σ.........	......	......	2	....	2.45	....

We cannot compare the figures directly. We do not even know that the measurements are the same. a may be expressed in feet, and b may mean years or something else. What have these two sets of figures in common? The deviations from their means may help us. Let us suppose that x represents the deviation of a from its mean, 6, and that y stands for the deviations of b from its mean, 4. Then we can compute the standard deviation of each set of figures, and call these σ_x and σ_y. These we find to be $\sqrt{4}$, or 2, and $\sqrt{6}$, or 2.45. We must now link together the two sets of observations and we do this by finding the products of their deviations, *i.e.*, xy, and the average of xy, *i.e.*, 4.6. This average value of the product of x and y, divided by the product of the standard variations, σ_x and σ_y, gives what Galton calls the coefficient of variation. It may be expressed by formula thus:

Coefficient of correlation $= \frac{\Sigma xy}{n\sigma_x\sigma_y}$, in which n is the number of observations, and Σxy the sum of all the xy's. In the example, $\frac{\Sigma xy}{n} = \frac{23}{5} = 4.6$, and the coefficient is $\frac{4.6}{2 \times 2.45} = 0.94$. This is a close correlation between a and b, because 1 represents perfect correlation and 0 no correlation at all.

Pearson's coefficient is not quite the same, but it is enough for practical purpose to remember Galton's.

Example of low correlation. — In the monthly bulletin of the Connecticut State Department of Health for Feb., 1918, a radial diagram is given showing that grippe outbreaks in one year are followed by measles the next year, and the statement is made that "the wheel of chance becomes a wheel of certainty." Let us see if these facts will stand the test of correlation. If we place the deaths from grippe in one year side by side with the deaths from measles the following year, we have the following twelve pairs of values for a and b.

TABLE 141

a	b	x	x^2	y	y^2	xy
(1)	(2)	(3)	(4)	(5)	(6)	(7)
8	32	−2.25	5.06	16.92	286.29	−38.07
4	8	−6.25	39.06	− 7.08	50.13	+44.25
15	26	4.75	22.56	− 9.08	82.45	−43.13
11	5	0.75	0.56	9.92	98.41	+ 7.44
6	6	−4.25	18.06	− 9.08	82.45	+38.59
8	22	−2.25	5.06	−13.08	171.09	+29.43
16	0	5.75	28.06	4.92	24.21	+28.29
7	3	−3.25	33.06	−12.08	145.93	+39.26
12	27	1.75	3.06	11.92	142.09	+20.86
2	12	−8.25	68.06	− 3.08	9.49	+25.41
9	6	−1.25	1.56	− 9.08	82.45	+11.35
25	34	14.75	217.56	18.92	357.97	+279.07
Sum 123	181	....	441.72	....	1532.96	442.75
Average 10.25	15.08	....	36.81	....	127.75	....
σ	......	....	6.07	....	11.30	....

It will be seen that the coefficient of correlation is

$$\frac{\Sigma xy}{n\sigma_x\sigma_y} = \frac{442.75}{12 \times 6.07 \times 11.30} = 0.54.$$

This is a low correlation. It is less than the coefficient of variation of either grippe or measles. It follows, therefore, that the statement that grippe is followed by measles a year later has little to substantiate it, if all the facts are considered. If we leave out a few exceptional years there does appear to be a general tendency for measles to follow grippe. But what right is there to leave out some of the facts? If they are mistakes they should be left out, otherwise they should be considered in drawing concessions.

A few years ago a sanitary chemist tried to show a relation between the color of water and the typhoid fever death-rates in Massachusetts water supplies. Computations of the coefficient of correlation for 54 places where surface water was used gave a figure of 0.16, while for 33 places where ground water was used it was 0.30. In other words there was very little correlation. In the same cities the correlations between the general death-rates and the typhoid fever death-rates were 0.59 and 0.56, respectively.

On the other hand, the Eldertons found the coefficient of correlation between the length and breadth of shells to be 0.95; that between the ages of husbands and wives, 0.91.

The student will find the use of the coefficient of correlation an admirable weapon for exploding false theories.

Correlation shown graphically. — In a general sense any graph with horizontal and vertical scales, in which pairs of observations are represented by a single point is a correlation plot. If the points fall on or near a straight line the correlation is high; if the points are so scattered that a straight line cannot be readily drawn to represent them the correlation is low. This needs no further illustration.

It is possible, however, to determine the coefficient of correlation graphically. In Bowley's Elements of Statistics we find the relation between the marriage-rate and the price of wheat. The first step is to select two suitable scales and plot the data as points. The second step is to find the mean marriage-rate and the mean price of wheat and plot these as

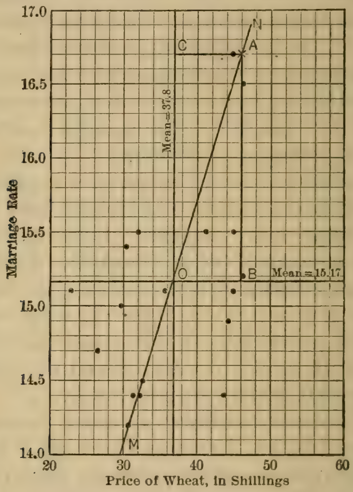


FIG. 61. — Correlation between Marriage-rate and Price of Wheat.

horizontal and vertical lines, respectively. The third step is to draw a line which will fairly well represent the points. In Fig. 61 this line is marked MN. It must pass through the intersection of the mean lines, 0. The fourth step is to select any point on MN, as, for example, A, and read from the vertical scale the value of AB. In the example AB is $16.7 - 15.17 = 1.53$. This is really the deviation of A

from the mean marriage-rate and $1.53 \div 15.17 = 10.15$ is the standard variation. In the same way AC is $46.0 - 37.8 = 8.2$, the deviation of A from the mean price of wheat, and $8.2 \div 37.8 = 21.7$ is the standard variation. The ratio of 10.15 to 21.7 gives us the coefficient of correlation, $10.15 \div 21.7 = 0.468$. By computation Bowley finds this to be 0.47. The graphical method is useful only when the correlation is fairly high, because if the correlation is low one cannot tell where to draw the line MN. In drawing this line an effort should be made to place it so that there will be as many points as possible near the line, with the other points as well balanced as possible on either side of the line. This requires experience and a sort of intuitive sense of distances.

A recent example of lack of correlation. — The Municipal Tuberculosis Sanitarium of Chicago, in its annual report for 1917, has published an interesting series of diagrams illustrative of the lack of correlation between housing and tuberculosis. Fig. 62 is one of these. The districts are arranged in order of occurrence of tuberculosis. The one-scale rectangles, appropriately divided according to the character of rooms, fail to show any progression coincident with tuberculosis under Chicago conditions.

The correlation table. — The correlation table is arranged much like a simple plot. There is a horizontal and a vertical arrangement of groups. This tabulation shows to the eye the relation between the two quantities. In Table 142 we see the correlation between the ages of husband and wife is fairly close. Of 669 wives in age-group 40–44, 309 were married to husbands in the same age-group. There is a slight tendency for husbands to be slightly older than their wives. The figures are not symmetrically arranged around the mode.

The correlation model is used but little. A description of its construction and use may be found in works on statistical methods.

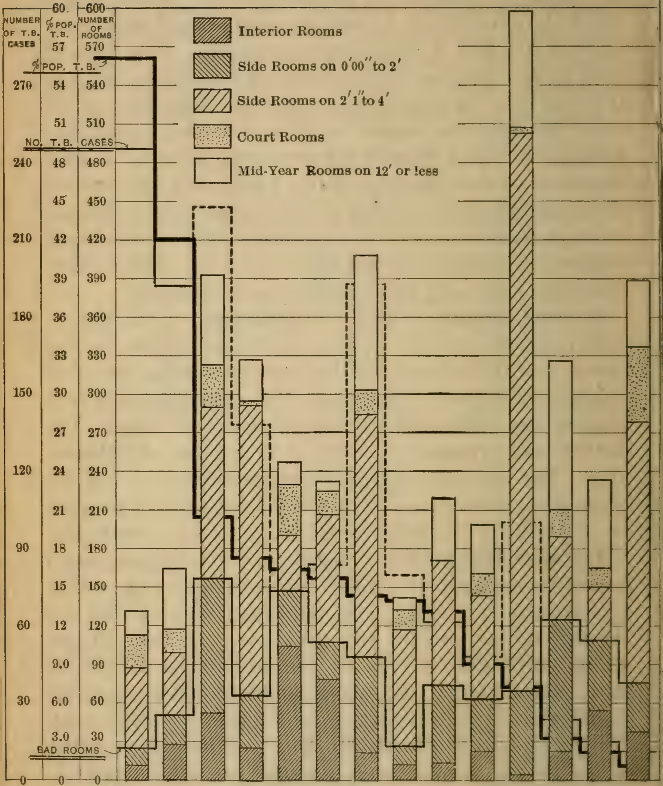


FIG. 62. — Diagram Showing Lack of Correlation between Interior Rooms in Certain Chicago Blocks and Tuberculosis Morbidity.

TABLE 142

CORRELATION BETWEEN (1) THE AGE OF WIFE, (2) THE AGE OF HUSBAND, FOR ALL HUSBANDS AND WIVES IN ENGLAND AND WALES WHO WERE RESIDING TOGETHER ON THE NIGHT OF THE CENSUS, 1901. (CENSUS, 1901, SUMMARY TABLES, P. 182.) TABLE BASED ON 5,317,520 PAIRS; CONDENSED BY OMITTING 000'S

(From Yule's Theory of Statistics, p. 159.)

Ages of husbands.	Ages of wives.														Total.
	15–	20–	25–	30–	35–	40–	45–	50–	55–	60–	65–	70–	75–	80–	85–	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
15–	2	2	....	....	....	....	....	....	....	....	....	....	....	....	....	4
20–	16	173	46	4	1	....	....	....	....	....	....	....	....	....	....	240
25–	4	185	402	84	10	2	1	....	....	....	....	....	....	....	....	688
30–	1	41	265	411	84	12	2	1	....	....	....	....	....	....	....	817
35–	....	9	69	251	369	80	12	2	1	....	....	....	....	....	....	793
40–	....	3	17	71	219	309	66	12	2	1	....	....	....	....	....	700
45–	....	1	6	20	66	178	252	59	10	2	1	....	....	....	....	595
50–	....	....	2	8	19	57	146	195	44	10	2	....	....	....	....	483
55–	....	....	1	3	8	18	46	110	141	35	6	1	....	....	....	369
60–	....	....	....	1	3	8	16	39	81	101	23	4	1	....	....	277
65–	....	....	....	1	1	3	6	11	26	53	58	13	2	1	....	175
70–	....	....	....	....	1	1	2	5	8	18	31	31	6	1	....	104
75–	....	....	....	....	....	1	1	2	3	5	10	14	12	2	....	50
80–	....	....	....	....	....	....	....	1	1	1	2	4	5	3	1	18
85–	....	....	....	....	....	....	....	....	....	....	1	1	1	1	....	4
Total	23	414	808	854	781	669	550	437	317	226	134	68	27	8	1	5317

Use of mathematical formulæ. — It is often desirable to find the equation of a straight line or curve drawn through a series of points. This is not difficult, but it requires a longer description than can be given here. The student can find good descriptions of the methods used in standard mathematical books.

Secondary correlation. — The correlation of two variables is often shown by plotting each against a third quantity, which latter varies in a regular manner. Thus in Fig. 52 we

have the number of cases of typhoid fever plotted as ordinates with months as abscissæ, and we have also the atmospheric temperature plotted as ordinates with months as abscissæ. Here we see that there is a general correspondence between the two curves and we say that there is correlation between the two. One must be very careful in using the graphic method in this way. We may have a diagram in which the correspondence between the two plotted lines is very definite *except occasionally*, yet these occasional lapses may be enough to upset the correlation. Again two lines may rise and fall together in point of time, and they may even rise and fall apparently the same amounts, yet this may be an incident depending on the scales used. Finally we must not forget that this sort of correlation — where two quantities vary as a third — does not establish causality.

In Fig. 53 we have typhoid fever death-rates and population supplied with filtered water, both plotted with time as the abscissæ; and we notice that as one line goes up the other goes down, giving a sort of inverse correspondence. We are not justified however in calling this a close correlation. Certainly we are not justified in saying that one is the cause of the other. It may be true, and few will dispute the fact that the filtration of polluted water tends to reduce the typhoid fever death-rate among the consumers of the water, but such a diagram as this does not prove it. As Phelps[1] says, we might plot a line showing the increase in the number of telephones which would very much resemble that of the population supplied with filtered water. Pearson does well to call this correlation based on comparison with a "common mutual," a "*spurious correlation*." A good many false conclusions have been based on statistics treated in this way.

[1] Am. Jour. Pub. Health, 1917, p. 23.

The lag. — When two lines are plotted with the scale of abscissæ in common to both variables it often happens that one line changes in curvature after the other; it lags behind it. Sometimes this lag is very regular, sometimes it is more or less irregular. This lag does not necessarily show lack of correlation. It may, on the other hand, result from cause and effect. It is obvious that a cause must precede in time the effect produced by that cause. It may require a certain interval of time for the cause to make itself felt, and this naturally would produce a lag. For example, let us suppose that it takes ten days after a typhoid infection for the victim to "come down" with the disease; then a plotted line showing by days the number of cases of typhoid fever would lag behind a line showing infection of the water-supply, — if we can imagine such facts to be plotted. Conversely, if we had the two plotted lines we might compute the length of the incubation period of typhoid fever by measuring the lag. If the comparison is between the dates of infection and the dates of deaths from typhoid fever, then, of course, the lag is much longer as it includes not only the period of incubation, but also the run of the disease, and this is not the same for all persons.

A device sometimes used is the "set back." If we are comparing two curves, one of which is supposed to represent the cause of the other, we may plot the causal curve on the true dates and we may set back the dates of the resulting curve by an amount equal to the lag. Correlation will then be indicated by the correspondence of the curves. This presupposes that the amount of the lag is known.

In comparing lagging curves which are apparently correlative it is important to distinguish between cause and effect. As we have reiterated, it is not the function of correlation to demonstrate causality.

Fig. 52 is an example of the use of the set back. This is a correlation between typhoid fever deaths and atmospheric temperatures, the deaths being set back two months.

Coefficient of correlation and the lag. — It is possible to deal with the lag analytically instead of graphically. We may find that by comparing two series of statistics, date for date, the coefficient of correlation is low; by setting one series back a day and recomputing the coefficient we may find it higher; by setting back two days the coefficient may be higher still; and by using greater set backs the coefficient may increase to a maximum, and beyond that point it may decrease. The set back which produces the highest correlation may be taken as a measure of the lag.

All such matters as these are fully discussed in the text-books of general statistics.

Other secondary correlations. — Sometimes the secondary character of a correlation is not as clearly revealed as in the case of two plotted lines with common abscissæ. It has been noticed that poliomyelitis cases seem to follow the river valleys; what is the real correlation here? It does not appear to be a direct correlation. One says that fleas are correlated with the river valleys, and that, secondarily, the disease is correlated with the fleas; another says that the lines of transportation are along the river valleys and that the real correlation is between poliomyelitis and the contact of people incident to intercommunication.

The whole matter of correlation is almost inseparable from the science of logic.

The epidemiologist's use of correlation. — Epidemiology, a branch of medical science, is based fundamentally on the laws of cause and effect. The epidemiologist is continually searching for the cause of outbreaks of disease in order that they may be checked and future outbreaks prevented. In his studies he uses statistics continually and is of necessity

mightily interested in correlation. The successful epidemiologist must have a nose for facts, must be able to analyze these facts skillfully and draw logical conclusions from them.

The influence of a particular factor as a cause of disease is often studied by means of statistics. For example, the filtration of a public water-supply may be followed by a reduction of the typhoid fever death-rate among the water takers. This is a sort of correlation, — one change being followed by another. We know, moreover, by inductive reasoning from many such occurrences in the past and also from experimental evidence that this is a correlation which implies causality. In using this method of reasoning, however, it is important to know that the change in the water-supply was the *only* change which occurred.

There are scores of instances where this method of reasoning has been used. In Panama the abolition of the mosquito reduced the death-rate from yellow fever. The evidence points to this as a clear-cut case not only of correlation, but causality. In Panama also the malaria has been greatly reduced since the anti-mosquito work was begun. But here we find that quinine has been used as an additional preventative. In this case therefore we have had two factors changing at about the same time. From experimental evidence there is no doubt in regard to the causal relations between malaria and the Anopheles mosquito, but statistically the evidence is not as strong as in the case of yellow fever.

In some of the old studies of typhoid fever it was found that the death-rate decreased after the introduction of a sewerage system. This was accompanied by an abolition of house privies. Now it was probably the abolition of the old privies, not the building of the new sewers which produced the result. In other cases a public water-supply

was installed at the same time that the sewers were built. A reduction in the typhoid death-rate following these events may have been due to either or to both.

The fact should not be overlooked that when epidemics occur there is not infrequently more than a single factor involved. Sometimes an outbreak can be traced to a single initial case, but just as in lighting a fire the match is applied to the paper, the burning paper sets fire to the kindling and the burning kindling sets fire to the coal, so a single case may start infections which may be scattered in various ways. It is important for the epidemiologist to find all of these methods of transmission.

Sometimes the epidemiologist is obliged to base his action upon statistics which show correlation without waiting to determine whether this correlation also means causation. For example, in the recent pandemic of influenza a certain vaccine, supposed to have a prophylactic value, was used upon several hundred persons. The question arose, "Shall this vaccine be distributed and generally used?" The data first collected showed a fair degree of correlation between the use of the vaccine and apparent protection against the disease, and on the strength of this finding the vaccine was distributed. Later studies, however, failed to corroborate the correlation at first noticed, and showed that there was no causal relation between the use of the vaccine and failure of persons to take the disease. It was really a case of correlation without causation, — *post hoc non propter hoc*. And yet the health authorities, compelled to take action one way or the other, were right in basing action on the supposed correlation.

EXERCISES AND QUESTIONS

1. Is there a correlation between epidemics of poliomyelitis and rainfall? [See Am. J. P. H., Sept., 1917, p. 813.]

2. Is there a higher correlation between flies and diarrhœal diseases among children than between diarrhœal diseases and other factors? [See Am. J. P. H., Feb., 1916, p. 143, also Mar., 1914, p. 184.]

3. Is there a correlation between pneumonia and influenza? Is there a causal relation? [See Am. J. P. H., Apr., 1916, p. 316.]

4. Is there a correlation between tuberculosis and housing? [See Am. A. J. P. H., Jan. 1913, p. 24.]

5. Look up Dr. Fulton's extravaganza on the subject of statistical logic as applied to the problem of prostitution. [See Am. J. P. H., July, 1913, p. 661.]

6. Study the correlation between plague and fleas. [Am. J. P. H., Aug., 1918, p. 572.] Is there strong presumptive evidence that infantile paralysis is spread by fleas?

7. Express Mill's three canons of logic in your own words.

8. Give examples of each in the field of epidemiology.

9. What is meant by quantitative induction? What part do statistics play in this? [See Jevon's Lessons in Logic, Chap. XXIX.]

CHAPTER XIV

LIFE TABLES

To the popular mind there is something mysterious and awesome about a life table. The insurance agent, wishing to sell you a policy, asks your age, consults a printed table and tells your "expectation of life" as so many years. What does this mean and how does he arrive at this expectation of life? It does not mean that *you* will live so many years and then die. It means that it has been found in the past that most men who have attained your age have lived so many years after reaching that age. It cannot apply to everyone. *You* may live to be a hundred years old or you may die to-morrow. The future is uncertain for every individual. But the probability of your future longevity can be determined by making a statistical study of a large group of people who have attained your age, to find out the average number of years which they lived after reaching that age. Instead of using the average, *i.e.*, the *mean* we might find the *median* number of years lived, or even the *mode*. All three methods have been suggested, but that based on the mean is the one commonly used. Thus we see that there is nothing mysterious about the "expectation of life"; it has no divine origin. It is merely the application of the ordinary methods of statistics to the experience of mankind in living beyond a given age.

Probability of living a year. — Although the expectation of life is used by insurance agents to impress the prospective purchaser with the fleeting character of human life, the rates

of insurance are not based directly on this expectation, but on the probability of a person of given age living to be one year older. It is this chance of living from year to year, coupled with the growth of money at compound interest which determines what premium the insured at any age must pay. These actuarial methods are too complicated to be entered into here. In the very early days life insurance was virtually a lottery; now it is based on experience. If, as a result of better living conditions, the longevity of the insured is greater than the experience upon which the rates were based, the insurance company is the gainer because the premiums are continued for a longer time and the final payment of the policy is postponed. If the company is a so-called mutual company, the benefit of increased longevity of the insured is distributed among the policy holders in the form of rebates. But should the longevity of the insured prove to be less than the experience upon which the rates were based the opposite condition would prevail.

What is the chance of a person living from year to year? Obviously it is one minus the chance of dying. The chance of dying within one year at any age is nothing else than our old friend the specific death-rate for the given age. Thus if at age 20 the specific death-rate is 7.80 per 1000, the chance of dying within the year is 780 in 100,000, 0.0078 in 1, or 1 chance in 128; at age 50 the chance is 0.01378, or 1 in 73; at age 70 it is 0.06199 or 1 in 16; at age 80 it is 0.14447, or 1 in 7; at age 90, it is 0.45454, or 1 in 2.2.

The chance of living through the year is 1 less the chance of dying. At age 20 the chance of living through the year is 99,220 in 100,000, *i.e.*, 0.9920; at age 50 it is 0.98622; at age 70, 0.93801; at age 80 it is 0.85553; at 90 it is 0.54546. Or, to put it in another form, — at age 20 the chance of living a year is 99.2 in a hundred; at age 50, 98.6; at age 70, 93.8; at age 80, 85.5; at age 90, 54.5 in a hundred.

Thus a column showing for each age of life the probability of living a year can be made by subtracting the yearly specific death-rates from unity, and expressing the results in decimal parts of 1. We might call these specific life-rates, as they are the converse of the specific death-rates.

This specific life-rate is never used in ordinary discussion, and there is little reason for using it, as it is probably better to think in terms of specific death-rates. It is the deaths which we are always trying to postpone. A table of specific death-rates and specific life-rates would look like this.

TABLE 143

SPECIFIC DEATH–RATES AND SPECIFIC LIFE–RATES

(Abridged from the American Experience Mortality Table.)

Age.	Population alive at mid-year.	Specific death-rate per 100,000 (number dying annually).	Specific life-rate per 100,000 (number living through the year).
(1)	(2)	(3)	(4)
10	100,000	749	99,251
20	100,000	780	99,220
30	100,000	843	99,157
40	100,000	979	99,021
50	100,000	1,378	98,622
60	100,000	2,669	97,331
70	100,000	6,199	93,801
80	100,000	14,447	85,553
90	100,000	45,454	54,546

One reason why specific death-rates are not used more commonly is because people do not clearly understand them. The base, *i.e.*, 100,000 persons, remains constant for all ages. Actually the number of persons alive is constantly decreasing as age advances. One says "you start with 100,000 persons at age 10 and kill off 749 in one year, but the next year you have 100,000 again. I don't understand it."

Now life tables are definitely related to specific death-rates and they take into account this decreasing population.

Mortality tables. — In order to make a life table we may first select some large class of people and determine the specific death-rates for each year of age. We start with a certain number of people alive at a certain age. The insurance companies commonly use age 10 because most insured persons are older than that, but we might use any other age. We might use age 0, and in making a life table for a general population this would be done. As an illustration, however, let us take the American Experience Mortality Table, which starts at age 10 and which is limited to males. Another reason for taking age 10 is that it is a round number not far from the age at which the specific death-rate is the lowest.

For convenience we start with 100,000 as a round number of persons alive at age 10. This number is called the *radix* of the computation. We might use a million or a thousand, but the former is hardly warranted by the precision of our specific death-rates, while the latter gives too many decimals.

In the table, column (1) gives the age, and column (5) the corresponding specific death-rates obtained from the original data. In column (2) we start with 100,000 persons alive at age 10, of these

92,637	lived to age		20
85,441	"	"	30
78,106	"	"	40
69,804	"	"	50
57,917	"	"	60
38,569	"	"	70
14,474	"	"	80
847	"	"	90
0	"	"	96

These figures were obtained as follows: — 100,000 were alive at the beginning of age 10 and 749 per 100,000 died

TABLE 144

AMERICAN EXPERIENCE MORTALITY TABLE

Age.	Number living.	Number dying.	No. of years expectation of life.	No. dying of each 100,000 annually.	Age.	Number living.	Number dying.	No. of years expectation of life.	No. dying of each 100,000 annually.
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
10	100,000	749	48.72	749	53	66,797	1,091	18.79	1,633
11	99,251	746	48.08	752	54	65,706	1,143	18.09	1,740
12	98,505	743	47.45	754	55	64,563	1,199	17.40	1,857
13	97,762	740	46.80	757	56	63,364	1,260	16.72	1,988
14	97,022	737	46.16	760	57	62,104	1,325	16.05	2,133
15	96,285	735	45.50	763	58	60,779	1,394	15.39	2,294
16	95,550	732	44.85	766	59	59,385	1,468	14.74	2,472
17	94,818	729	44.19	769	60	57,917	1,546	14.10	2,669
18	94,089	727	43.53	773	61	56,371	1,628	13.47	2,888
19	93,362	725	42.87	776	62	54,743	1,713	12.86	3,129
20	92,637	723	42.20	780	63	53,030	1,800	12.26	3,394
21	91,914	722	41.53	785	64	51,230	1,889	11.67	3,687
22	91,192	721	40.85	791	65	49,341	1,980	11.10	4,013
23	90,471	720	40.17	796	66	47,361	2,070	10.54	4,371
24	89,751	719	39.49	801	67	45,291	2,158	10.00	4,765
25	89,032	718	38.81	806	68	43,133	2,243	9.47	5,200
26	88,314	718	38.12	813	69	40,890	2,321	8.97	5,676
27	87,596	718	37.43	820	70	38,569	2,391	8.48	6,199
28	86,878	718	36.73	826	71	36,178	2,448	8.00	6,766
29	86,160	719	36.03	834	72	33,730	2,487	7.55	7,373
30	85,441	720	35.33	843	73	31,243	2,505	7.11	8,018
31	84,721	721	34.63	851	74	28,738	2,501	6.68	8,703
32	84,000	723	33.92	861	75	26,237	2,476	6.27	9,437
33	83,277	726	33.21	872	76	23,761	2,431	5.88	10,231
34	82,551	729	32.50	883	77	21,330	2,369	5.49	11,106
35	81,822	732	31.78	895	78	18,961	2,291	5.11	12,083
36	81,090	737	31.07	909	79	16.670	2,196	4.74	13,173
37	80,353	742	30.35	923	80	14,474	2,091	4.39	14,447
38	79,611	749	29.62	941	81	12,383	1,964	4.05	15,860
39	78,862	756	28.90	959	82	10,419	1,816	3.71	17,430
40	78,106	765	28.18	979	83	8,603	1,648	3.39	19,156
41	77,341	774	27.45	1,001	84	6,955	1,470	3.08	21,136
42	76,567	785	26.72	1,025	85	5,485	1,292	2.77	23,555
43	75,782	797	26.00	1,052	86	4,193	1,114	2.47	26,568
44	74,985	812	25.27	1,083	87	3,079	933	2.18	30,302
45	74,173	828	24.54	1,116	88	2,146	744	1.91	34,669
46	73,345	848	23.81	1,156	89	1,402	555	1.66	39,586
47	72,497	870	23.08	1,200	90	847	385	1.42	45,454
48	71,627	896	22.36	1,251	91	462	246	1.19	53,247
49	70,731	927	21.63	1,311	92	216	137	0.98	63,426
50	69,804	962	20.91	1,378	93	79	58	0.80	73,418
51	68,842	1,001	20.20	1,454	94	21	18	0.64	85,714
52	67,841	1,044	19.49	1,539	95	3	3	0.50	100,000

during the year. Consequently, the number alive at age 11 was $100{,}000 - 749 = 99{,}251$. In the next year the specific death-rate was 752 per 100,000. The number dying was, therefore, $99{,}251 \times \frac{752}{100{,}000} = 746$, and the number alive at age 12 was $99{,}251 - 746 = 98{,}505$. And so on. The number dying each year is given in column (3), the number living in column (2). At age 96 all were dead.

These are the facts of the case, now how shall we use them? There are three ways, which correspond to the mode, the median and the mean, and they are called respectively the "most probable life-time," the "Vie Probable," and the "Expectation of Life."

The "most probable life-time." — The figures in column 3 form a frequency curve, the mode of which is 2505. There are more deaths at age 73 (*i.e.*, age 73–74) than at any other age. 73 is the fashionable, modish age to die. The chance of dying at that age is greater than at any other age.

The difference between a given age and 73 years is called "the most probable life-time." At age 10, it is $73 - 10 = 63$; at age 20 it is 53; and so on. Above the age 73 the "most probable life-time" becomes a negative quantity, and this is the objection to the use of this computation. It is applicable only to the first part of the frequency curve.

The "Vie Probable." — The "Vie Probable" is the number of years which a person (at a stated age) has an even chance of living. It is the difference between a given age and the age at which the number of persons alive is one-half the number alive at the given age. The latter is the median age to which the persons who passed the given age lived.

At age 10 there are 100,000 persons alive. One-half of this number, *i.e.*, 50,000, are alive at age $64.5\pm$. Hence $64.5 - 10 = 54.5$ is the "vie probable." In this period of time the chance of living or dying is just even.

At age 20, there are 92,637 persons alive. One-half of this number, *i.e.*, 46,318, were still alive at age 66.5. Hence the "vie probable," for age 20 is 66.5 − 20 = 46.5 years.

The "Expectation of Life." — The "Expectation of life" means the average number of years that persons of a given age will probably survive. It is obtained by finding the average of the lengths of life of all the persons who lived beyond the given age.

Thus of the 100,000 alive at age 10, 3 lived to the age of 95, that is, they lived for (95 − 10 =) 85 years after the age of 10. 21 lived to age 94, *i.e.*, 84 years. But these 21 include the 3 who lived to age 95, so there were 18 who lived 84 years. 79 lived 83 years, but these include both the 3 and the 18, so in addition to them (79 − 21 =) 58 lived 82 years. And so on. The weighted averages of all of these lives gives what is called the expectation of life. These results are given in column (4).

In obtaining the figures for column (4) it is most convenient to begin at the higher ages and work backward.

At the beginning of age 95, there were 3 persons alive; at the end there were none alive. Not knowing at what part of the year they died the best assumption is that they died (on an average) at the middle of the year, *i.e.*, they lived one-half year. Hence at age 95, the average length of the lives was $\frac{3 \times \frac{1}{2}}{3} = 0.50$ year. This is the expectation of life at age 95.

At age 94, 21 persons were alive. 3 of these lived $1\frac{1}{2}$ years each; the other 18 died within the year, and may be said to have lived one-half year. Hence we have:

$$\begin{array}{rl} 3 \times 1.5 = & 4.5 \\ \underline{18} \times 0.5 = & \underline{9.0} \\ 21 & 13.5 \end{array} \quad \text{and } 13.5 \div 21 = 0.64 \text{ yr.}$$

Hence at age 94 the expectation of life is 0.64 year.

At age 93 we have:

$$\begin{array}{rl} 3 \times 2.5 = & 7.5 \\ 18 \times 1.5 = & 27.0 \\ \underline{58} \times 0.5 = & \underline{29.0} \\ 79 & 63.5, \text{ and } 63.5 \times 79 = 0.80 \text{ year.} \end{array}$$

In this way we find that at age 10, the average number of years lived by those who passed age 10, was 48.72 years. At age 20 it was 42.20 years; at age 30 it was 35.33 years, etc.

Comparison of the three results. — The U. S. Life Tables for 1910 give the "complete expectation of life" (computed on the basis of the mean), and from the tables may be obtained the "most probable life-time" (based on the mode) and the "vie probable" (based on the median). The following figures give the results for age zero, that is, they show the expectation of life at birth.

TABLE 145

COMPARISON OF "EXPECTATION OF LIFE," "VIE PROBABLE" AND "MOST PROBABLE LIFE-TIME"

Original registration states.	Expectation of life. (Mean.)	"Vie probable."[1] (Median.)	Most probable[1] life-time. (Mode.)
(1)	(2)	(3)	(4)
White males	50.23	59.30	74.0
White females	53.62	63.27	73.5
Negro males	34.05	34.85	59.5
Negro females	37.67	40.58	65.5
White males in cities	47.32	55.00	68.5
White males in rural part	55.06	65.33	76.5
White females in cities	51.39	60.73	71.5
White females in rural part	57.35	67.38	76.5
Males in Massachusetts	49.33	58.82	69.5
Females in Massachusetts	53.06	62.74	74.5

[1] Approximate.

Life tables based on living population. — Life tables are usually computed in another way. They are based on the population living at each age as shown by the census returns or by data collected by the insurance companies. Thus we may assume that the figures in column (2) have been obtained in this way. If we start with 100,000 persons alive at age 10 and find that 99,251 were alive at age 11 then the number of deaths during the year must have been 100,000 − 99,251, or 749. Between ages 11 and 12 the deaths were 99,251 − 98,505, or 746; and so on. By this method we compute the deaths, and we may also compute the specific death-rates for each age. This method justifies the use of the term *life tables*, as the results are based on the living and not on the dying. It is obvious that migrations of population interfere somewhat with this method. It is obvious, also, that concentrations of population on the round numbers present another difficulty. As a matter of practice the ragged data must be smoothed out before a life table can be constructed; otherwise the computed expectations of life would themselves be erratic. These errors of round numbers creep into the computations of specific death-rates, so that in any case it is necessary to do a certain amount of "smoothing" before computing life tables. One method commonly used is that known as "osculatory interpolation," which may be found described in such books as Vital Statistics Explained, by Burn.

Still another method of computing a life table is to base it wholly on the distribution of deaths, making use of certain mathematical formulas for frequency curves.[1]

Mathematical formula for computing the expectation of life. — There is a mathematical formula for the computation of the expectation of life by the use of which the labor may be shortened. It is usually stated as follows:

[1] Arne Fisher. Note on the Construction of Mortality Tables by means of Compound Frequency Curves. Proc. Casualty Actuarial and Statistical Society of America, Vol. IV, Pt. 1, No. 9.

$$\overset{\circ}{e}_x = \frac{\frac{1}{2} l_x + l_{(x+1)} + l_{(x+2)} + l_{(x+3)} + \cdots}{l_x} = \frac{1}{2} + Q.$$

$$Q = \frac{l_{(x+1)} + l_{(x+2)} + l_{(x+3)} + \cdots}{l_x}.$$

$\overset{\circ}{e}_x$ = expectation of life, in years, at age x.
l_x = number of persons living at age x.
$l_{(x+1)}$ = number of persons living at age $x + 1$.
$l_{(x+2)}$ = number of persons living at age $x + 2$, etc.

For a more detailed description of these methods the reader is referred to such books as United States Life Tables, 1910, Bureau of the Census, prepared by Prof. James W. Glover and published in 1916; Life Assurance Primer, by Henry Moir; Vital Statistics, by Newsholme; Mortality Laws and Statistics, by Robert Henderson.

Early history of life tables. — It is not surprising that most of the life tables which have been computed have been confined to males of insurable age. Halley, the British astronomer, famous for the comet which bears his name, was the first to use the method. This was in 1692 and related to the town of Breslau. Other famous tables are the Northampton Table of 1762, the Carlisle Table of 1815 and Dr. Farr's English Table of 1851.

In 1843 seventeen American insurance companies combined their experiences and published a table known as the Actuaries or Combined Experience Table. It was based on 84,000 policies. The American Experience Table of Mortality, now recognized by the insurance companies as the standard for America, was formed by Sheppard Homans in 1868. It is supposed to have been based on the experience of the Mutual Life Insurance Company of New York.

In 1869 the H^M Table was published in England. H^M means Healthy Males. It was based on 180,000 policies. Then there is an O^M Table (ordinary life, males) based on over 400,000 lives. This is the Canadian standard.

Recent life tables. — In 1898 Dr. Samuel W. Abbott published in the annual report of the Massachusetts State Board of Health for that year,[1] a life table for Massachusetts. This is one of our best American papers on the subject.

Dr. Guilfoy, the statistician of the New York City Board of Health, has published the following interesting comparison between the expectations of life in 1879–81 and 1909–11. The changes which have taken place during the interval are striking. The figures are as follows:

TABLE 146

APPROXIMATE LIFE TABLES FOR THE CITY OF NEW YORK BASED ON MORTALITY RETURNS FOR THE TRIENNIA 1879–1881 AND 1909–1911

Years of mortality, ages.	Expectation of life, 1879 to 1881.			Expectation of life, 1909 to 1911.			Gain (+) or loss (−) in years of expectancy.		
	Males.	Females.	Persons.	Males.	Females.	Persons.	Males.	Females.	Persons.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
−5	39.7	42.8	41.3	50.1	53.8	51.9	+10.4	+11.0	+10.6
5	44.9	47.7	46.3	49.4	52.9	51.1	+ 4.5	+ 5.2	+ 4.8
10	42.4	45.3	43.8	45.2	48.7	46.9	+ 2.8	+ 3.4	+ 3.1
15	38.2	41.2	39.7	40.8	44.2	42.5	+ 2.6	+ 3.0	+ 2.8
20	34.4	37.3	35.8	36.6	40.0	38.3	+ 2.2	+ 2.7	+ 2.5
25	31.2	34.0	32.6	32.7	36.0	34.3	+ 1.5	+ 2.0	+ 1.7
30	28.2	31.0	29.6	28.9	32.1	30.5	+ 0.7	+ 1.1	+ 0.9
35	25.3	28.1	26.7	25.4	28.4	26.9	+ 0.1	+ 0.3	+ 0.2
40	22.5	25.2	23.9	22.1	24.7	23.4	− 0.4	+ 0.5	− 0.5
45	19.8	22.4	21.1	18.9	21.1	20.0	− 0.9	− 1.1	− 1.1
50	17.2	19.4	18.3	15.9	17.7	16.8	− 1.3	− 1.7	− 1.5
55	14.5	16.4	15.4	13.2	14.6	13.9	− 1.3	− 1.8	− 1.5
60	12.2	13.8	13.0	10.8	11.8	11.3	− 1.4	− 2.0	− 1.7
65	9.9	11.2	10.5	8.8	9.4	9.1	− 1.1	− 1.8	− 1.4
70	8.5	9.3	8.9	6.9	7.5	7.2	− 1.6	− 1.8	− 1.7
75	7.1	7.5	7.3	5.3	5.7	5.5	− 1.8	− 1.8	− 1.8
80	6.2	6.5	6.4	4.1	4.5	4.3	− 2.1	− 2.0	− 2.1
85+	5.4	5.5	5.5	2.0	2.4	2.2	− 3.4	− 3.1	− 3.3
Balance							+24.8	+28.7	+26.6
							−15.3	−17.6	−16.6
							+ 9.5	+11.1	+10.0

[1] See State Sanitation, Vol. II, p. 300, by G. C. Whipple.

United States life tables. — In 1916 the Bureau of the Census published a special report entitled United States Life Tables, 1910, prepared under the direction of Prof. James W. Glover of the University of Michigan. This was the first report of its kind in America. The tables are based on the general unselected population, and, therefore, differ from the life tables of the insurance companies. The radix is 100,000 at age 0. The data were obtained from the U. S. Census of 1910. Expectations of life are computed by months up to one year of age, and after that by years up to age 106. Separate tables are given for males, for females and for both sexes combined; there are separate tables also for negroes and whites, and for native and foreign born whites; for cities and for rural districts, — all of these relating to the population of the original registration states, namely, the New England states, New York, New Jersey, Indiana, Michigan and the District of Columbia. Separate tables for males and for females are given for the states of Indiana, Massachusetts, Michigan, New Jersey and New York.

These tables are well prepared and their results are of much interest. Besides giving the expectations of life computed in the usual way, computations are made on the assumption of a stationary population, that is one where the general death-rate is equal to the general birth-rate. These have the advantage of excluding the effect of emigration results and immigration, and from them one can compare the death-rates of different communities for the population above a given age. For these results the reader is referred to the original report.

A few comparisons. — It will be interesting to make a few comparisons of the expectations of life at certain ages for different classes of people and at different ages. For greater details the reader should consult Professor Glover's report.

TABLE 147

EXPECTATIONS OF LIFE, 1910

Age. Original registration states.	0	10	20	30	40	50	70
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Native white males	50.58	51.93	43.32	35.61	28.33	21.20	9.09
Native white females	54.19	54.43	45.76	37.98	30.33	22.78	9.80
Foreign-born white males	.....	50.30	41.75	33.71	26.03	19.08	8.40
Foreign-born white females	.....	52.24	43.50	35.31	27.55	20.09	8.67
Negro males	34.05	40.65	33.46	27.33	21.57	16.21	8.00
Negro females	37.67	42.84	36.14	29.61	23.34	17.65	9.22
White males in cities	47.32	49.13	40.51	32.61	25.32	18.59	8.14
White males in rural part	55.06	54.53	45.92	38.10	30.20	22.43	9.36
White females in cities	51.39	52.22	43.51	35.52	27.88	20.53	8.99
White females in rural part	57.35	55.54	46.86	39.05	31.15	23.27	9.76
Males in Indiana	54.70	53.91	45.44	37.76	29.99	22.38	9.29
Males in Michigan	53.86	54.09	45.57	37.76	29.81	22.10	9.17
Males in Massachusetts	49.33	51.14	42.48	34.55	26.97	19.79	8.58
Males in New Jersey	49.08	50.31	41.66	33.86	26.57	19.67	8.65
Males in New York	47.89	49.40	40.79	33.01	25.88	19.28	8.58

The greater longevity of females as compared with males is evident throughout the tables. It is greater for native whites than for foreign born whites, greater for whites than for negroes, greater for rural districts than for cities. The differences between the states depend upon differences in the composition of the population, and upon urban and rural conditions.

It is interesting also to compare the specific death-rates for the corresponding ages. The relations between these and the expectations of life are in a general way reciprocal. The specific death-rates are lower in the rural districts than in the cities, especially in the early and the later years; in middle life there is less difference. The differences between whites and negroes are very striking.

TABLE 148

SPECIFIC DEATH-RATES

Age. Registration area.	0	10	20	30	40	50	70
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Native white males	126.02	2.37	4.82	7.14	10.02	21.20	57.20
Native white females	104.60	2.06	4.40	6.13	7.76	11.68	50.24
Foreign-born white males	......	2.47	5.10	5.80	10.53	17.92	70.79
Foreign-born white females	......	2.09	3.65	5.84	8.55	14.42	67.87
Negro males	219.35	5.02	11.96	14.96	21.03	31.42	83.98
Negro females	185.07	5.18	10.74	12.02	17.50	25.52	71.27
White males in cities	133.80	2.59	4.93	7.22	12.10	19.17	74.20
White males in rural part	103.26	2.07	4.83	5.39	7.06	10.65	52.93
White females in cities	111.23	2.23	4.10	6.33	8.83	14.44	63.50
White females in rural part	84.97	1.80	4.41	5.46	6.65	9.91	49.92

EXERCISES AND QUESTIONS

1. Compare the life table for New Haven with that for the U. S. Registration Area. [See Am. J. P. H., Aug., 1918, p. 580.]

2. Compute a life table for some city, to be assigned by the instructor.

3. Find your own "probability of living a year," "vie probable," "most probable life-time," and "expectation of life."

CHAPTER XV

A COMMENCEMENT CHAPTER

This last chapter is to be something like the day after college commencement. On the day before the student regards his work as finished; his exercises are all completed, he has passed his examinations, he is to be graduated. But on the day after commencement he finds himself plunging into a world of problems yet unsolved; he sees that most of the things he is called upon to do were not in his curriculum; that he must learn to do these things for himself. Little by little he comes to realize that what his stupid old professors had been trying to do was not to tell him all there was to know in the world but to teach him how to think and how to use tools. He had heard much of principles, and laws and formulæ and synopses and all that, and had regarded them as the dry parts of his courses — the necessary evils. But little by little he finds that these general principles, these almost self-evident ideas, help him to solve his problems; that his systematic methods of going at a thing help him to do his work more easily and quickly; that by following the dry old laws of logic, his conclusions are somehow better than those of the other fellow who does not take the trouble to see that all the steps in the problem are "necessary and sufficient." In short, he comes to realize that his education has enabled him to do his work easier and better and has given him intellectual confidence. If it doesn't do this for him he has wasted his opportunities in college.

In the preceding chapters of this book the author has endeavored to place the emphasis not on the subject matter but on methods of procedure, to outline the simpler principles of the statistical method as applied to studies in demography, to warn against the common fallacies which so often creep into discussions of vital statistics, and to urge students and health officers not to be content with such things as general rates but to seek the answers to their problems by methods of statistical analysis and the use of specific rates and ratios.

Let us now take an outlook upon some of the problems of demography as they come piling in upon the health officer from day to day. And if, for convenience' sake, we take them at random, one after the other, without order or system we shall simulate more nearly every-day practice. If we can solve this and that problem or if we can see the steps in the solution we shall know that we have acquired the use of the tools of the statistician, and will have confidence in our own studies. This chapter will also include certain subjects which have not logically found a place in the preceding chapters. Several of these subjects might easily be expanded into chapters of their own.

Military statistics. — In general the vital statistics of armies are computed in the same way as those of civil populations, but instead of using the mid-year estimated population, the mean strength for the year is used as a basis of rates. An army does not increase in numbers as a population grows, slowly by geometrical progression, but is kept up to a fairly constant strength or is suddenly increased or decreased according to demands made upon it. An army represents a selected population, — males between certain age limits, and above set standards of health and physique. Rates computed for armies are therefore specific rates and they must not be compared with general

rates. The health of the soldiers is carefully looked after by the surgeons, who are obliged to keep records; hence the morbidity records are more complete than in the case of the civil population.

Since 1894, when an international commission for the unification of medical statistics met at Budapest, tables of statistics made up according to certain schedules have been published for most armies. These may be found in the annual reports of the Surgeon General of the U. S. A. In the report for 1916 we find that in the entire U. S. army of 93,262 enlisted men in 1915 the sick admissions "to quarters" and "to hospitals" amounted to 745 per 1000. This does not mean 745 different men, for sometimes the same man was admitted more than once. Of these 96 per cent returned to duty, *i.e.* recovered, 0.65 per cent died, and 3.4 per cent were "otherwise disposed of." The death-rate for the mean strength was 4.6 per 1000. The annual number of days lost through sickness was 9.44 for each soldier, or 12.7 for each "admission." In the published tables the figures are classified according to the location of the troups, the arms of the service, the season, the larger garrisons, and according to the cause of the sickness or death. It should be observed that in the international tables for the army the international list of diseases, as given on page 257, is not followed. The Surgeon General of the United States uses it, however, in the body of his report.

In 1915 in the entire U. S. army (103,842 officers and enlisted men) the following were the rates per 1000 of mean strength:

TABLE 149

VITAL STATISTICS OF U. S. ARMY: 1915

	Death-rates per 1000.		
	From disease.	From injury.	Total.
(1)	(2)	(3)	(4)
Admissions	597.0	129.2	726.2
Discharged on certificate of disability	12.6	1.4	14.0
Died	2.5	1.9	4.4
Total losses	15.1	3.3	18.4

The percentage of soldiers constantly non-effective was 2.5 per cent.

If we look back a few years we find that the health of the army has been improving.

TABLE 150

HOSPITAL ADMISSION RATES AND PERCENTAGE OF NON-EFFECTIVES, U. S. A.

Year.	Admission-rate per 1000	Non-effectives, per cent.
(1)	(2)	(3)
1906	1118	4.8
1907	1102	4.4
1908	1079	4.2
1909	964	4.1
1910	870	3.5
1911	858	3.2
1912	806	2.9
1913	666	2.4
1914	660	2.4
1915	726	2.5
1916	597	2.5

Army diseases. — In the consideration of army diseases one must distinguish between peace times and war times; one must also distinguish between the diseases which cause death and those which render the men non-effective.

In 1915 the specific death-rates among the American enlisted men in the U. S. A. were, in order of their importance, as follows:

	Per 100,000
Tuberculosis	33
Pneumonia (lobar)	31
Organic heart disease	23
Measles	23
Appendicitis	13
Epidemic cerebro-spinal meningitis	11

The principal causes of discharge were:

	Per 1000
Mental alienation	3.30
Tuberculosis	1.79
Flat foot	1.25
Venereal disease	0.82
Epilepsy	0.69
Organic heart disease	0.50

The admission and non-effective rates for white enlisted men were:

TABLE 151

ADMISSION RATES AND PERCENTAGE OF NON-EFFECTIVES FROM PARTICULAR DISEASES, U. S. A.

	Admission rate, per 1000	Non-effectives, per cent.
(1)	(2)	(3)
Venereal diseases	106	0.47
Tuberculosis	3	0.17
Mental alienation	4	0.09
Bronchitis	35	0.06
Tonsilitis	47	0.07
Appendicitis	9	0.06
Malaria	24	0.05
Mumps	10	0.05
Influenza	35	0.05
Diarrhœa and enteritis	32	0.04
Measles	7	0.05
Articular rheumatism	6	0.04
Hernia	4	0.04

In war times we have to consider the venereal diseases, syphilis, gonorrhœa, etc.; the diarrhœal diseases, typhoid fever, cholera, dysentery; the insect-borne diseases, typhus fever, relapsing fever, trench fever, malaria, etc.; scurvy — besides all sorts of diseases associated with wounds. No attempt will be made here to discuss these war diseases, because the Great War will yield statistics better and more complete than any which we now have. Some day it will be in order to make comparisons between the Civil war, the Spanish war and the present Great War. We shall then see what enormous strides have been taken in sanitation, in the use of antitoxins, in providing proper food, in the enforcement of the rules of personal hygiene, in the treatment of the sick and wounded, in the ambulance and hospital service, in the protection of the health of the civil population in war time in factory and home. One

gratifying result of the war seems assured — a world-wide up-lift in public health. We shall hereafter need world-wide vital statistics, that is, we shall need the science of demography.

Effect of the Great War on demography. — A thousand and one questions have arisen as a result of the war.

What are we to do with the enormous number of non-resident males in the United States? How are we to compute death-rates? Will our usual methods have to be modified as an emergency measure?

What effect has the war had on the marriage-rates, birth-rates and death-rates? A big hole is sure to be made in the male population for the ages of youth and early manhood; fewer young men of twenty in 1920 will mean fewer men of thirty in 1930 and fewer men of forty in 1940. How will this alter the general death-rate? Will the birth-rate rise as a natural reaction to war's destruction or will hard economic conditions keep it low? Can we learn anything from past wars on this matter?

Typhoid fever, the past scourge of armies, has been almost completely conquered. Will the venereal diseases also be conquered? Will the Great War point out the way to this end?

What has been the effect of reduced food rations on health and physique? Will the loss of the most vigorous young men lower the standards of physique by hereditary influences?

Will the lessons in hygiene and sanitation be so well learned that their benefits will offset other baneful influences?

We knew approximately the standing of the nations before the war as to population, natural rates of growth, migrations, death-rates, and so on — how will these nations stand after the war? Who will be the greatest losers? What will be their most serious losses?

Such questions as these force themselves upon us. Demography will be the science looked to for the answers.

Hospital statistics. — There are many hospitals in the country and they are an increasingly important factor in the control of disease. Some of these hospitals keep good records of their cases and some publish them. Other hospitals keep very inadequate records and publish nothing. Uniformity in this matter is most desirable, as a good opportunity for collecting facts in regard to certain non-reportable diseases and in regard to the fatality of these diseases is being lost.

Several plans for unifying hospital statistics have been suggested. Dr. Charles F. Bolduan,[1] of the New York City Health Department, suggested the idea of a discharge certificate, to be filled out for each case on leaving a hospital, — a certificate comparable to the ordinary death certificate. Another method is to have the annual reports (or monthly reports) made out on some fixed schedule of statistics and submitted to some central authority.[2] Perhaps the U. S. Public Health Service may some day take the lead in the collection of the important data to be secured from hospitals. See also page 471.

Statistics of industrial disease. — Statistical studies of industrial diseases are becoming increasingly numerous. It is a most complex and difficult branch of the subject. At the outset we are met with the fundamental difficulty of defining occupations. The extent of this difficulty may be appreciated from the fact that in 1915 the U. S. Bureau of the Census published an "Index to Occupations" which covered over four hundred pages and included 9000 occupational designations. The report makes 215 main classes, 84 of which are subdivided. This list has been given in Chapter VIII.

[1] N. Y. Medical Journal, Mar. 29, 1913.
[2] Amer. Jour. Pub. Health, Apr., 1918.

A second difficulty is due to the migration of laborers from place to place, and from one class to another. A third, which grows out of the other two, is the difficulty of getting constant, well-defined classes to serve as the basis of the computation of rates and ratios. A fourth is the oft repeated error of concealed classification. These and other minor difficulties have compelled us to resort to the use of specially gathered statistics, which are often not truly representative of the conditions discussed.

For example, the Massachusetts General Hospital recently made a study of lead poisoning in its Industrial Clinic. During the first year of this clinic 148 cases of lead poisoning were diagnosed in the hospital as against 147 during the previous five years.

This was found by sifting out of the hospital admissions by a trained worker those suspected of being exposed to special industrial hazard. A study of these 148 cases gave an industrial distribution as follows:

TABLE 152

Occupation.	Number exposed.	Number cases.	Per cent poisoned.
(1)	(2)	(3)	(4)
Painters	217	68	31
House	....	56	....
Others	....	12	....
Shipyard and navy yard	54	16	30
Rubber workers	169	11	7
Brass foundrymen	9	4	44
Lead and lead oxide worker	....	6	....
Plumbers	42	8	19
Printers	64	11	17
Miscellaneous	135	14	10
Non-industrial	....	10	....
Total	....	148	....

An attempt to ascertain the rate of attack was made by ascertaining as well as possible the number of persons exposed. These rates are, of course, far too high; 31 per cent of all painters did not get lead poisoning, but only 31 per cent of the exposed persons who were sorted out in this industrial clinic. The report does not err in this respect but the reader may get a false impression unless he reads thoughtfully. The underlying idea of this clinic is excellent and the work, unfortunately interrupted, was already yielding excellent results. The danger of lead poisoning of men engaged in certain occupations in ship yards was clearly shown.

Economic conditions and health. — Poverty and disease mutually influence each other. We cannot expect to solve the problem by attacking either alone. It is most difficult to separate cause from effect. In fact, there is a third major factor which we may call ignorance — and all three are mutually dependent. Then there are many minor factors.

We can correlate these things by statistics, and that is worth while because it calls attention to the problems; but the plan of attack must rest upon the fact that the different conditions are mutually related. If we help only a little to raise the economic and hygienic conditions the result is an accelerating social advance; to aid one without the other does not bring about permanent betterment.

A glimpse at these mutual relations, as shown by Warren and Sydenstricker,[1] is instructive. They classified the health of certain garment workers with respect to the annual earnings of the heads of families as follows:

[1] Pub. Health Reports, May 26, 1916, p. 1298.

TABLE 153

HEALTH OF GARMENT WORKERS

	Annual earnings.		
	$500	$500-$699	$700
(1)	(2)	(3)	(4)
Number of persons	381	581	462
Ave. annual earnings	$382	$577	$866
Ave. rate of weekly earnings	$19	$23	$27
Per cent which actual earnings were of maximum possible earnings	38%	48%	61%
Maximum possible earnings for year	$988	$1196	$1404
Ave. number of persons per family	5.36	5.38	4.88
Ave. number of children born per family	3.78	3.34	2.75
Ave. number of children living per family	2.99	2.78	2.43
Ave. number of children dead per family	0.78	0.56	0.32
Infant mortality rate	206.9	167.2	116.5
Per cent of male married garment workers who were poorly nourished	25.00	15.02	12.72
Ave. hæmoglobin index, Talquist	85.94	86.99	87.35
Per cent with hæmoglobin index under 80	9.94	5.65	4.42
Per cent of family heads tuberculous	5.64	5.30	0.44

Accidents and accident-rates. — Injuries and deaths from accidental causes are attracting much attention nowadays, and rightly so. The death-rate from accidents in the United States is far greater than from typhoid fever. Only a few years ago it was more than 100 per 100,000 of population. Some of the principal causes are railroad accidents, falls, drowning and burns, but there are many accidents associated with different industries. All of these present interesting problems for study and each should be studied by itself.

Taking accidents as a general class, we find that the

specific death-rates follow closely the death-rates from all causes, decreasing from the first year to a minimum between ages 10–14 and then increasing steadily to the highest ages. Owing to the age distribution of population we find the mode of the accident distribution curve occurring somewhere in age group 25–29 years.

In the case of railroad accidents among males the mode is found in age-group 25–29 years, that is, the largest number of accidents occurs among males at that period; in the case of falls the mode is in age-group 45–49; in the case of drowning it is at age 20–24. The specific death-rate from railroad accidents is low until the age of twenty, when it rises to above 30 per 100,000 and fluctuates between 30 and 50 for all higher age-groups. The specific death-rate from falls rises steadily from the tenth year and above 75 years of age exceeds 100 per 100,000. The specific death-rate from drowning on the other hand is highest at about twenty years of age. Except for falls the accident-rates from the major causes are higher for males than for females.

If time permitted it would be interesting to follow up this subject of accidents and find the seasonal distribution and classify them in other ways.

In studying accidents in industrial establishments we must ask the usual questions, — where, when, what, how, who, and answer them by collecting the necessary statistics. It does not do to follow popular impressions in these matters. Thus it is sometimes said that most accidents occur "at the end of a tired day," yet statistics collected in Massachusetts by the Industrial Accident Board showed that it is between 9 and 10 A.M. and 2 and 3 P.M. that accidents are most frequent. Yet this general statement is not enough. We need to know what kinds of accidents are meant. Perhaps some kinds of accidents do occur at the end of the working day. Then there are daily differences to be con-

sidered, and seasonal differences, as well as differences due to the weather. In the case of the English munition factories, which run night and day, the accident mode occurs in the evening. One runs a great risk in generalizing from composite statistics.

There are various ways of expressing accident rates. One is the ratio between annual accidents and number of employees. Another is between annual accidents and the number of full time workers, *i.e.*, 300 days per year. Another is between days lost through accident and full time workers. Differences in the severity of the accidents are also important from an economic point of view.

Age distribution of cases of poliomyelitis. — One of the diseases which has recently attracted attention is Anterior Poliomyelitis, commonly known as infantile paralysis. Many attempts have been made to correlate the occurrences of this disease with factors which might point to the manner of its communicability. There is an excellent opportunity here for original statistical work based on recently accumulated data. As bearing on the theory of contact as a major element in its communicability the age distribution of the cases is important. The disease is essentially one of the early ages. A recent study by the author appears to indicate that the median age is inversely proportional to the density of population. This is likewise true for measles, whooping cough and similar diseases.

It has been noticed that if the cases of poliomyelitis are plotted on logarithmic probability paper they tend to fall on a straight line, except that above the upper decentile there is an irregular divergence from the straight line. From this diagram it is easy to read off the median age or the per cent of cases below any age or between given ages. Fig. 63 shows that in the populous city of New York the median age was 2.5 years, in Boston 3.7 years,

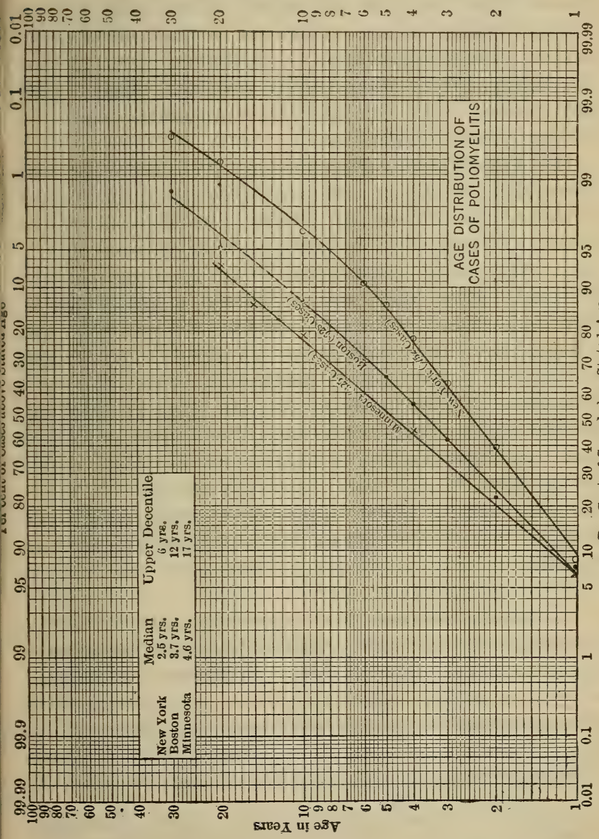


FIG. 63. — Example of Plotting on Logarithmic Probability Paper.

and in Minnesota 4.6 years. Similar differences were observed in the upper decentiles. These data are not strictly comparable as they were not for the same year and are presented merely to show the advantage of this method of plotting. It is interesting to note that scarlet fever cases plotted by ages on logarithmic probability paper also fall nearly on a straight line.

The Mills-Reincke Phenomenon. — Problems like this offer excellent opportunities to apply the principles of statistics. In 1893–94 Mr. Hiram F. Mills found that at Lawrence, Mass., after the introduction of the sand filter to purify the public water-supply taken from the polluted Merrimac River, there was a material reduction in the general death-rate of the city. Notably typhoid fever was reduced, but this reduction was not sufficient to account for the fall in the general death-rate. About the same time Dr. J. J. Reincke found the same thing in Hamburg. In 1904 Hazen studied these and other records and stated that "where one death from typhoid fever had been avoided by the use of better water, a certain number of deaths, probably two or three, from other causes have been avoided." In 1910 Sedgwick and MacNutt[1] published an elaborate study in which Hazen's statement was dignified with the rank of "theorem."

The natural inference from such statements is that the purification of a polluted water-supply reduces deaths from causes other than typhoid fever. In Lawrence if one considers short periods before and after the introduction of the filter a decrease is observed in several diseases, — as, for example, pneumonia, tuberculosis, cholera infantum and so on. Some have, without sufficient thought, extended the

[1] Sedgwick, W. T. and J. Scott MacNutt. On the Mills-Reincke Phenomenon and Hazen's Theorem. Jour. Infectious Diseases, Aug., 1910, pp. 489–564.

idea back of Hazen's "theorem" to undue limits, and have argued that pure water has the effect of raising the general health, of lifting the health tone of individuals, and so has a value beyond that of preventing the spread of diseases of the intestinal tract. This is unwarranted and to that extent Dr. Chapin[1] has rightly criticized the "theorem." The idea may be correct, but the vital statistics available do not demonstrate it. The correlation between the decreased typhoid-fever rate and the general death-rate in cities which have introduced water filtration or otherwise bettered their supply is not high. It is more frequently true where the original water-supply has been very badly polluted, as was the case at Lawrence. Even at Lawrence it is probable that the pneumonia death-rate was abnormally high just before the filter was built and that the reason for its subsequent decrease had little or nothing to do with water filtration. Yet to condemn the "theorem" altogether is to take too extreme a view. Without doubt infant mortality was reduced by filtration, chiefly through the reduction in diarrhœal diseases. McLaughlin has shown that this has occurred in many places.

The trouble with this whole problem has grown out of the use of general rates. If we want to find the effect of filtration we must compare the morbidity and mortality rates for particular diseases before and after filtration, with due regard to changes in population. Somebody who has time ought to restudy this whole matter in the light of recent data.

The sanitary index. — Many attempts have been made to devise a "sanitary index," to select and combine certain specific death-rates so as to get for a given place a single figure which, when compared with similar figures for other places, will correlate health and sanitary conditions. We

[1] Chapin, Chas. V., "Modes of Infection."

know that the general death-rate will not serve this purpose. Even the death-rate adjusted to a standard population is inadequate. The infant mortality has been claimed as the best index. Dr. Wilmer R. Batt,[1] the Registrar of the Pennsylvania State Department of Health, has suggested a composite index which illustrates this striving to get an index. It is computed as follows:

$$\text{Sanitary index} = \frac{\text{Deaths from causes No. 1 to No. 15 plus all infant deaths}}{\text{Population}}.$$

The ratio of all the other deaths to the population is called the residual death-rate. Hence the sum of the two gives the general death-rate.

He found that from 1906 to 1915 the general death-rate of the state declined from 16.0 to 13.8 per 1000 *i.e.*, 13.8 per cent. The "sanitary index," however, declined from 6.5 to 4.5, or 30.8 per cent, while the residual death-rate declined from 9.5 to 9.3 or only 2.1 per cent. This index, it will be observed, takes no account of the changing composition of the population.

Others have suggested that the index ought to be based on social and economic factors as well as vital statistics, and their point seems to be well taken. This only emphasizes the complexity of the problem. The author believes that it is too early to attempt the establishment of a health index, and that better results will be secured by the critical use of specific rates.

Current use of vital statistics. — Vital statistics have their historic uses, but their greatest value lies in their immediate use. It is interesting and ultimately most valuable to know that a baby has been born at a certain place, on a certain day, of such and such parentage, but it is more

[1] Penn. Monthly Health Bulletin, No. 70, Feb., 1916.

important that the baby shall live and grow up well. No baby should be allowed to come unnoticed into the world; boards of health or other proper authorities should see to it that every baby born has a good chance to live. In most cases the parents, the physician and the nurse are sufficient caretakers and the public authorities should not be unnecessarily intrusive or over-zealous; on the other hand their advice and aid should be prompt where occasion warrants, and immediate knowledge of the facts is the only basis of wise action.

In reported cases of diseases dangerous to the public health the need for prompt action is even greater. It is by the daily study of such reports that pending epidemics or local outbreaks of disease may be headed off. Every local health officer should keep on the walls of his office, or on a suitable frame, or in shallow drawers, a series of local maps — one for each important communicable disease. The maps should show the names of the streets. There should be a street index at hand, with the street numbers given for each intersection, and with information as to which side of the street has the odd (or even) numbers. On these maps, with the aid of the index, each case of communicable disease should be marked with a pin immediately on receipt of the report. There are many little devices involving the use of pins of different colors for different dates, the removal of pins after recovery, the additions of pins to indicate death, and so on; the details of which are bound to vary according to local conditions. But the main thing is to study the pins *daily*. In the case of state departments of health the required maps are of course on a different scale and the cases are arranged by cities and towns instead of streets. Both local and state studies are necessary.

In addition to the location maps the health officer needs

to keep up chronological charts for each disease — a separate chart for each. Pins may be used for this work also, or lines may be drawn, black or colored. These charts, together with the maps, answer the questions *where* and *when* did the cases occur.

For state work another device is convenient, — namely, a summary of cases by cities, towns, or other geographical divisions, and by weeks or months. These should be made up regularly for comparison with past records. All cities have certain numbers of cases of communicable diseases which occur with a fair degree of regularity — and what the health officer needs most to know is whether there is at any time an abnormally large number of cases of any disease. In order to quickly tell this he needs to have at hand certain generalized results of past experience. In New York City Dr. Bolduan has been in the habit of finding the average number of cases of typhoid fever, for example, in each ward and for each week of the year, — but omits from these averages any local outbreak or epidemics. He has called this the "normalized average." [1] In the author's opinion what is needed here is not the average, with the unusual conditions omitted, but the median. The Massachusetts State Department of Health is using the median under the name of the "endemic index." A better name would be the *endemic median*. This can be very easily found for a five- or ten-year period and would serve admirably as a standard of comparison. It would of course need occasional revision.

Card systems are generally found most convenient for keeping records of individual reports, and the punched-card system with mechanical devices for sorting and counting is the best of all.

[1] Bolduan, Chas. F., Typhoid Fever in New York City, No. 3 Monograph Series, Aug., 1912.

Publication of reports. — The author will perhaps be regarded as a heretic on the subject of published reports. He believes, however, that thousands of pages of useless tables of reported cases of disease are printed every year in the United States at enormous expense and that the same amount of money spent in maintaining more complete and more accurate records in state and local health departments and in studying and using the records from day to day would bring better results. The object of reporting diseases dangerous to the public health is not to pile up records but to prevent the diseases from spreading. Statements of the occurrences of communicable diseases published monthly, or even weekly, usually reach their readers too late to be of any practical use, while as historical records such frequent publication is wholly unnecessary. Some publication is desirable, however, but only that which is of real use.

Let us consider the case of communicable diseases, for example, as reported to a state department of health. If the number of cases of measles in a city is less than the endemic median, that is, less than the ordinary number of cases, no announcement is necessary; but should the number of cases rise above the endemic median a prompt announcement of that fact in the local paper [1] might be of positive benefit as it would sound a warning. If the fire bells were ringing very gently all the time except when a fire occurred and then rang loudly, the public would not heed the warning; and in the same way the constant publication of figures which are of little moment blunts the sense of caution. Arrangements might well be made, however, for the immediate publication of notices of all unusual occurrences of disease in local papers or wherever such notices would do the most good. So far as communicable

[1] Daily paper preferred.

diseases are concerned the general principle of publication should be to publish at once or not at all and to publish only the unusual occurrences. The preparation of such notices would by reflex action stimulate the health officers themselves, and would assist physicians in making diagnosis of suspected cases.

The problem of annual reports is different. Here the object is to establish a record for permanent preservation, useful alike to health officers, to physicians, and to the interested public. The calendar year with its subdivisions is the most convenient unit of time. The vital statistics of every political subdivision in the country should be published annually, and as soon after the end of the year as possible. Here we find a great amount of unnecessary duplication. It is a waste of money to have the local Board of Health of Cambridge, Mass., publish certain facts (usually a year or two late), to have the same facts published by the State Registrar and perhaps by the State Department of Health, and finally to have them published again by the U. S. Bureau of the Census, and perhaps by the U. S. Public Health Service. It is worse than wasteful, because the various tables often fail to agree and all sorts of distressing statistical errors creep in. On the other hand, while the figures for Cambridge may be found in several places, there may be other places where it is difficult to find any statistics at all. Uniformity in this matter is very greatly needed, and this must come through federal control or state coöperation, with uniform minimum schedules to serve as a basis of record.

The author believes that no systematic attempt should be made every year to publish specific rates or minute analyses of rates, for the reason that such studies are based necessarily on estimated populations. Such studies are of course very necessary for the study of special problems as

they arise, but these results should be published as special studies and not as a part of a systematic schedule. It would be better to wait for the census years, when the *facts* of population can be used instead of *estimates* and to then make a most careful analysis of all vital statistics. Such an analysis made once in five years in Massachusetts would serve every useful purpose, would save much time and expense, would avoid the need of revision and would prevent the publication of figures which contain annoying variations. The principle should be to wait for the facts, and then make a careful analysis based on the facts. Of course, general rates should be published annually, based on estimated populations, but no one need take these very seriously, as in any event they mean little. If it is thought worth while to publish specific rates for each post censal year, these should be recomputed after the next census has been taken.

Various attempts to establish standards have been made. One of these may be found in the American Journal of Public Health.[1] Another in the annual report of the N. Y. State Department of Health for 1912, another in the Quarterly Publication of the American Statistical Association[2] and so on. In establishing standards it will be necessary to determine what shall be the geographical units, what subdivision of the year, what data and in what combinations. The usual facts secured in regard to deaths are (1) place of death, (2) time of death, (3) sex, (4) age, (5) race or color, (6) cause of death, (7) birthplace, (8) birthplace of father, (9) birthplace of mother, (10) marital condition, (11) occupation. The possible number of combinations of these eleven items two at a time is 55, three at a time 165, and four at a time 330. No wonder therefore that there is lack of uniformity in published reports. Any

[1] 1913, p. 595. [2] 1911, p. 510.

standard tables must of necessity be arbitrary. The time has come when uniformity of report is necessary in the interest of both economy and efficiency.

EXERCISES AND QUESTIONS

1. Distinguish between the environments represented by the following terms:

- *a.* A felt hat and a straw hat factory.
- *b.* A paper box and a wooden box factory.
- *c.* An iron and a brass foundry.
- *d.* A wholesale and a retail merchant or dealer.
- *e.* A farm laborer on his home farm and one working out.
- *f.* A clerk in a store and a salesman.
- *g.* A dressmaker in a factory or shop and one working elsewhere.
- *h.* A cook and a servant.
- *i.* A paid housekeeper and a servant girl.
- *j.* A practical and a trained nurse.

2. To what extent do these terms conceal other important differences in age or sex or nationality?

3. What data were collected in the industrial clinic of the Massachusetts General Hospital? [Monthly Review (Dec., 1917), U. S. Bureau of Labor Statistics. Edsall, David J.: The Study of Occupational Diseases in Hospitals.]

4. How would you explain the alleged fact that more cases of infectious diseases are reported to the New York City Department of Health on Monday than on any other day, and the fewest on Saturday?

5. How are the medical and vital statistics of the U. S. Navy kept? [See Am. J. P. H., June, 1918, p. 442.]

6. How are the medical and vital statistics of the U. S. army kept? [See Am. J. P. H., Jan., 1918, p. 14.]

7. What facts are needed in the registration of still-births? [See Am. J. P. H., Jan., 1917, p. 46.]

8. Describe the epidemic of poliomyelitis in New York and New England in 1916. [See Am. J. P. H., Feb., 1917, p. 117.]

9. What proportion of children "take" the common children's diseases at some time? [See Am. J. P. H., Sept., 1916, p. 971.]

APPENDIX I

REFERENCES

To study demography, or even vital statistics, seriously one must have at hand several of the standard textbooks on the statistical method, and certain of the more recent federal, state and municipal reports. One must also have access to files of certain periodicals. The following is a list of some of the more important of these references. It is far from being complete, and is intended merely to pave the way for further searches in the library.

A complete list of references to books and articles on the many phases of the subject would be overwhelming. The most recent writings on vital statistics are not necessarily the best for the beginner to study, as some of the soundest and most logical monographs were written many years ago. Of course, the most recent data are the most interesting — but that is another matter.

Many references to particular articles will be found scattered through the footnotes of this book and printed in connection with the Exercises and Questions.

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There are scores of annual reports which deal with vital statistics. The following are illustrative:

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ENGLAND AND WALES. Annual reports of Registrar-General. (79th report in 1916.)

MASSACHUSETTS STATE REGISTRATION REPORTS. Annually since 1842.

MASSACHUSETTS STATE BOARD OF HEALTH (now Department of Health). Annually since 1870.

STATE DEPARTMENTS OF HEALTH of New York, New Jersey, Pennsylvania, Ohio, Michigan, Maine, New Hampshire, Connecticut, etc.

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APPENDIX II

THE MODEL STATE LAW FOR MORBIDITY REPORTS

ADOPTED BY THE ELEVENTH ANNUAL CONFERENCE OF STATE AND TERRITORIAL HEALTH AUTHORITIES WITH THE UNITED STATES PUBLIC HEALTH SERVICE, MINNEAPOLIS, JUNE 16, 1913.

A Bill To provide for the notification of the occurrence and prevalence of certain diseases.

Be it enacted by the Senate and General Assembly of the State of ———:

Section 1. It shall be, and is hereby, made the duty of the State department of health (or commissioner or board of health) to keep currently informed of the occurrence, geographic distribution, and prevalence of the preventable diseases throughout the State, and for this purpose there shall be established in the State department of health a bureau (or division) of sanitary reports which shall, under the direction of the State commissioner of health (State health officer or secretary of the State board of health), be in charge of an assistant commissioner of health who shall receive an annual salary of ——— dollars and the necessary expenses incurred in the performance of his duties. The State department of health shall provide such clerical and other assistance as may be necessary for the establishment and maintenance of said bureau.

Sec. 2. The following-named diseases and disabilities are hereby made notifiable and the occurrence of cases shall be reported as herein provided:

GROUP I. — INFECTIOUS DISEASES

Actinomycosis.
Anthrax.
Chicken-pox.
Cholera. Asiatic (also cholera nostras when Asiatic cholera is present or its importation threatened).
Continued fever lasting seven days.
Dengue.
Diphtheria.
Dysentery:
 (*a*) Amebic.
 (*b*) Bacillary.
Favus.
German measles.

GROUP I. — INFECTIOUS DISEASES — *Continued*

Glanders.
Hookworm disease.
Leprosy.
Malaria.
Measles.
Meningitis:
(*a*) Epidemic cerebrospinal.
(*b*) Tuberculous.
Mumps.
Ophthalmia neonatorum (conjunctivitis of new born infants).
Paragonimiasis (endemic hemoptysis).
Paratyphoid fever.
Plague.
Pneumonia.
Poliomyelitis (acute infectious).
Rabies.
Rocky Mountain spotted, or tick, fever.
Scarlet fever.
Septic sore throat.
Smallpox.
Tetanus.
Trachoma.
Trichinosis.
Tuberculosis (all forms, the organ or part affected in each case to be specified).
Typhoid fever.
Typhus fever.
Whooping cough.
Yellow fever.

GROUP II. — OCCUPATIONAL DISEASES AND INJURIES.

Arsenic poisoning.
Brass poisoning.
Carbon monoxide poisoning.
Lead poisoning.
Mercury poisoning.
Natural gas poisoning.
Phosphorous poisoning.
Wood alcohol poisoning.
Naphtha poisoning.
Bisulphide of carbon poisoning.
Dinitrobenzine poisoning.
Caisson disease (compressed-air illness).
Any other disease or disability contracted as a result of the nature of the person's employment.

GROUP III. — VENEREAL DISEASES

Gonococcus infection.
Syphilis.

GROUP IV. — DISEASES OF UNKNOWN ORIGIN.

Pellagra.
Cancer.

Provided, That the State department of health (or board of health) may from time to time, in its discretion, declare additional diseases notifiable and subject to the provisions of this act.

SEC. 3. Each and every physician practicing in the State of ——— who treats or examines any person suffering from or afflicted with, or suspected to be suffering from or afflicted with, any one of the notifiable diseases shall immediately report such case of notifiable disease in writing to the local health authority having jurisdiction. Said report shall

be forwarded either by mail or by special messenger and shall give the following information:

1. The date when the report is made.

2. The name of the disease or suspected disease.

3. The name, age, sex, color, occupation, address, and school attended or place of employment of patient.

4. Number of adults and children in the household.

5. Source or probable source of infection or the origin or probable origin of the disease.

6. Name and address of the reporting physician.

Provided, That if the disease is, or is suspected to be, smallpox the report shall, in addition, show whether the disease is of the mild or virulent type and whether the patient has ever been successively vaccinated, and, if the patient has been successfully vaccinated, the number of times and dates or approximate dates of such vaccination; and if the disease is, or is suspected to be, cholera, diphtheria, plague, scarlet fever, smallpox, or yellow fever, the physician shall, in addition to the written report; give immediate notice of the case to the local health authority in the most expeditious manner available; and if the disease is, or is suspected to be, typhoid fever, scarlet fever, diphtheria or septic sore throat the report shall also show whether the patient has been, or any member of the household in which the patient resides is, engaged or employed in the handling of milk for sale or preliminary to sale: *And provided further*, That in the reports of cases of the venereal diseases the name and address of the patient need not be given.

Sec. 4. The requirements of the preceding section shall be applicable to physicians attending patients ill with any of the notifiable diseases in hospitals, asylums, or other institutions, public or private: *Provided*, That the superintendent or other person in charge of any such hospital, asylum, or other institution in which the sick are cared for may, with the written consent of the local health officer (or board of health) having jurisdiction, report in the place of the attending physician or physicians the cases of notifiable diseases and disabilities occurring in or admitted to said hospital, asylum, or other institution in the same manner as that prescribed by physicians.

Sec. 5. Whenever a person is known, or is suspected, to be afflicted with a notifiable disease, or whenever the eyes of an infant under two weeks of age become reddened, inflamed, or swollen, or contain an unnatural discharge, and no physician is in attendance, an immediate report of the existence of the case shall be made to the local health officer

by the midwife, nurse, attendant, or other person in charge of the patient.

SEC. 6. Teachers or other persons employed in, or in charge of, public or private schools, including Sunday Schools, shall report immediately to the local health officer each and every known or suspected case of a notifiable disease in persons attending or employed in their respective schools.

SEC. 7. The written reports of cases of the notifiable disease required by this act of physicians shall be made upon blanks supplied for the purpose, through the local health authorities, by the State department of health. These blanks shall conform to that adopted and approved by the State and Territorial health authorities in conference with the United States Public Health Service.

SEC. 8. Local health officers or boards of health shall within seven days after the receipt by them of reports of cases of the notifiable diseases forward by mail to the State department of health the original written reports made by physicians, after first having transcribed the information given in the respective reports in a book or other form of record for the permanent files of the local health office. On each report thus forwarded the local health officer shall state whether the case to which the report pertains was visited or otherwise investigated by a representative of the local health office and whether measures were taken to prevent the spread of the disease or the occurrence of additional cases.

SEC. 9. Local health officers or boards of health shall, in addition to the provisions of section 8, report to the State department of health in such manner and at such times as the State department of health may require by regulation the number of new cases of each of the notifiable diseases reported to said local health officers or boards of health.

SEC. 10. Whenever there occurs within the jurisdiction of a local health officer or board of health an epidemic of a notifiable disease, the local health officer or board of health shall, within 30 days after the epidemic shall have subsided, make a report to the State department of health of the number of cases occurring in the epidemic, the number of cases terminating fatally, the origin of the epidemic, and the means by which the disease was spread: *Provided,* That whenever the State department of health has taken charge of the control and suppression or undertaken the investigation of the epidemic, the local health authority having jurisdiction need not make the report otherwise required.

SEC. 11. No person shall be appointed to the position of local health

officer in any city, town, or county until after the qualifications of said person have been approved by the State department of health.

SEC. 12. In localities in which there are no local health officers or boards of health, and in localities in which, although there are health officers or boards of health, adequate provision has not, in the opinion of the State department of health, been made for the proper notification, investigation, and control of notifiable disease, and in localities in which the local health authorities fail to carry out the provisions of this act, the State department of health shall appoint properly qualified sanitary officers to act as local health officers and to prevent the spread of disease in and from such localities and to enforce the provisions of this act: *Provided,* That salaries and other expenses incurred under the provisions of this section shall be paid by the local authorities.

SEC. 13. Any physician or other person or persons who shall fail, neglect or refuse to comply with, or who shall violate any of the provisions of this act shall be guilty of a misdemeanor, and upon conviction thereof shall be sentenced to pay a fine of not less than ——— dollars nor more than ——— dollars or to imprisonment for not less than ——— days nor more than ——— days for each offense: *Provided,* That in the case of a physician his license to practice medicine within the State may be revoked in accordance with existing statutory provisions.

SEC. 14. No license to practice medicine shall be issued to any person until after the applicant shall have filed with the State licensing board a statement, signed and sworn to before a notary or other officer qualified to administer oaths, that said applicant has familiarized himself with the requirements of this act, a copy of which sworn statement shall be forwarded to the State department of health.

SEC. 15. Each and every person engaged in the practice of medicine shall display in a prominent place in his or her office a card upon which sections 2, 3, 4, 7, 13, 14, and 15 of this act have been printed with type not smaller than 10-point. A similar card shall be displayed in a prominent place in the office of each and every hospital, asylum, or other public or private institution for the treatment of the sick. These cards shall each be not less than 1 square foot in size and shall be furnished to institutions and licensed physicians without cost by the State department of health.

SEC. 16. The sum of ——— dollars is hereby appropriated from any money in the State treasury not otherwise appropriated for carrying out the provisions of this act.

SEC. 17. This act shall take effect immediately, and all acts or parts of acts inconsistent with the provisions of this act are hereby repealed.

THE STANDARD MORBIDITY NOTIFICATION BLANK

The following model notification blank was also adopted by the conference of state and territorial health authorities with the United States Public Health Service, June 16, 1913, as the standard notification blank referred to in section 7 of the Model Law as the one to be used in the reporting of cases of the notifiable diseases. This blank is intended to be printed on a post card:

[Face of card.]

........................, 191..
(Date.)

Disease or suspected disease..
Patient's name...................., age.............., sex............, color............
Patient's address.................................., occupation....................
School attended or place of employment...
Number in household: Adults...................., children........................
Probable source of infection or origin of disease.......................................
If disease is smallpox, type..., number of times successfully vaccinated and approximate dates.....................................
If typhoid fever, scarlet fever, diphtheria, or septic sore throat, was patient, or is any member of household engaged in the production of handling of milk ....
Address of reporting physician...
Signature of physician ....

[Reverse of card.]

For use of local health department.

....................191 ,....
(Date.)
Was case investigated by health department?
....
....
Was nature of disease verified?.............................
....
What measures were taken to prevent the spread or the occurrence of additional cases from same origin?........................
....
....

Health Department,
(City)..........
(State)..........

HOSPITAL DISCHARGE CERTIFICATE

Suggested by Bolduan for use in connection with hospital morbidity reports.

DISCHARGE CERTIFICATE.

Name of hospital..........................	Hospital admission No.................... Sex..........Age.........................
How admitted — Ambulance or own application or (Tabulation No.) transfer from other hospital.	White. Hebrew. Colored. Gentile. Mongolian. Place of birth..............................
Patient's address.......................... ....Borough.................. Date admitted............................ Date discharged.......................... Days in hospital........months............ days............... (If over a year, omit the days and give only years and months.)	Single or married or widowed or divorced or unknown. Discharged to — Home. Other hospital. Convalescent retreat. Coroner.

Occupation — (*a*) Trade, profession, or particular kind of work.

....

(*b*) General nature of the industry, business, or establishment in which employed (or employer).

....

Diagnosis..
and
Complications..
If operated upon, state nature of operation..
Condition on discharge: Cured. Improved. Unimproved.
Died — Autopsy.
No autopsy.
Signed................
House Physician — Surgeon.

APPENDIX III

THE MODEL STATE LAW FOR THE REGISTRATION OF BIRTHS AND DEATHS

A Bill[1] To provide for the registration of all births and deaths in the State of———.

Note.—After the bill has been prepared for presentation to the legislature of a State, the title should be carefully revised by competent legal authority.

Be it enacted by the legislature of the State of ———

Section 1. That the State board of health shall have charge of the registration of births and deaths; shall prepare the necessary instructions, forms, and blanks for obtaining and preserving such records and shall procure the faithful registration of the same in each primary registration district as constituted in section 3 of this act, and in the central bureau of vital statistics at the capital of the State. The said board shall be charged with the uniform and thorough enforcement of the law throughout the State, and shall from time to time recommend any additional legislation[2] that may be necessary for this purpose.

Sec. 2. That the secretary of the State board of health shall have general supervision over the central bureau of vital statistics, which is hereby authorized to be established by said board, and which shall be under the immediate direction of the State registrar of vital statistics, whom the State board of health shall appoint within thirty days after the taking effect of this law, and who shall be a medical practitioner of not less than five years' practice in his profession, and a competent vital statistician. The State registrar of vital statistics shall hold office for four years and until his successor has been appointed and has qualified, unless such office shall sooner become vacant by death, disqualification,

[1] Before introducing this bill in any legislature it should be carefully redrafted by a competent lawyer and submitted to the Bureau of the Census for criticism.

[2] The words "and shall promulgate any additional rules or regulations" may be inserted in bills prepared for States in which the State board of health has power to make rules and regulations having the effect of law.

operation of law, or other causes. Any vacancy occurring in such office shall be filled for the unexpired term by the State board of health. At least ten days before the expiration of the term of office of the State registrar of vital statistics, his successor shall be appointed by the State board of health. The State registrar of vital statistics shall receive an annual salary at the rate of ——— dollars from the date of his entering upon the discharge of the duties of his office. The State board of health shall provide for such clerical and other assistants as may be necessary for the purposes of this act, who shall serve during the pleasure of the board, and shall fix the compensation of persons thus employed within the amount appropriated therefor by the legislature. The custodian of the capitol shall provide for the bureau of vital statistics in the State capitol at ——— suitable offices, which shall be properly equipped with fireproof vault and filing cases for the permanent and safe preservation of all official records made and returned under this act.

SEC. 3. That for the purposes of this act the State shall be divided into registration districts as follows: Each city, each incorporated town, and each township[1] shall constitute a primary registration district: *Provided*, That the State board of health may combine two or more primary registration districts when necessary to facilitate registration.

SEC. 4. That within ninety days after the taking effect of this act, or as soon thereafter as possible, the State board of health shall appoint a local registrar of vital statistics for each registration district in the State.[2] The term of office of each local registrar so appointed shall be

[1] Or other primary political unit, as "town," "precinct," "civil district," "hundred," etc. When there are no such units available, the following substitutes for section 3 may be employed: Section 3. That for the purposes of this act the State shall be divided into registration districts as follows: Each city and each incorporated town shall constitute a primary registration district; and for that portion of each county outside of the cities and incorporated towns therein the State board of health shall define and designate the boundaries of a sufficient number of rural registration districts, which districts it may change or combine from time to time as may be necessary to insure the convenience and completeness of registration.

[2] This method of appointment of local registrars by the State board of health — or perhaps by the State registrar or upon his nomination — with a reasonably long term of service and subject to removal for neglect of duty, is the preferable one for efficient service. Should there be objection, however, to the creation of new offices, the section may be redrafted so that it will provide that township, village, or city clerks, or other suitable officials, shall be the local registrars.

four years, and until his successor has been appointed and has qualified, unless such office shall sooner become vacant by death, disqualification, operation of law, or other causes: *Provided*, That in cities where health officers or other officials are, in the judgment of the State board of health, conducting effective registration of births and deaths under local ordinances at the time of the taking effect of this act such officials may be appointed as registrars in and for such cities, and shall be subject to the rules and regulations of the State registrar and to all of the provisions of this act. Any vacancy occurring in the office of local registrar of vital statistics shall be filled for the unexpired term by the State board of health. At least ten days before the expiration of the term of office of any such local registrar his successor shall be appointed by the State board of health.

Any local registrar who, in the judgment of the State board of health, fails or neglects to discharge efficiently the duties of his office as set forth in this act, or to make prompt and complete returns of births and deaths as required thereby, shall be forthwith removed by the State board of health, and such other penalties may be imposed as are provided under section 22 of this act.

Each local registrar shall, immediately upon his acceptance of appointment as such, appoint a deputy, whose duty it shall be to act in his stead in case of his absence or disability; and such deputy shall in writing accept such appointment and be subject to all rules and regulations governing local registrars. And when it appears necessary for the convenience of the people in any rural district the local registrar is hereby authorized, with the approval of the State registrar, to appoint one or more suitable persons to act as subregistrars, who shall be authorized to receive certificates and to issue burial or removal permits in and for such portions of the district as may be designated; and each subregistrar shall note on each certificate, over his signature, the date of filing, and shall forward all certificates to the local registrar of the district within ten days, and in all cases before the third day of the following month: *Provided*, That each subregistrar shall be subject to the supervision and control of the State registrar and may be by him removed for neglect or failure to perform his duty in accordance with the provisions of this act or the rules and regulations of the State registrar, and shall be subject to the same penalties for neglect of duty as the local registrar.

Sec. 5. That the body of any person whose death occurs in this State, or which shall be found dead therein, shall not be interred, deposited in a vault or tomb, cremated or otherwise disposed of, or re-

moved from or into any registration district, or be temporarily held pending further disposition more than seventy-two hours after death, unless a permit for burial, removal, or other disposition thereof shall have been properly issued by the local registrar of the registration district in which the death occurred or the body was found.[1] And no such burial or removal permit shall be issued by any registrar until, wherever practicable, a complete and satisfactory certificate of death has been filed with him as hereinafter provided: *Provided*, That when a dead body is transported from outside the State into a registration district in ——— for burial, the transit or removal permit, issued in accordance with the law and health regulations of the place where the death occurred, shall be accepted by the local registrar of the district into which the body has been transported for burial or other disposition, as a basis upon which he may issue a local burial permit; he shall note upon the face of the burial permit the fact that it was a body shipped in for interment, and give the actual place of death; and no local registrar shall receive any fee for the issuance of burial or removal permits under this act other than the compensation provided in section 20.

Sec. 6. That a stillborn child shall be registered as a birth and also as a death, and separate certificates of both the birth and the death shall be filed with the local registrar, in the usual form and manner, the certificate of birth to contain in place of the name of the child, the word "stillbirth": *Provided*, That a certificate of birth and a certificate of death shall not be required for a child that has not advanced to the fifth month of uterogestation. The medical certificate of the cause of death shall be signed by the attending physician, if any, and shall state the cause of death as "stillborn," with the cause of the stillbirth, if known, whether a premature birth, and, if born prematurely, the period of uterogestation, in months, if known; and a burial or removal permit of the prescribed form shall be required. Midwives shall not sign certificates of death for stillborn children; but such cases, and stillbirths occurring without attendance of either physician or midwife, shall be treated as deaths without medical attendance, as provided for in section 8 of this act.

Sec. 7. That the certificate of death shall contain the following items, which are hereby declared necessary for the legal, social, and sanitary purposes subserved by registration records:[2]

[1] A special proviso may be required for sparsely settled portions of a State.

[2] The following items are those of the United States standard certificate of death, approved by the Bureau of the Census.

(1) Place of death, including State, county, township, village, or city. If in a city, the ward, street, and house number; if in a hospital or other institution, the name of the same to be given instead of the street and house number. If in an industrial camp, the name of the camp to be given.

(2) Full name of decedent. If an unnamed child, the surname preceded by "Unnamed."

(3) Sex.

(4) Color or race, as white, black, mulatto (or other negro descent), Indian, Chinese, Japanese, or other.

(5) Conjugal condition, as single, married, widowed, or divorced.

(6) Date of birth, including the year, month, and day.

(7) Age, in years, months, and days. If less than one day, the hours or minutes.

(8) Occupation. The occupation to be reported of any person, male or female, who had any remunerative employment, with the statement of (*a*) trade, profession or particular kind of work; (*b*) general nature of industry, business, or establishment in which employed (or employer).

(9) Birthplace; at least State or foreign country, if known.

(10) Name of father.

(11) Birthplace of father; at least State or foreign country, if known.

(12) Maiden name of mother.

(13) Birthplace of mother; at least State or foreign country, if known.

(14) Signature and address of informant.

(15) Official signature of registrar, with the date when certificate was filed, and registered number.

(16) Date of death, year, month, and day.

(17) Certification as to medical attendance on decedent, fact and time of death, time last seen alive, and the cause of death, with contributory (secondary) cause of complication, if any, and duration of each, and whether attributed to dangerous or insanitary conditions of employment; signature and address of physician or official making the medical certificate.

(18) Length of residence (for inmates of hospitals and other institutions; transients or recent residents) at place of death and in the State, together with the place where disease was contracted, if not at place of death, and former or usual residence.

(19) Place of burial or removal; date of burial.

(20) Signature and address of undertaker or person acting as such.

The personal and statistical particulars (items 1 to 13) shall be authenticated by the signature of the informant, who may be any competent person acquainted with the facts.

The statement of facts relating to the disposition of the body shall be signed by the undertaker or person acting as such.

The medical certificate shall be made and signed by the physician, if any, last in attendance on the deceased, who shall specify the time in attendance, the time he last saw the deceased alive, and the hour of the day at which death occurred. And he shall further state the cause of death, so as to show the course of disease or sequence of causes resulting in the death, giving first the name of the disease causing death (primary cause), and the contributory (secondary) cause, if any, and the duration of each. Indefinite and unsatisfactory terms, denoting only symptoms of disease or conditions resulting from disease, will not be held sufficient for the issuance of a burial or removal permit; and any certificate containing only such terms as defined by the State Registrar shall be returned to the physician or person making the medical certificate for correction and more definite statement. Causes of death which may be the result of either disease or violence shall be carefully defined; and if from violence, the means of injury shall be stated and whether (probably) accidental, suicidal, or homicidal.[1] And for deaths in hospitals, institutions, or of nonresidents the physician shall supply the information required under this head (item 18), if he is able to do so, and may state where, in his opinion, the disease was contracted.

Sec. 8. That in case of any death occurring without medical attendance it shall be the duty of the undertaker to notify the local registrar of such death, and when so notified the registrar shall, prior to the issuance of the permit, inform the local health officer and refer the case to him for immediate investigation and certification: *Provided*, That when the local health officer is not a physician, or when there is no such official, and in such cases only, the registrar is authorized to make the certificate and return from the statement of relatives or other persons having adequate knowledge of the facts: *Provided further*, That if the registrar has reason to believe that the death may have been due to unlawful act or neglect he shall then refer the case to the coroner or other proper officer for his investigation and certification. And the coroner or other proper officer whose duty it is to hold an inquest on the

[1] In some States the question whether a death was accidental, suicidal, or homicidal must be determined by the coroner or medical examiner and the registration law must be framed to harmonize.

body of any deceased person and to make the certificate of death required for a burial permit shall state in his certificate the name of the disease causing death, or if from external causes, (1) the means of death and (2) whether (probably) accidental, suicidal, or homicidal, and shall in any case furnish such information as may be required by the State Registrar in order properly to classify the death.

Sec. 9. That the undertaker or person acting as undertaker shall file the certificate of death with the local registrar of the district in which the death occurred and obtain a burial or removal permit prior to any disposition of the body. He shall obtain the required personal and statistical particulars from the person best qualified to supply them, over the signature and address of his informant. He shall then present the certificate to the attending physician, if any, or to the health officer or coroner, as directed by the local registrar, for the medical certificate of the cause of death and other particulars necessary to complete the record, as specified in sections 7 and 8. And he shall then state the facts required relative to the date and place of burial or removal, over his signature and with his address, and present the completed certificate to the local registrar in order to obtain a permit for burial, removal, or other disposition of the body. The undertaker shall deliver the burial permit to the person in charge of the place of burial before interring or otherwise disposing of the body, or shall attach the removal permit to the box containing the corpse, when shipped by any transportation company, said permit to accompany the corpse to its destination, where, if within the State of ———, it shall be delivered to the person in charge of the place of burial.

[Every person, firm, or corporation selling a casket shall keep a record showing the name of the purchaser, purchaser's post-office address, name of deceased, date of death, and place of death of deceased, which record shall be open to inspection of the State Registrar at all times. On the first day of each month the person, firm, or corporation selling caskets shall report to the State Registrar each sale for the preceding month, on a blank, provided for that purpose: *Provided, however*, That no person, firm, or corporation selling caskets to dealers or undertakers only shall be required to keep such record, nor shall such report be required from undertakers when they have direct charge of the disposition of a dead body.

Every person, firm, or corporation selling a casket at retail, and not having charge of the disposition of the body, shall inclose within the casket a notice furnished by the State Registrar calling attention to the requirements of the law, a blank certificate of death, and the rules

and regulations of the State board of health concerning the burial or other disposition of a dead body.][1]

SEC. 10. That if the interment or other disposition of the body is to be made within the State, the wording of the burial or removal permit may be limited to a statement by the registrar, and over his signature, that a satisfactory certificate of death having been filed with him, as required by law, permission is granted to inter, remove, or dispose otherwise of the body, stating the name, age, sex, cause of death, and other necessary details upon the form prescribed by the State registrar.

SEC. 11. That no person in charge of any premises on which interments are made shall inter or permit the interment or other disposition of any body unless it is accompanied by a burial, removal, or transit permit, as herein provided. And such person shall indorse upon the permit the date of interment, over his signature, and shall return all permits so indorsed to the local registrar of his district within ten days from the date of interment, or within the time fixed by the local board of health. He shall keep a record of all bodies interred or otherwise disposed of on the premises under his charge, in each case stating the name of each deceased person, place of death, date of burial or disposal, and name and address of the undertaker; which record shall at all times be open to official inspection: *Provided,* That the undertaker, or person acting as such, when burying a body in a cemetery or burial ground having no person in charge, shall sign the burial or removal permit, giving the date of burial, and shall write across the face of the permit the words "No person in charge," and file the burial or removal permit within ten days with the registrar of the district in which the cemetery is located.

SEC. 12. That the birth of each and every child born in this State shall be registered as hereinafter provided.

SEC. 13. That within ten days after the date of each birth there shall be filed with the local registrar of the district in which the birth occurred a certificate of such birth, which certificate shall be upon the form adopted by the State board of health with a view to procuring a full and accurate report with respect to each item of information enumerated in section 14 of this act.[2]

In each case where a physician, midwife, or person acting as midwife was in attendance upon the birth, it shall be the duty of such physician,

[1] The provisions in brackets may be useful in States in which many funerals are conducted without regular undertakers.

[2] A proviso may be added that shall require the registration, or notification, at a shorter interval than ten days, of births that occur in cities.

midwife, or person acting as midwife to file in accordance herewith the certificate herein contemplated.

In each case where there was no physician, midwife, or person acting as midwife in attendance upon the birth, it shall be the duty of the father or mother of the child, the householder or owner of the premises where the birth occurred, or the manager or superintendent of the public or private institution where the birth occurred, each in the order named, within ten days after the date of such birth, to report to the local registrar the fact of such birth. In such case and in case the physician, midwife, or person acting as midwife, in attendance upon the birth is unable, by diligent inquiry, to obtain any item or items of information contemplated in section 14 of this act, it shall then be the duty of the local registrar to secure from the person so reporting, or from any other person having the required knowledge, such information as will enable him to prepare the certificate of birth herein contemplated, and it shall be the duty of the person reporting the birth, or who may be interrogated in relation thereto, to answer correctly and to the best of his knowledge all questions put to him by the local registrar which may be calculated to elicit any information needed to make a complete record of the birth as contemplated by said section 14, and it shall be the duty of the informant as to any statement made in accordance herewith to verify such statement by his signature, when requested so to do by the local registrar.

Sec. 14. That the certificate of birth shall contain the following items, which are hereby declared necessary for the legal, social, and sanitary purposes subserved by registration records:[1]

(1) Place of birth, including State, county, township or town, village, or city. If in a city, the ward, street, and house number; if in a hospital or other institution, the name of the same to be given, instead of the street and house number.

(2) Full name of child. If the child dies without a name, before the certificate is filed, enter the words "Died unnamed." If the living child has not yet been named at the date of filing certificate of birth, the space for "Full name of child" is to be left blank, to be filled out subsequently by a supplemental report, as hereinafter provided.

(3) Sex of child.

(4) Whether a twin, triplet, or other plural birth. A separate certificate shall be required for each child in case of plural births.

[1] The following items are those of the United States standard certificate of birth, approved by the Bureau of the Census.

(5) For plural births, number of each child in order of birth.

(6) Whether legitimate or illegitimate.[1]

(7) Date of birth, including the year, month, and day.

(8) Full name of father.

(9) Residence of father.

(10) Color or race of father.

(11) Age of father at last birthday, in years.

(12) Birthplace of father; at least State or foreign country, if known.

(13) Occupation of father. The occupation to be reported if engaged in any remunerative employment, with the statement of (*a*) trade, profession, or particular kind of work; (*b*) general nature of industry, business, or establishment in which employed (or employer).

(14) Maiden name of mother.

(15) Residence of mother.

(16) Color or race of mother.

(17) Age of mother at last birthday, in years.

(18) Birthplace of mother; at least State or foreign country, if known.

(19) Occupation of mother. The occupation to be reported if engaged in any remunerative employment, with the statement of (*a*) trade, profession, or particular kind of work; (*b*) general nature of industry, business, or establishment in which employed (or employer).

(20) Number of children born to this mother, including present birth.

(21) Number of children of this mother living.

(22) The certification of attending physician or midwife as to attendance at birth, including statement of year, month, day (as given in item 7), and hour of birth, and whether the child was born alive or stillborn. This certification shall be signed by the attending physician or midwife, with date of signature and address; if there is not physician or midwife in attendance, then by the father or mother of the child, householder, owner of the premises, or manager or superintendent of public or private institution where the birth occurred, or other competent person, whose duty it shall be to notify the local registrar of such birth, as required by section 13 of this act.

(23) Exact date of filing in office of local registrar, attested by his official signature, and registered number of birth, as hereinafter provided.

Sec. 15. That when any certificate of birth of a living child is presented without the statement of the given name, then the local registrar

[1] This question may be omitted if desired, or provision may be made so that the identity of parents will not be disclosed.

shall make out and deliver to the parents of the child a special blank for the supplemental report of the given name of the child, which shall be filled out as directed, and returned to the local registrar as soon as the child shall have been named.

SEC. 16. That every physician, midwife, and undertaker shall, without delay, register his or her name, address, and occupation with the local registrar of the district in which he or she resides, or may hereafter establish a residence; and shall thereupon be supplied by the local registrar with a copy of this act, together with such rules and regulations as may be prepared by the State registrar relative to its enforcement. Within thirty days after the close of each calendar year each local registrar shall make a return to the State registrar of all physicians, midwives, or undertakers who have been registered in his district during the whole or any part of the preceding calendar year: *Provided*, That no fee or other compensation shall be charged by local registrars to physicians, midwives, or undertakers for registering their names under this section or making returns thereof to the State registrar.[1]

SEC. 17. That all superintendents or managers, or other persons in charge of hospitals, almshouses, lying-in, or other institutions, public or private, to which persons resort for treatment of diseases, confinement, or are committed by process of law, shall make a record of all the personal and statistical particulars relative to the inmates in their institutions at the date of approval of this act, which are required in the forms of the certificates provided for by this act, as directed by the State registrar; and thereafter such record shall be, by them, made for all future inmates at the time of their admittance. And in case of persons admitted or committed for treatment of disease, the physician in charge shall specify for entry in the record, the nature of the disease, and where, in his opinion, it was contracted. The personal particulars and information required by this section shall be obtained from the individual himself if it is practicable to do so; and when they can not be so obtained, they shall be obtained in as complete a manner as possible from relatives, friends, or other persons acquainted with the facts.

SEC. 18. That the State registrar shall prepare, print, and supply to all registrars all blanks and forms used in registering, recording, and preserving the returns, or in otherwise carrying out the purposes of this act; and shall prepare and issue such detailed instructions as may be required to procure the uniform observance of its provisions and the

[1] This section may be omitted if deemed expedient and the duty of supplying instructions may be assumed by the State officer.

maintenance of a perfect system of registration; and no other blanks shall be used than those supplied by the State registrar. He shall carefully examine the certificates received monthly from the local registrars, and if any such are incomplete or unsatisfactory he shall require such further information to be supplied as may be necessary to make the record complete and satisfactory. And all physicians, midwives, informants, or undertakers, and all other persons having knowledge of the facts, are hereby required to supply, upon a form provided by the State registrar or upon the original certificate, such information as they may possess regarding any birth or death upon demand of the State registrar, in person, by mail, or through the local registrar: *Provided*, That no certificate of birth or death, after its acceptance for registration by the local registrar, and no other record made in pursuance of this act, shall be altered or changed in any respect otherwise than by amendments properly dated, signed, and witnessed. The State registrar shall further arrange, bind, and permanently preserve the certificates in a systematic manner, and shall prepare and maintain a comprehensive and continuous card index of all births and deaths registered; said index to be arranged alphabetically, in the case of deaths, by the names of decendents, and in the case of births, by the names of fathers and mothers. He shall inform all registrars what diseases are to be considered infectious, contagious, or communicable and dangerous to the public health, as decided by the State board of health, in order that when deaths occur from such diseases proper precautions may be taken to prevent their spread.

If any cemetery company or association, or any church or historical society or association, or any other company, society, or association, or any individual, is in possession of any record of births or deaths which may be of value in establishing the genealogy of any resident of this State, such company, society, association, or individual may file such record or a duly authenticated transcript thereof with the State registrar, and it shall be the duty of the State registrar to preserve such record or transcript and to make a record and index thereof in such form as to facilitate the finding of any information contained therein. Such record and index shall be open to inspection by the public, subject to such reasonable conditions as the State registrar may prescribe. If any person desires a transcript of any record filed in accordance herewith, the State registrar shall furnish the same upon application, together with a certificate that it is a true copy of such record, as filed in his office, and for his services in so furnishing such transcript and certificate he shall be entitled to a fee of (ten cents per folio) (fifty cents

per hour or fraction of an hour necessarily consumed in making such transcript) and to a fee of twenty-five cents for the certificate, which fees shall be paid by the applicant.

Sec. 19. That each local registrar shall supply blank forms of certificates to such persons as require them. Each local registrar shall carefully examine each certificate of birth or death when presented for record in order to ascertain whether or not it has been made out in accordance with the provisions of this act and the instructions of the State registrar; and if any certificate of death is incomplete or unsatisfactory, it shall be his duty to call attention to the defects in the return, and to withhold the burial or removal permit until such defects are corrected. All certificates, either of birth or of death, shall be written legibly, in durable black ink, and no certificate shall be held to be complete and correct that does not supply all of the items of information called for therein, or satisfactorily account for their omission. If the certificate of death is properly executed and complete, he shall then issue a burial or removal permit to the undertaker; provided, that in case the death occurred from some disease which is held by the State board of health to be infectious, contagious, or communicable and dangerous to the public health, no permit for the removal or other disposition of the body shall be issued by the registrar, except under such conditions as may be prescribed by the State board of health. If a certificate of birth is incomplete, the local registrar shall immediately notify the informant and require him to supply the missing items of information if they can be obtained. He shall number consecutively the certificates of birth and death, in two separate series, beginning with number 1 for the first birth and the first death in each calendar year, and sign his name as registrar in attest of the date of filing in his office. He shall also make a complete and accurate copy of each birth and each death certificate registered by him in a record book supplied by the State registrar, to be preserved permanently in his office as the local record, in such manner as directed by the State registrar. And he shall, on the tenth day of each month, transmit to the State registrar all original certificates registered by him for the preceding month. And if no births or no deaths occurred in any month, he shall, on the tenth day of the following month, report that fact to the State registrar, on a card provided for such purpose.

Sec. 20. That each local registrar shall be paid the sum of twenty-five cents for each birth certificate and each death certificate properly and completely made out and registered with him, and correctly recorded and promptly returned by him to the State registrar, as required by

this act.[1] And in case no births or no deaths were registered during any month, the local registrar shall be entitled to be paid the sum of twenty-five cents for each report to that effect, but only if such report be made promptly as required by this act. All amounts payable to a local registrar under the provisions of this section shall be paid by the treasurer of the county in which the registration district is located, upon certification by the State registrar. And the State registrar shall annually certify to the treasurers of the several counties the number of births and deaths properly registered, with the names of the local registrars and the amounts due each at the rates fixed herein.[2]

Sec. 21. That the State registrar shall, upon request, supply to any applicant a certified copy of the record of any birth or death registered under provisions of this act, for the making and certification of which he shall be entitled to a fee of fifty cents, to be paid by the applicant. And any such copy of the record of a birth or death, when properly certified by the State registrar, shall be prima facie evidence in all courts and places of the facts therein stated. For any search of the files and records when no certified copy is made, the State registrar shall be entitled to a fee of fifty cents for each hour or fractional part of an hour of time of search, said fee to be paid by the applicant. And the State registrar shall keep a true and correct account of all fees by him received under these provisions, and turn the same over to the State treasurer: *Provided,* That the State registrar shall, upon request of any parent or guardian, supply, without fee, a certificate limited to a statement as to the date of birth of any child when the same shall be necessary for admission to school, or for the purpose of securing employment: *And provided further,* That the United States Census Bureau may obtain, without expense to the State, transcripts, or certified copies of births and deaths without payment of the fees herein prescribed.

Sec. 22. That any person, who for himself or as an officer, agent, or employee of any other person, or of any corporation or partnership (*a*) shall inter, cremate, or otherwise finally dispose of the dead body of a human being, or permit the same to be done, or shall remove said body from the primary registration district in which the death occurred or

[1] A proviso may be inserted at this point relative to fees of city registrars who are already compensated by salary for their services. See laws of Missouri, Ohio, and Pennsylvania.

[2] Provision may be made in this section for the payment of sub-registrars and also, if desired, for the payment of physicians and mid-wives. See Kentucky law.

the body was found without the authority of a burial or removal permit issued by the local registrar of the district in which the death occurred or in which the body was found; or (*b*) shall refuse or fail to furnish correctly any information in his possession, or shall furnish false information affecting any certificate or record, required by this act; or (*c*) shall willfully alter, otherwise than is provided by section 18 of this act, or shall falsify any certificate of birth or death, or any record established by this act; or (*d*) being required by this act to fill out a certificate of birth or death and file the same with the local registrar, or deliver it, upon request, to any person charged with the duty of filling the same, shall fail, neglect, or refuse to perform such duty in the manner required by this act; or (*e*) being a local registrar, deputy registrar, or subregistrar, shall fail, neglect, or refuse to perform his duty as required by this act and by the instructions and direction of the State registrar thereunder, shall be deemed guilty of a misdemeanor, and upon conviction thereof shall for the first offense be fined not less than five dollars ($5) nor more than fifty dollars ($50), and for each subsequent offense not less than ten dollars ($10) nor more than one hundred dollard ($100), or be imprisoned in the county jail not more than sixty days, or be both fined and imprisoned in the discretion of the court.[1]

SEC. 23. That each local registrar is hereby charged with the strict and thorough enforcement of the provisions of this act in his registration district, under the supervision and direction of the State registrar. And he shall make an immediate report to the State registrar of any violation of this law coming to his knowledge, by observation or upon complaint of any person or otherwise.

The State registrar is hereby charged with the thorough and efficient execution of the provisions of this act in every part of the State, and is hereby granted supervisory power over local registrars, deputy local registrars, and subregistrars to the end that all of its requirements shall be uniformly complied with. The State registrar, either personally or by an accredited representative, shall have authority to investigate cases of irregularity or violation of law, and all registrars shall aid him upon request, in such investigations. When he shall deem it necessary he shall report cases of violation of any of the provisions of this act to the prosecuting attorney of the county, with a statement of the facts

[1] Provision may be made whereby compliance with this act shall constitute a condition of granting licenses to physicians, midwives, and embalmers.

and circumstances; and when any such case is reported to him by the State registrar the prosecuting attorney shall forthwith initiate and promptly follow up the necessary court proceedings against the person or corporation responsible for the alleged violation of law. And upon request of the State registrar, the attorney general shall assist in the enforcement of the provisions of this act.

NOTE. — Other sections should be added giving the date on which the act is to go into effect, if not determined by constitutional provisions of the State; providing for the financial support of the law; and repealing prior statutes inconsistent with the present act.

It is desirable that the entire bill should be reviewed by competent legal authority for the purpose of discovering whether it can be made more consistent in any respect with the general form of legislation of the State in which the bill is to be introduced, without material change or injury to the effectiveness of registration.

THE STANDARD BIRTH AND DEATH CERTIFICATES

The following are facsimile reproductions of the standard birth and death certificates. They have been reduced in size to meet the requirements of the printed page. The size of the birth certificate is $6\frac{1}{8}$ by $7\frac{7}{8}$ inches, and of the death certificate $7\frac{1}{4}$ by $8\frac{1}{2}$ inches. Copies can be obtained from the Director of the Census upon request.

UNITED STATES STANDARD CERTIFICATE OF BIRTH

8–334 *a*

V. S. No. 109

(Instructions on certain points may be printed on the back Size of certificate, $6\frac{1}{8} \times 7\frac{7}{8}$ inches.)

MARGIN RESERVED FOR BINDING

WRITE PLAINLY, WITH UNFADING INK — THIS IS A PERMANENT RECORD

N. B. — In case of more than one child at a birth, a SEPARATE RETURN must be made for each, and the number of each, in order of birth, stated

PLACE OF BIRTH

DEPARTMENT OF COMMERCE AND LABOR
BUREAU OF THE CENSUS

County of............ STANDARD CERTIFICATE OF BIRTH

Township of.......... ....

or

Village of............ Registered No...............

or

City of............... (No....., ...St.; ....Ward)

FULL NAME OF CHILD.................... { If child is not yet named, make supplemental report as directed

Sex of Child	Twin, triplet, or other?	Number in order of birth	Legitimate?	Date of birth ...., ...19. (Month) (Day) (Year)
	(To be answered only in event of plural births)			

FATHER	MOTHER
FULL NAME	FULL MAIDEN NAME
RESIDENCE	RESIDENCE
COLOR AGE AT LAST BIRTHDAY......... (Years)	COLOR AGE AT LAST BIRTHDAY........ (Years)
BIRTHPLACE	BIRTHPLACE
OCCUPATION	OCCUPATION
Number of children born to this mother, including present birth...	Number of children of this mother now living......................

CERTIFICATE OF ATTENDING PHYSICIAN OR MIDWIFE[1]

I hereby certify that I attended the birth of this child, who was ....at......M., on the date above stated.
(Born alive or Stillborn)

[1] *When there was no attending physician or midwife, then the father, householder, etc., should make this return. A stillborn child is one that neither breathes nor shows other evidence of life after birth.*

(Signature)..................

......................,
(Physician or Midwife)

Given name added from a supplemental report............................., 19

Address.....................

Filed........, 19 ....
Registrar

11—385 H

....
Registrar

MARGIN RESERVED FOR BINDING

This supplemental report is to be pasted beneath the original

8–442

V. S. No. 108

SUPPLEMENTAL REPORT OF BIRTH

(STATE)

(This return should preferably be made by the person who made the original)

Registered Number [1] ....

Place of birth [1] ....*No*............................ *St.*

(Registration district)

SEX OF CHILD [1]	Twin,[1] triplet, or other? and	Number [1] in order of birth
DATE OF BIRTH [1] ....190.. (Month) (Day) (Year)		
FULL [1] NAME	FATHER	
FULL [1] MAIDEN NAME	MOTHER	

[1] These items to be entered by the Registrar before giving out this form.

I HEREBY CERTIFY that the child described herein has been named:

....

(Give name in full) (Surname)

(Signature)...................

....

(Physician or midwife)

11–3

8-209d

V.S. No. 98

MARGIN RESERVED FOR BINDING

N. B.—WRITE PLAINLY, WITH UNFADING INK—THIS IS A PERMANENT RECORD. Every item of information should be carefully supplied. AGE should be stated EXACTLY. PHYSICIANS should state CAUSE OF DEATH in plain terms, so that it may be properly classified. Exact statement of OCCUPATION is very important. See instructions on back of certificate.

DEPARTMENT OF COMMERCE
BUREAU OF THE CENSUS

STANDARD CERTIFICATE OF DEATH

1 PLACE OF DEATH

County........................State..............Registered No.....

Township...................or Village..............................or

City................No....,.........................St.,......Ward

(If death occurred in a hospital or institution, give its NAME instead of street and number.)

2 FULL NAME..

(a) Residence. No..............St.,....Ward. ....
(Usual place of abode.) (If nonresident give city or town and State.)

Length of residence in city or town where death occurred. yrs. mos. ds. How long in U. S., if of foreign birth? yrs. mos. ds.

PERSONAL AND STATISTICAL PARTICULARS	MEDICAL CERTIFICATE OF DEATH
3 SEX | 4 COLOR OR RACE | 5 Single, Married, Widowed, or Divorced (*Write* the word)	16 DATE OF DEATH (month, day, and year) 19
5a If married, widowed, or divorced HUSBAND of (or) WIFE of	17 I HEREBY CERTIFY, That I attended deceased from ...., 19...., to........., 19.. that I last saw h...alive on....., 19.. and that death occurred, on the date stated above, at...........m. The CAUSE OF DEATH* was as follows:
6 DATE OF BIRTH (month, day, and year)	....
7 AGE Yrs. | Mos. | Ds. | If LESS than 1 day,... hrs. *or*......min.?	....
8 OCCUPATION OF DECEASED (a) Trade, profession, or particular kind of work.......... (b) General nature of industry, business, or establishment in which employed (or employer).... (c) Name of employer..........	......(duration)...yrs....mos...ds. CONTRIBUTORY..................... (Secondary) ....(duration)...yrs...mos...ds.
9 BIRTHPLACE (city or town).......... (State or country)	18 Where was disease contracted if not at place of death?.......... Did an operation precede death? ....Date of.........
PARENTS: 10 NAME OF FATHER	Was there an autopsy?........... What test confirmed diagnosis?....
PARENTS: 11 BIRTHPLACE OF FATHER (city or town)..................... (State or country)	(Signed)..................., M.D. , 19 (Address)
PARENTS: 12 MAIDEN NAME OF MOTHER	* State the Disease Causing Death, or in deaths from Violent Causes, state (1) Means and Nature of Injury; and (2) whether Accidental, Suicidal, or Homicidal. (See reverse side for additional space.)
PARENTS: 13 BIRTHPLACE OF MOTHER (city or town)..................... (State or country)	19 PLACE OF BURIAL, CREMATION, OR REMOVAL | DATE OF BURIAL 19
14 Informant....................... (Address)	
15 Filed........, 19 .... Registrar	20 UNDERTAKER | ADDRESS

TABLE VI.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
1	0432	0475	0518	0561	0604	0647	0689	0732	0775	0817
2	0860	0903	0945	0988	1030	1072	1115	1157	1199	1242
3	1284	1326	1368	1410	1452	1494	1536	1578	1620	1662
4	1703	1745	1787	1828	1870	1912	1953	1995	2036	2078
5	2119	2160	2202	2243	2284	2325	2366	2407	2449	2490
6	2531	2572	2612	2653	2694	2735	2776	2816	2857	2898
7	2938	2979	3019	3060	3100	3141	3181	3222	3262	3302
8	3342	3383	3423	3463	3503	3543	3583	3623	3663	3703
9	3743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
1	4532	4571	4610	4650	4689	4727	4766	4805	4844	4883
2	4922	4961	4999	5038	5077	5115	5154	5192	5231	5269
3	5308	5346	5385	5423	5461	5500	5538	5576	5614	5652
4	5690	5729	5767	5805	5843	5881	5918	5956	5994	6032
5	6070	6108	6145	6183	6221	6258	6296	6333	6371	6408
6	6446	6483	6521	6558	6595	6633	6670	6707	6744	6781
7	6819	6856	6893	6930	6967	7004	7041	7078	7115	7151
8	7188	7225	7262	7298	7335	7372	7408	7445	7482	7518
9	7555	7591	7628	7664	7700	7737	7773	7809	7846	7882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
1	8279	8314	8350	8386	8422	8458	8493	8529	8565	8600
2	8636	8672	8707	8743	8778	8814	8849	8884	8920	8955
3	8991	9026	9061	9096	9132	9167	9202	9237	9272	9307
4	9342	9377	9412	9447	9482	9517	9552	9587	9621	9656
5	9691	9726	9760	9795	9830	9864	9899	9934	9968	10003
6	10037	10072	10106	10140	10175	10209	10243	10278	10312	0346
7	0380	0415	0449	0483	0517	0551	0585	0619	0653	0687
8	0721	0755	0789	0823	0857	0890	0924	0958	0992	1025
9	1059	1093	1126	1160	1193	1227	1261	1294	1327	1361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
1	1727	1760	1793	1826	1860	1893	1926	1959	1992	2024
2	2057	2090	2123	2156	2189	2222	2254	2287	2320	2352
3	2385	2418	2450	2483	2516	2548	2581	2613	2646	2678
4	2710	2743	2775	2808	2840	2872	2905	2937	2969	3001
5	3033	3066	3098	3130	3162	3194	3226	3258	3290	3322
6	3354	3386	3418	3450	3481	3513	3545	3577	3609	3640
7	3672	3704	3735	3767	3799	3830	3862	3893	3925	3956
8	3988	4019	4051	4082	4114	4145	4176	4208	4239	4270
9	4301	4333	4364	4395	4426	4457	4489	4520	4551	4582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
1	4922	4953	4983	5014	5045	5076	5106	5137	5168	5198
2	5229	5259	5290	5320	5351	5381	5412	5442	5473	5503
3	5534	5564	5594	5625	5655	5685	5715	5746	5776	5806
4	5836	5866	5897	5927	5957	5987	6017	6047	6077	6107
5	6137	6167	6197	6227	6256	6286	6316	6346	6376	6406
6	6435	6465	6495	6524	6554	6584	6613	6643	6673	6702
7	6732	6761	6791	6820	6850	6879	6909	6938	6967	6997
8	7026	7056	7085	7114	7143	7173	7202	7231	7260	7289
9	7319	7348	7377	7406	7435	7464	7493	7522	7551	7580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869

N	0	1	2	3	4	5	6	7	8	9
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
1	7898	7926	7955	7984	8013	8041	8070	8099	8127	8156
2	8184	8213	8241	8270	8298	8327	8355	8384	8412	8441
3	8469	8498	8526	8554	8583	8611	8639	8667	8696	8724
4	8752	8780	8808	8837	8865	8893	8921	8949	8977	9005
5	9033	9061	9089	9117	9145	9173	9201	9229	9257	9285
6	9312	9340	9368	9396	9424	9451	9479	9507	9535	9562
7	9590	9618	9645	9673	9700	9728	9756	9783	9811	9838
8	9866	9893	9921	9948	9976	20003	20030	20058	20085	20112
9	20140	20167	20194	20222	20249	0276	0303	0330	0358	0385
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656
1	0683	0710	0737	0763	0790	0817	0844	0871	0898	0925
2	0952	0978	1005	1032	1059	1085	1112	1139	1165	1192
3	1219	1245	1272	1299	1325	1352	1378	1405	1431	1458
4	1484	1511	1537	1564	1590	1617	1643	1669	1696	1722
5	1748	1775	1801	1827	1854	1880	1906	1932	1958	1985
6	2011	2037	2063	2089	2115	2141	2167	2194	2220	2246
7	2272	2298	2324	2350	2376	2401	2427	2453	2479	2505
8	2531	2557	2583	2608	2634	2660	2686	2712	2737	2763
9	2789	2814	2840	2866	2891	2917	2943	2968	2994	3019
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
1	3300	3325	3350	3376	3401	3426	3452	3477	3502	3528
2	3553	3578	3603	3629	3654	3679	3704	3729	3754	3779
3	3805	3830	3855	3880	3905	3930	3955	3980	4005	4030
4	4055	4080	4105	4130	4155	4180	4204	4229	4254	4279
5	4304	4329	4353	4378	4403	4428	4452	4477	4502	4527
6	4551	4576	4601	4625	4650	4674	4699	4724	4748	4773
7	4797	4822	4846	4871	4895	4920	4944	4969	4993	5018
8	5042	5066	5091	5115	5139	5164	5188	5212	5237	5261
9	5285	5310	5334	5358	5382	5406	5431	5455	5479	5503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
1	5768	5792	5816	5840	5864	5888	5912	5935	5959	5983
2	6007	6031	6055	6079	6102	6126	6150	6174	6198	6221
3	6245	6269	6293	6316	6340	6364	6387	6411	6435	6458
4	6482	6505	6529	6553	6576	6600	6623	6647	6670	6694
5	6717	6741	6764	6788	6811	6834	6858	6881	6905	6928
6	6951	6975	6998	7021	7045	7068	7091	7114	7138	7161
7	7184	7207	7231	7254	7277	7300	7323	7346	7370	7393
8	7416	7439	7462	7485	7508	7531	7554	7577	7600	7623
9	7646	7669	7692	7715	7738	7761	7784	7807	7830	7852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
1	8103	8126	8149	8171	8194	8217	8240	8262	8285	8307
2	8330	8353	8375	8398	8421	8443	8466	8488	8511	8533
3	8556	8578	8601	8623	8646	8668	8691	8713	8735	8758
4	8780	8803	8825	8847	8870	8892	8914	8937	8959	8981
5	9003	9026	9048	9070	9092	9115	9137	9159	9181	9203
6	9226	9248	9270	9292	9314	9336	9358	9380	9403	9425
7	9447	9469	9491	9513	9535	9557	9579	9601	9623	9645
8	9667	9688	9710	9732	9754	9776	9798	9820	9842	9863
9	9885	9907	9929	9951	9973	9994	30016	30038	30060	30081
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298

N	0	1	2	3	4	5	6	7	8	9
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298
1	0320	0341	0363	0384	0406	0428	0449	0471	0492	0514
2	0535	0557	0578	0600	0621	0643	0664	0685	0707	0728
3	0750	0771	0792	0814	0835	0856	0878	0899	0920	0942
4	0963	0984	1006	1027	1048	1069	1091	1112	1133	1154
5	1175	1197	1218	1239	1260	1281	1302	1323	1345	1366
6	1387	1408	1429	1450	1471	1492	1513	1534	1555	1576
7	1597	1618	1639	1660	1681	1702	1723	1744	1765	1785
8	1806	1827	1848	1869	1890	1911	1931	1952	1973	1994
9	2015	2035	2056	2077	2098	2118	2139	2160	2181	2201
210	32222	32243	32263	32284	32305	32325	32346	32366	32387	32408
1	2428	2449	2469	2490	2510	2531	2552	2572	2593	2613
2	2634	2654	2675	2695	2715	2736	2756	2777	2797	2818
3	2838	2858	2879	2899	2919	2940	2960	2980	3001	3021
4	3041	3062	3082	3102	3122	3143	3163	3183	3203	3224
5	3244	3264	3284	3304	3325	3345	3365	3385	3405	3425
6	3445	3465	3486	3506	3526	3546	3566	3586	3606	3626
7	3646	3666	3686	3706	3726	3746	3766	3786	3806	3826
8	3846	3866	3885	3905	3925	3945	3965	3985	4005	4025
9	4044	4064	4084	4104	4124	4143	4163	4183	4203	4223
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420
1	4439	4459	4479	4498	4518	4537	4557	4577	4596	4616
2	4635	4655	4674	4694	4713	4733	4753	4772	4792	4811
3	4830	4850	4869	4889	4908	4928	4947	4967	4986	5005
4	5025	5044	5064	5083	5102	5122	5141	5160	5180	5199
5	5218	5238	5257	5276	5295	5315	5334	5353	5372	5392
6	5411	5430	5449	5468	5488	5507	5526	5545	5564	5583
7	5603	5622	5641	5660	5679	5698	5717	5736	5755	5774
8	5793	5813	5832	5851	5870	5889	5908	5927	5946	5965
9	5984	6003	6021	6040	6059	6078	6097	6116	6135	6154
230	36173	36192	36211	36229	36248	36267	36286	36305	36324	36342
1	6361	6380	6399	6418	6436	6455	6474	6493	6511	6530
2	6549	6568	6586	6605	6624	6642	6661	6680	6698	6717
3	6736	6754	6773	6791	6810	6829	6847	6866	6884	6903
4	6922	6940	6959	6977	6996	7014	7033	7051	7070	7088
5	7107	7125	7144	7162	7181	7199	7218	7236	7254	7273
6	7291	7310	7328	7346	7365	7383	7401	7420	7438	7457
7	7475	7493	7511	7530	7548	7566	7585	7603	7621	7639
8	7658	7676	7694	7712	7731	7749	7767	7785	7803	7822
9	7840	7858	7876	7894	7912	7931	7949	7967	7985	8003
240	38021	38039	38057	38075	38093	38112	38130	38148	38166	38184
1	8202	8220	8238	8256	8274	8292	8310	8328	8346	8364
2	8382	8399	8417	8435	8453	8471	8489	8507	8525	8543
3	8561	8578	8596	8614	8632	8650	8668	8686	8703	8721
4	8739	8757	8775	8792	8810	8828	8846	8863	8881	8899
5	8917	8934	8952	8970	8987	9005	9023	9041	9058	9076
6	9094	9111	9129	9146	9164	9182	9199	9217	9235	9252
7	9270	9287	9305	9322	9340	9358	9375	9393	9410	9428
8	9445	9463	9480	9498	9515	9533	9550	9568	9585	9602
9	9620	9637	9655	9672	9690	9707	9724	9742	9759	9777
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950

N	0	1	2	3	4	5	6	7	8	9
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950
1	9967	9985	40002	40019	40037	40054	40071	40088	40106	40123
2	40140	40157	0175	0192	0209	0226	0243	0261	0278	0295
3	0312	0329	0346	0364	0381	0398	0415	0432	0449	0466
4	0483	0500	0518	0535	0552	0569	0586	0603	0620	0637
5	0654	0671	0688	0705	0722	0739	0756	0773	0790	0807
6	0824	0841	0858	0875	0892	0909	0926	0943	0960	0976
7	0993	1010	1027	1044	1061	1078	1095	1111	1128	1145
8	1162	1179	1196	1212	1229	1246	1263	1280	1296	1313
9	1330	1347	1363	1380	1397	1414	1430	1447	1464	1481
260	41497	41514	41531	41547	41564	41581	41597	41614	41631	41647
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9	8632	8637	8641	8646	8650	8655	8659	8664	8668	8673
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717
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